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COLLECTION OF PROBLEMS AND EXERCISES IN MATHEMATICS

MIR PUBLISHERS
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Для
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Preface to the English Edition

This collection is in four chapters: 1. Arithmetic; 2. Algebra; Elementary Functions and Fundamentals of Analysis; 3. Trigonometrical Functions; and 4. Geometry.

This edition was written to watch the mathematics syllabus for high schools and the preparatory faculties of institutions for higher education. The problems are intended for use in the class and to be solved by the student at home. The exercises and problems are presented in order of increasing difficulty starting from the level a student should have reached at school.

At present, mathematics is used in every sphere of human activity, and therefore we have included additional problems which reflect the relation between mathematics and practice and should help the student solve problems in applied mathematics.

Most exercises and problems are supplied with answers and the basic formulas are given at the end of the book. This study aid is also intended for those entering college.

We wish to express our deep appreciation to colleagues who helped prepare the text. All criticisms and suggestions are welcome.

The Authors

Mathematical Symbols

$\Rightarrow$	The sign of logical consequence
$\Leftrightarrow$	The sign of logical equivalence
$\in$	The membership symbol
$\subset$	The inclusion sign
$D \cup E$	The union (or join or sum) of the sets D and E
$D \cap E$	The intersection of the sets D and E
$\mathbf{N}$	The set of all natural numbers
$\mathbf{Z}$	The set of all whole numbers
$\mathbf{R}$	The set of all real numbers
$\mathbf{R}_+$	The set of all positive real numbers
$\mathbf{R}_-$	The set of all negative real numbers
$\mathbf{R}^2$	The number plane
$[a, b]$	A closed interval
(a, b)	An open interval
$[a, b)$; $(a, b]$	A half-open interval
$[AB]$	A line segment
(CD)	A straight line passing through the points C and D
$\mid AB \mid$	The length of a line segment
$[x]$	The integral part of the number x
$\{x\}$	The fractional part of the number x
$\mid x \mid$	Absolute value (modulus) of the number x
$P_n = n!$	Factorial (or n factorial)—the number of permutations
C_n^m	The number of combinations of n elements taken m at a time
A_n^m	The number of arrangements of n elements taken m at a time
$D(f)$	The domain (of definition) of a function
$E(f)$	The range (of values) of a function
f^{-1}	The inverse of a function
Δx	An increment of x
$\Delta f(x_0)$	An increment of a function at x_0
$\lim_{x \to a} f(x)$	The limit of a function $f(x)$ as x approaches a
$f'(x_0)$	The derivative of a function at the point x_0

$\angle AOB$	The angle AOB
$\widehat{AOB}$	The size of the angle AOB
$R\alpha_0$	Rotation of a plane, a ray, a vector through an angle of α
$\int f(x)\,dx$	The indefinite integral or antiderivative of f with respect to x
$\int_a^b f(x)\,dx$	The definite integral of f between the limits a and b

CHAPTER 1

Arithmetic

§ 1. Whole Numbers. Operations on Whole Numbers

1.1. Find the sum:

(1) $27 + 39 + 13 + 11$; (2) $18 + 39 + 27 + 12 + 23$;
(3) $38 + 94 + 12 + 16$; (4) $116 + 37 + 14 + 43$;
(5) $29 + 87 + 21 + 51 + 13$; (6) $155 + 46 + 5 + 54 + 30$;
(7) $198 + 36 + 102 + 64 + 75$;
(8) $122 + 193 + 27 + 78 + 15$;
(9) $527 + 108 + 11 + 73 + 39 + 92$;
(10) $401 + 223 + 49 + 77 + 54$;
(11) $217 + 125 + 49 + 111 + 83 + 75$;
(12) $326 + 157 + 74 + 43 + 229$;
(13) $303 + 144 + 97 + 56 + 137$;
(14) $773 + 111 + 27 + 89 + 201$;
(15) $845 + 136 + 155 + 164 + 208$.

1.2. Find the difference:

(1) $357 - (257 + 89)$; (2) $(357 + 476) - 257$;
(3) $234 - (134 - 35)$; (4) $(826 - 438) - 126$;
(5) $(525 + 169) - (325 + 69)$;
(6) $(733 + 328) - (533 + 288)$;
(7) $(529 + 173 + 126) - (76 + 73 + 429)$;
(8) $(249 + 576 + 138) - (376 + 249 + 38)$;
(9) $(425 - 252 + 133) - (48 + 33 + 225)$.

1.3. Find x:

(1) $x + 394 = 512$; (2) $5894 + x = 6282$;
(3) $2401 + x = 7202$; (4) $4206 + x = 5208$;
(5) $x + 1244 = 3245$; (6) $x - 786\,957 = 446\,789$;

(7) $15\,036 - x = 7204$; (8) $9\,987\,768 - x = 25\,609$;
(9) $100\,000 - x = 25\,609$; (10) $75\,833 - (31\,200 + x) = 999$;
(11) $(x - 7756) - 12\,000 = 4896$;
(12) $4284 - (x - 378) = 100$;
(13) $(5376 - x) - 3877 = 904$;
(14) $(66\,625 - x) + 655 = 10\,006$;
(15) $(x - 12\,500) + 26\,448 = 92\,225$.

1.4. Find the product:

(1) $4 \cdot 8 \cdot 3 \cdot 25 \cdot 125$; (2) $2 \cdot 14 \cdot 25 \cdot 5$; (3) $17 \cdot 25 \cdot 4 \cdot 30$;
(4) $45 \cdot 8 \cdot 4 \cdot 25 \cdot 2$; (5) $36 \cdot 25 \cdot 44 \cdot 0$; (6) $0 \cdot 243 \cdot 11 \cdot 63$;
(7) $(40 \cdot 7) \cdot 3 \cdot 25$; (8) $8 \cdot (125 \cdot 7 \cdot 3) \cdot 4$; (9) $298 \cdot 4 \cdot 50 \cdot 20$;
(10) $498 \cdot 4 \cdot 125 \cdot 8$; (11) $44 \cdot 75 \cdot 16 \cdot 125$; (12) $28 \cdot 50 \cdot 250 \cdot 16$.

1.5. (Orally). Replace addition by multiplication:

(1) $4 + 4 + 4 + 4 + 4$; (2) $7 + 7 + 7 + 7$;
(3) $3 + 3 + 3 + 3 + 3$;
(4) $5 + 5 + 5 + 5 + 5 + 5 + 5$;
(5) $12 + 12 + 12 + 12 + 12$.

1.6. (Orally). Find the result:

(1) $484 \div 4$; (2) $483 \div 7$; (3) $960 \div (4 \cdot 6 \cdot 8)$;
(4) $960 \div 30$; (5) $0 \div 25$; (6) $(428 \cdot 75) \div 25$;
(7) $(845 \cdot 48) \div 16$; (8) $(360 \cdot 215) \div 8$);
(9) $(84 \cdot 35 \cdot 18) \div 9$.

1.7. Perform the indicated operations:

(1) $78 + 23\,(81 - 69)$; (2) $(78 + 23)\,(81 - 69)$;
(3) $106 + 22\,(92 - 69)$; (4) $(78 + 32) \div (81 - 70)$;
(5) $5871 \div 103 + (247 - 82) \div 5 - 1$;
(6) $840 + 357 \cdot 527 + 481$; (7) $(840 + 357) \cdot 527 + 481$;
(8) $78 \cdot 507 - 19 \cdot 97 + 927 \div (2081 - 1978)$;
(9) $79 \cdot 68 + (1400 - (777 - 687) \cdot 5) \div 19$;
(10) $805\,001 + (908 \cdot 307 - 65 \cdot (403 - 289)) \div 3$;
(11) $25 \cdot (28 \cdot 105 + 7236 \div 18) - (4217 - 1823) \div 6 \cdot 25$.

1.8. Find x:

(1) $x + (114 - 25) = 729$; (2) $2x - (67 + 34) = 121$;
(3) $(6400 + 2600) - 3x = 1200$;
(4) $(x + 2598) \div 87 = 134$;
(5) $(2x - 5385) \div 107 = 9$; (6) $(2088 - 1188) \div x = 45$;
(7) $(35\,400 - 24\,800) \div 2x = 53$;
(8) $(34\,500 - 291\,400) \div x = 7$.

1.9. (a) (Orally). Mention all the prime numbers from 1 to 50; (b) mention all the prime numbers from 51 to 100; (c) mention two composite numbers which will be mutually prime numbers; (d) mention two numbers mutually prime with the number: (1) 8; (2) 20; (3) 84; (4) 24.

1.10. Write all the even numbers: (a) (1) from 60 to 75; (2) from 200 to 215; (b) mention the least and the greatest even numbers; (c) mention all the odd numbers: (1) from 74 to 78; (2) from 450 to 466; (d) mention the least and the greatest odd numbers.

§ 2. Tests for Divisibility of Numbers

1.11. (a) Write out the numbers divisible by 2; 5; 10; 25: 80; 105; 115; 126; 155; 175; 208; 235; 240; 249; 255; 280; 295; 302; 315; 340; 355; 375; 400; 445; 448; 482; 500; 560; 575; 605; 612; 625; 648; 720; 1000; 1400; 20 288; 62 125; 70 125.

(b) Write out the numbers divisible by 3; 9: 78; 87; 93; 96; 99; 123; 135; 183; 225; 288; 570; 576; 600; 981; 2400; 4233; 8136; 54 090.

1.12. Write out the products divisible by 2; 3; 5; 9:

(a) (1) $24 \cdot 37 \cdot 53$; (2) $60 \cdot 25 \cdot 17$; (3) $61 \cdot 44 \cdot 70$;
(4) $37 \cdot 121 \cdot 57$; (5) $123 \cdot 207 \cdot 41$; (6) $43 \cdot 50 \cdot 11$;

(b) (1) $6 \cdot 23 \cdot 75$; (2) $55 \cdot 32 \cdot 27$; (3) $64 \cdot 128 \cdot 32$;
(4) $177 \cdot 22 \cdot 13$; (5) $235 \cdot 75 \cdot 17$; (6) $175 \cdot 16 \cdot 47$;
(7) $11 \cdot 29 \cdot 43$; (8) $117 \cdot 31 \cdot 19$; (9) $711 \cdot 47 \cdot 61$.

1.13. Write out the sums divisible by 2; 3; 9:

(1) $117 + 72 + 711$; (2) $123 + 57 + 111$;
(3) $225 + 207 + 801$; (4) $250 + 75 + 155$;

(5) 35 + 135 + 85; (6) 405 + 105 + 40;
(7) 65 + 215 + 720; (8) 315 + 400 + 60;
(9) 45 + 306 + 504; (10) 33 + 237 + 123;
(11) 219 + 411 + 87; (12) 2700 + 1836 + 729;
(13) 7200 + 6300; (14) 3636 + 4800;
(15) 6075 + 123 + 110.

§ 3. Factorizing Numbers into Prime Factors. Greatest Common Divisor (G.C.D.). Least Common Multiple (L.C.M.)

1.14. Factorize the given numbers into prime factors: 48; 52; 63; 72; 78; 87; 90; 92; 100; 105; 111; 117; 125; 245; 290; 300; 306; 312; 375; 400; 425; 426; 432; 500; 630; 720; 1080; 1155; 10 000; 11 700.

1.15. Find all the prime and composite divisors of the following numbers: 12; 16; 18; 28; 48; 56; 68; 87; 96; 102; 111; 117; 189; 196; 205; 390; 510; 828; 2280.

1.16. (a) Write three numbers simultaneously multiple of the numbers: 3; 5; 12.

(b) Write three numbers simultaneously multiple of the numbers: 15; 20; 75; 80.

(c) Write several common multiples of the given numbers: (1) 7; 14; 98; 112; (2) 18; 36; 54; 108; (3) 15; 25; 30; 50.

1.17. (a) Find a least common multiple (L.C.M.) of each of the following groups of numbers: (39, 169); (51, 170); (58, 145); (56, 70, 126); (54, 90, 162); (84, 98, 140); (52, 91, 182); (35, 105, 280); (38, 3364).

(b) Find a greatest common divisor (G.C.D.) of each of the following groups of numbers: (56, 104); (63, 140); (85, 102, 68, 34); (936, 1128); (375, 360, 90); (174, 145); (225, 720, 1080); (240, 960, 2160).

§ 4. Common Fractions

1.18. (a) If the numerator of a proper fraction is 4, then what may the denominator of this fraction be equal to? Write three such fractions.

(b) If the numerator of an improper fraction is 5, then what may the denominator of this fraction be equal to? Write these fractions.

(c) If the denominator of a proper fraction is 3, then what may the numerator of this fraction be equal to? Write these fractions.

(d) Write out separately proper and improper fractions: (1) 3/5; 5/7; 7/6; 6/5; 4/9; 5/3; 8/8; 8/11; 12/10; 11/11; 13/5; 3/4; (2) 15/16; 16/9; 25/24; 24/23; 31/35; 36/17; 19/19; 54/59; 47/46; 89/85; 99/90; (3) 101/101; 125/126; 204/201; 305/302; 500/501. Read these fractions.

1.19. (1) Write five improper fractions whose numerator is by 2 greater than the denominator. Read these fractions.
(2) Write four proper fractions whose numerator is by 7 less than the denominator. Read these fractions.
(3) (Orally). What fraction will be obtained if unity is divided into five equal parts, and one such part is taken four times? Read this fraction.
(4) (Orally). What fraction will be obtained if unity is divided into four equal parts, and one such part is taken five times? Read this fraction.

1.20. (Orally).
(1) What part of the meter is: 1 cm; 5 cm; 26 cm; 3 dm; 5 dm; 8 dm; 9 dm; 24 cm; 36 cm; 11 dm; 65 cm; 75 cm; 25 dm?
(2) What part of the kilogram is represented by: 3 g; 5 g; 17 g; 309 g; 500 g; 605 g; 720 g; 905 g; 880 g; 725 g; 250 g; 400 g; 125 g?
(3) Write the following quotients in the form of fractions: $3 \div 5$; $7 \div 10$; $5 \div 8$; $13 \div 15$; $5 \div 18$; $43 \div 47$; $21 \div 20$; $52 \div 41$; $120 \div 17$. Read these fractions.

1.21. Write the following numbers in the form of improper fractions:

$2\frac{1}{2}$; $1\frac{5}{7}$; $3\frac{2}{3}$; $2\frac{15}{16}$; $3\frac{5}{8}$; $5\frac{2}{7}$; $10\frac{3}{7}$; $11\frac{2}{3}$; $15\frac{1}{2}$; $20\frac{1}{3}$; $35\frac{5}{12}$; $101\frac{3}{7}$; $407\frac{5}{11}$; $502\frac{7}{15}$; $607\frac{1}{20}$; $720\frac{3}{50}$; $825\frac{3}{24}$; $800\frac{4}{125}$; $900\frac{7}{36}$.

1.22. Write the following fractions in the form of mixed numbers: 11/8; 12/5; 15/7; 50/9; 86/17; 107/35; 125/312; 145/36; 206/3; 643/40; 1201/55; 3256/421; 16 507/2000; 20 236/150.

§ 5. The Fundamental Property of the Fraction

1.23. (1) Double each of the following numbers: 1/4; 2/3; 3/8; 11; 6/13; 1/15; 15/16; 7/12; 35/42; 14/18; 21/36; 91/108. Read these numbers.

(2) Reduce by a factor of 4 each of the following numbers: 1/4; 8/11; 12/13; 23/27; 16/51; 49/20; 64/125; 7/1000; $2\frac{2}{5}$; $4\frac{4}{7}$; $7\frac{1}{5}$; $24\frac{4}{7}$. Read these numbers.

(3) Increase by a factor of 3 each of the following fractions: 7/8; 5/23; 21/29; 7/30; 17/36; 41/16; 49/24; 56/27; 45/17; 29/39; 17/135; 29/144; 53/225; 61/240; 77/324; 83/540; 92/315.

(4) Reduce by a factor of 5 each of the following fractions: 8/13; 17/19; 21/22; 25/7; 24/11; 22/43; 36/17; 27/26; 45/23; 95/103; 175/131; 245/321; 805/921; 165/730; 295/691; 415/711.

1.24. (1) The denominator of a fraction is reduced by a factor of 3. How should the numerator be changed to keep the fraction unchanged?

(2) The numerator of a fraction is doubled. How should the denominator be changed to retain the fraction unchanged?

(3) Write three equal fractions having different denominators.

(4) Write three equal fractions having different numerators.

1.25. (1) Write the numbers equal to one-third of each of the following fractions: 1/3; 2/5; 1/7; 4/9; 11/12; 7/8; 15/22; 9/4; 61/9.

(2) Write the numbers five times as large as each of the following numbers: $\frac{7}{5}$; $\frac{4}{15}$; $\frac{22}{25}$; $\frac{31}{45}$; $\frac{57}{60}$; $1\frac{1}{5}$; $3\frac{3}{5}$; $4\frac{7}{20}$; $5\frac{7}{65}$.

(3) How will the fraction change if:

(a) its numerator is increased 2 times? 5 times? 16 times?

(b) its numerator is reduced by a factor of 3? 12? 20?

(c) its denominator is reduced by a factor of 3? 10? 30?

(d) its denominator is reduced by a factor of 5? 7? 25?

(e) how will the following fractions change if the numerator of each of them is replaced by unity: 4/7; 5/11; 11/15; 22/23; 30/31?

(f) how will the following fractions change if the denominator of each of them is replaced by unity: 4/7; 5/11; 11/15?

1.26. Reduce the given fractions to lower terms and single out a whole number where it is possible:

(a) 2/4; 4/10; 6/9; 3/15; 8/18; 20/24; 28/40; 150/200; 125/375; 500/750; 450/480;

(b) 27/999; 66/440; 117/1300; 300/525; 1680/2640; 1250/1625; 840/950; 264/312; 128/192; 501/1002;

(c) $\frac{17\cdot3\cdot9}{6\cdot51\cdot15}$; $\frac{19\cdot8\cdot3\cdot11}{22\cdot4\cdot20\cdot19}$; $\frac{15\cdot13\cdot6}{6\cdot9\cdot5\cdot2\cdot6}$; $\frac{49\cdot77\cdot56\cdot100}{33\cdot70\cdot42\cdot28}$; $\frac{8\cdot14\cdot50\cdot27}{18\cdot6\cdot8}$;

(d) $\frac{231\cdot177\cdot63\cdot25}{77\cdot59\cdot100\cdot3}$; $\frac{64\cdot22\cdot49\cdot15}{66\cdot42\cdot16\cdot56}$; $\frac{75\cdot102\cdot130\cdot108}{162\cdot78\cdot68\cdot114}$; $\frac{210\cdot85\cdot300}{147\cdot1000\cdot65}$.

§ 6. Reducing Several Fractions to a Least Common Denominator

1.27. Reduce the given fractions to a least common fraction:

(a) (1) 1/12 and 1/7; (2) 7/15 and 3/4; (3) 5/11 and 2/13; (4) 4/15 and 7/11; (5) 5/96 and 7/24; (6) 5/57 and 1/114; (7) 1/74 and 2/111;

(b) (1) 7/10; 2/3, and 10/11; (2) 3/4; 5/9, and 3/7; (3) 4/5; 5/7, and 7/9; (4) 4/57; 5/19, and 5/38; (5) 3/38; 7/19, and 2/57;

(c) (1) 3/20; 1/5, and 7/130; (2) 3/8; 19/20, and 8/15; (3) 11/50; 7/10, and 27/100; (4) 2/105 and 11/35; (5) 5/43; 7/172, and 4/215;

(d) (1) $1\frac{5}{36}$; $2\frac{8}{9}$, and $5\frac{7}{144}$; (2) $4\frac{17}{65}$; $3\frac{3}{10}$, and $5\frac{1}{130}$; (3) $2\frac{7}{45}$; $1\frac{13}{90}$, and $\frac{7}{900}$.

1.28. (a) Compare two factors: (1) 2/3 and 160/240; (2) 3/4 and 72/96; (3) 5/6 and 30/42; (4) 1/2 and 20/35; (5) 7/12 and 25/36; (6) 2/7 and 5/21; (7) 11/5 and 7/6.

(b) Write the given fractions in ascending order: (1) 3/4; 5/6; 11/12; 8/9; 7/8; 4/81; 17/36; 19/27; 23/162; 11/72; 13/324; 7/18.

(2) 5/6; 7/8; 16/17; 2/3; 3/4; 11/68; 7/34; 23/102; 29/51; 58/153; 37/136.

(c) Write the given numbers in descending order:

$1\frac{2}{7}$; $\frac{5}{42}$; $\frac{679}{343}$; $3\frac{1}{7}$; $\frac{15}{98}$; $\frac{680}{294}$; $3\frac{1}{21}$; $1\frac{5}{84}$; $12\frac{1}{49}$; $12\frac{153}{343}$.

1.29. (a) Which of the given numbers is greater:

(1) 7/10; 11/15; (2) 2/3; 1/2; (3) 7/9; 11/13; 17/39; (4) $3\frac{7}{8}$; $3\frac{11}{12}$; $3\frac{14}{15}$; (5) $11\frac{7}{8}$; $11\frac{89}{120}$; $11\frac{121}{240}$; (6) $35\frac{4}{5}$; $35\frac{19}{25}$; $35\frac{71}{75}$; $35\frac{151}{150}$?

(b) Which of the given numbers is less:

1) 5/6; 8/9; 13/18; (2) 5/8; 7/12; 9/15; (3) 5/6; 1/8; 7/24; 11/15; (4) 6/7; 11/21; 15/49; 121/243; (5) $8\frac{7}{13}$; $8\frac{11}{26}$; $8\frac{37}{91}$; $8\frac{153}{182}$; (6) $25\frac{13}{20}$; $25\frac{17}{75}$; $25\frac{91}{150}$; $25\frac{31}{50}$?

§ 7. Addition and Subtraction of Fractions

1.30. Find the sum:

(a) (1) 1/11 + 2/11 + 3/11 + 5/11; (2) 1/3 + 7/9 + 5/6 + 17/18 + 13/36; (3) 7/15 + 14/45 + 2/9 + 5/18 + 1/3; (4) $5\frac{13}{42} + 2\frac{13}{28} + 5/24$;

(b) (1) 2/17 + 10/17 + 3/17 + 12/17; (2) 59/180 + 14/15 + 8/9 + 23/30 + 9/20;

(c) (1) $1\frac{2}{7}+3\frac{1}{7}+5\frac{4}{7}+\frac{14}{7}$; (2) $8\frac{19}{60}+10\frac{17}{40}+15\frac{1}{24}$; (3) $3\frac{7}{12}+2\frac{11}{18}+2\frac{1}{24}$; (4) $3\frac{1}{4}+2\frac{11}{15}+1\frac{3}{5}+4\frac{7}{12}+\frac{1}{3}$;

(d) (1) $2\frac{2}{9}+1\frac{5}{9}+3\frac{4}{9}$; (2) $4\frac{3}{4}+1\frac{7}{9}+2\frac{5}{12}+5\frac{2}{9}+\frac{7}{12}$.

1.31. Find the difference:

(1) $3/4 - 1/4$; (2) $2 - 15/16$; (3) $15 - 1\frac{31}{36}$;

(4) $1\frac{5}{18} - \frac{11}{15}$;

(5) $504\frac{11}{14} - 385\frac{15}{28}$; (6) $\left(17\frac{3}{4} + 16\frac{5}{6}\right) - \left(16\frac{5}{9} - 15\frac{7}{12}\right)$;

(7) $50\frac{11}{14} - 35\frac{15}{224}$; (8) $17 - 12\frac{4}{23}$; (9) $29\frac{5}{13} - 11\frac{12}{143}$;

(10) $125\frac{23}{27} - 123\frac{37}{108}$; (11) $302\frac{11}{69} - 297\frac{47}{92}$;

(12) $622\frac{12}{85} - 596\frac{7}{340}$.

1.32. Verify the following equalities;

(a) (1) $3\frac{17}{24} + 2\frac{8}{15} + 1\frac{7}{8} = 4\frac{3}{4} + 3\frac{11}{30}$;

(2) $8\frac{19}{60} + 10\frac{17}{40} + 15\frac{11}{24} = 17\frac{5}{8} + 16\frac{23}{40}$;

(3) $20\frac{19}{120} + 15\frac{9}{40} + 1\frac{91}{300} = 21\frac{47}{150} + 15\frac{28}{75}$;

(b) (1) $\left(3\frac{5}{8} + \frac{1}{4} + 2\frac{7}{12}\right) - \left(\left(4\frac{8}{15} + \frac{11}{30} + \frac{17}{45}\right) - \left(\frac{3}{4} + 12\frac{1}{6} + 10\frac{2}{15}\right)\right) = 10\frac{53}{72}$;

(2) $105 - \left(\left(12\frac{1}{2} + 28\frac{6}{7}\right) - \left(\frac{19}{21} + 34\frac{5}{21}\right)\right) - \left(103\frac{4}{21} - 72\frac{5}{18}\right) = 67\frac{55}{63}$;

(3) $56\frac{2}{21} = \left(\left(1\frac{5}{6} + 2\frac{13}{14}\right) + \left(27\frac{13}{30} - \left(15\frac{5}{12} - 12\frac{13}{20}\right)\right)\right) = 26\frac{2}{3}$.

1.33. Find x:

(a) (1) $x + \frac{3}{10} = 5\frac{7}{10}$; (2) $6\frac{11}{24} - x = 5\frac{5}{18}$; (3) $\frac{123}{144} + x = 6\frac{121}{360}$; (4) $x - \frac{11}{90} = \frac{5}{18}$; (5) $x + 4\frac{1}{6} = 5\frac{4}{45}$; (6) $1\frac{23}{30} - x = 1\frac{1}{6}$;

(b) (1) $1\frac{1}{7}-x=\frac{23}{49}$; (2) $\left(\frac{3}{5}+\frac{1}{8}\right)-x=\frac{1}{200}$; (3) $2\frac{11}{18}+x=3\frac{1}{12}+2\frac{1}{24}$; (4) $x+\left(\frac{1}{6}+\frac{1}{10}\right)=1\frac{1}{15}$; (5) $(2/5+49/55)-x=3/11$.

§ 8. Multiplication and Division of Fractions

1.34. Find the product:

(a) (1) $\frac{17}{18}\cdot\frac{15}{34}$; (2) $\frac{16}{85}\cdot\frac{17}{32}$; (3) $\frac{5}{32}\cdot\frac{18}{35}$; (4) $\frac{17}{40}\cdot\frac{10}{11}$; (5) $\frac{35}{48}\cdot\frac{16}{35}$; (6) $\frac{225}{343}\cdot\frac{98}{135}$; (7) $\frac{216}{625}\cdot\frac{375}{1080}$; (8) $\frac{121}{144}\cdot\frac{576}{726}$; (9) $\frac{729}{961}\cdot\frac{155}{243}$;

(b) (1) $1\frac{1}{2}\cdot 1\frac{1}{5}$; (2) $1\frac{1}{3}\cdot 2\frac{1}{4}$; (3) $13-\frac{5}{6}\cdot 1\frac{1}{3}$; (4) $8\frac{12}{31}\cdot 9\frac{7}{13}$; (5) $6\frac{4}{9}\cdot 2\frac{42}{29}$; (6) $7\frac{5}{11}\cdot 1\frac{14}{41}$; (7) $7\frac{23}{40}\times 1\frac{59}{101}$.

1.35. Find the indicated part:

(a) (1) 3/4 of 12; (2) 2/3 of 15; (3) 7/8 of 64; (4) 4/15 of 225; (5) 19/21 of 105; (6) 11/15 of 5/22; (7) 24/25 of 25/72; (8) 1/3 of 7/15; (9) 31/32 of 16/93; (10) 15/16 of 4/5; (11) 5/8 of 4/15; (12) 1/2 of 2/3;

(b) (1) $\frac{1}{2}$ of $1\frac{1}{2}$; (2) $\frac{1}{3}$ of $2\frac{3}{4}$; (3) $\frac{2}{3}$ of $4\frac{1}{3}$; (4) $\frac{2}{3}$ of $2\frac{1}{4}$; (5) $\frac{11}{19}$ of $4\frac{3}{4}$; (6) $\frac{9}{25}$ of $20\frac{5}{8}$; (7) $\frac{11}{15}$ of $3\frac{9}{22}$; (8) $2\frac{7}{8}$ of 120; (9) $\frac{9}{35}$ of 14; (10) $\frac{5}{27}$ of 108; (11) $\frac{4}{15}$ of 0; (12) $1\frac{7}{8}$ of $\frac{7}{120}$.

1.36. Which is greater:

(1) 3/4 of 60 or 5/8 of 80? (2) 5/7 of 49 or 1/2 of 70? (3) $\frac{3}{4}$ or $\frac{5}{6}$ of 60? (4) $\frac{7}{8}$ of 40 or $\frac{7}{18}$ of 126? (5) $2\frac{2}{3}$

of $1\frac{7}{8}$ or $1\frac{3}{5}$ of $3\frac{1}{8}$? (6) $2\frac{7}{8}$ of $2\frac{19}{23}$ or $4\frac{2}{7}$ of $1\frac{19}{30}$? (7) $3\frac{2}{11}$ or $2\frac{1}{3}$ of $2\frac{34}{49}$? (8) $4\frac{5}{8}$ or $2\frac{5}{16}$ of $2\frac{6}{37}$? (9) $3\frac{5}{6}$ or $1\frac{11}{12}$ of $1\frac{5}{46}$?

1.37. Find the quotient:

(a) (1) $\frac{5}{6} \div 5$; (2) $\frac{4}{5} \div 2$; (3) $\frac{8}{11} \div 4$; (4) $\frac{21}{25} \div 7$; (5) $\frac{12}{25} \div 6$; (6) $\frac{3}{4} \div 2$; (7) $56 \div \frac{14}{15}$; (8) $49 \div \frac{21}{25}$; (9) $135 \div \frac{45}{51}$; (10) $108 \div \frac{54}{59}$;

(b) (1) $1\frac{1}{2} \div 2$; (2) $1\frac{1}{2} \div 3$; (3) $3\frac{1}{3} \div 5$; (4) $2\frac{5}{6} \div 17$; (5) $1\frac{1}{35} \div 18$; (6) $5\frac{4}{9} \div 8$; (7) $6\frac{3}{7} \div 3$; (8) $7\frac{14}{15} \div 7$; (9) $27\frac{5}{6} \div 9$; (10) $15\frac{25}{19} \div 5$;

(c) (1) $0 \div 4\frac{8}{9}$; (2) $45 \div 3\frac{1}{3}$; (3) $120 \div 1\frac{4}{5}$; (4) $320 \div 3\frac{1}{5}$; (5) $1 \div 1\frac{1}{15}$; (6) $22 \div 4\frac{1}{8}$; (7) $77 \div 3\frac{1}{7}$; (8) $125 \div 3\frac{1}{8}$; (9) $153 \div 1\frac{5}{26}$; (10) $360 \div 1\frac{1}{35}$;

(d) (1) $\frac{17}{30} \div \frac{7}{10}$; (2) $\frac{28}{29} \div \frac{4}{29}$; (3) $\frac{17}{36} \div \frac{51}{72}$; (4) $\frac{11}{108} \div \frac{22}{27}$; (5) $\frac{13}{43} \div \frac{299}{989}$; (6) $\frac{4}{7} \div \frac{10}{21}$; (7) $\frac{15}{22} \div \frac{15}{33}$; (8) $\frac{37}{70} \div \frac{333}{385}$;

(e) (1) $1\frac{1}{2} \div 2\frac{1}{2}$; (2) $1\frac{2}{3} \div 1\frac{1}{9}$; (3) $8\frac{2}{5} \div 1\frac{1}{20}$; (4) $12\frac{2}{5} \div 3\frac{1}{10}$; (5) $5\frac{5}{6} \div 3\frac{8}{9}$; (6) $3\frac{1}{8} \div 6\frac{2}{3}$;

(f) (1) $8\frac{13}{16} \div \frac{47}{64} \div 1\frac{1}{35} \div 3\frac{1}{2}$; (2) $10\frac{5}{6} \div \frac{13}{18} \div 7\frac{1}{2} \div \frac{3}{4} \div \frac{1}{3}$; (3) $14\frac{2}{2} \div 5\frac{1}{2} \div \frac{7}{24} \div 1\frac{2}{7}$.

1.38. Find the number:

(a) (1) 2/5 of which is equal to 10; (2) 3/7 of which is equal to 21; (3) 6/11 of which is equal to 66; (4) 15/22 of

which is equal to 45; (5) 12/13 of which is equal to 288; (6) 15/23 of which is equal to 75; (7) 1/2 of which is equal to 2/5;

(b) (1) $3\frac{1}{2}$ of which is equal to $1\frac{2}{3}$; (2) $1\frac{1}{20}$ of which is equal to $12\frac{3}{5}$; (3) $\frac{3}{5}$ of which is equal to $\frac{3}{5}$; (4) $9\frac{3}{4}$ of which is equal to $7\frac{1}{11}$; (5) $3\frac{3}{4}$ of which is equal to $22\frac{1}{2}$; (6) $\frac{2}{5}$ of which is equal to $\frac{3}{4}$ of 160; (7) $\frac{2}{5}$ of which is equal to $\frac{5}{6}$ of 240.

§ 9. Operations on Common Fractions

1.39. Perform the indicated operations:

(1) $2/5+2\frac{4}{9}\div\left(\left(7\frac{5}{12}-5\frac{3}{4}\right)\div 22\frac{1}{2}+10\cdot\frac{5}{18}\right)-\frac{4}{5}$;

(2) $\left(20\frac{8}{15}\cdot 7\frac{1}{2}-54\frac{3}{5}\div 2\frac{1}{2}\right)\div\left(3\frac{13}{21}\cdot 8\frac{2}{5}-29\frac{2}{5}\right)-\frac{5}{6}\cdot 1\frac{1}{5}+21/25$;

(3) $7/9\cdot 1\frac{2}{7}+43\frac{3}{4}\div 1\frac{12}{3}-3\frac{18}{25}+1\frac{1}{45}\cdot\left(37\frac{1}{2}\div 2\frac{1}{2}-1\frac{3}{23}\cdot 9\right)+47/100$;

(4) $11\frac{2}{5}+7\frac{1}{2}\cdot\left(285\frac{3}{5}\div 14-1\frac{23}{30}+13/50\right)\div\left(24\frac{2}{5}-10\frac{23}{100}\right)$;

(5) $\left(3\frac{1}{2}-2\frac{2}{3}+5\frac{5}{6}+4\frac{3}{5}\right)\cdot 24+\left(5\frac{3}{8}+18\frac{1}{2}-7\frac{5}{24}\right)\div 16\frac{2}{3}$;

(6) $\left(12\frac{5}{12}+1\frac{2}{3}-3\frac{5}{6}+2\frac{3}{4}\right)\div\left(2\frac{1}{2}\cdot\frac{2}{5}-\frac{7}{9}\right)$;

(7) $\left(\left(\frac{15}{28}-\frac{11}{36}\right)\cdot\frac{21}{29}+6\frac{6}{7}\div\frac{16}{21}\right)\div 16\frac{1}{2}$;

(8) $\left(\left(4\frac{5}{7}-1\frac{11}{14}\right)\cdot 4\frac{2}{3}+\left(3\frac{2}{9}-1\frac{5}{6}\right)\cdot\frac{8}{25}\right)\div 2\frac{3}{4}$;

(9) $1\frac{9}{40}\cdot\left(7\frac{5}{7}\div 3\frac{3}{5}-(53/56-29/35)\div 33/40\right)$;

(10) $\left(\left(5\frac{5}{9}-7/18\right)\div 35+(40/63-8/21)\div 20+\right.$
$\left.\left(\frac{83}{90}-\frac{41}{50}\right)\div 2\right)\cdot 3\frac{1}{2};$

(11) $\dfrac{14\frac{4}{5}-6\frac{11}{12}+12\frac{3}{4}-7\frac{2}{15}}{10\frac{2}{3}-3\frac{11}{12}}+2\frac{2}{3}\cdot 3\frac{3}{4}+$
$\dfrac{36\frac{2}{3}\div 15+8-\frac{2}{3}\cdot 7}{12\frac{1}{3}+8\frac{6}{7}\div 2\frac{4}{7}}+\dfrac{2\frac{3}{8}\div\frac{3}{4}+24\cdot\frac{7}{9}}{7\frac{2}{3}-157\frac{4}{5}\div 24};$

(12) $\dfrac{1\frac{9}{16}\cdot 3\frac{1}{5}+16\frac{2}{3}-9\div 2\frac{2}{5}}{17\frac{7}{12}-6\frac{1}{3}}+\dfrac{12\frac{2}{3}-61\frac{1}{2}\div 6\frac{3}{4}}{2\frac{2}{3}}+$
$\dfrac{9\frac{1}{4}-7\frac{2}{5}\cdot 2\frac{1}{2}-1\frac{1}{2}}{\frac{1}{8}+4\frac{3}{20}-1\frac{5}{48}-5\frac{2}{5}\right)\div 3\frac{1}{2}}+\dfrac{3-4\cdot\frac{1}{10}}{7+1\div\frac{3}{7}};$

(13) $\dfrac{\left(9-\frac{3}{8}\right)\cdot\left(4\frac{5}{12}-\frac{4}{2\frac{2}{3}}+\left(\frac{3}{10}-\frac{1}{2}\div 4\right)\cdot\frac{4}{7}\right)}{1\frac{1}{24}+\frac{1}{4}\div 13\frac{1}{3}};$

(14) $\left(\dfrac{\left(3\frac{2}{5}+1\frac{5}{7}\right)\cdot 11\frac{2}{3}}{1\frac{2}{9}-1\frac{1}{18}}-\dfrac{\left(10\frac{1}{4}-1\frac{5}{6}\right)\cdot 12}{\left(5\frac{3}{20}-4\frac{1}{4}\right)\cdot 1\frac{1}{9}}\right)\div 85\frac{2}{3};$

(15) $\dfrac{\left(\left(\frac{23}{36}+\frac{31}{63}\right)-\left(\frac{3}{4}+\frac{5}{21}\right)\right)\cdot 48\div\left(\frac{3}{5}\div\frac{7}{8}\right)}{\left(\frac{19}{26}+\frac{14}{39}-\frac{1}{6}\right)\cdot 54\frac{1}{6}\div\left(8\frac{4}{7}\div\frac{12}{35}\right)}.$

1.40. Find x if:

(1) $\frac{3}{8}x+25=100$; (2) $\frac{1}{9}x-20=56$;

(3) $\frac{7}{15}x-50\frac{3}{4}=19\frac{1}{4}$; (4) $40-3/8x=35\frac{1}{2}$;

(5) $\left(4\frac{1}{2}-2x\right)\cdot 3\frac{2}{3}=11/15$; (6) $\left(2\frac{4}{5}x-50\right)\div 2/3=51$;

(7) $3\frac{4}{15} \div \left(3\frac{1}{2} + x\right) \div 21\frac{3}{7} - 1\frac{3}{8} = \frac{5}{8}$;

(8) $\left(2\frac{3}{4} - 5/6 \cdot 1\frac{1}{5}\right) \div \left(\left(5\frac{1}{35} - x\right) \cdot 12\frac{1}{4}\right) = 5.$

§ 10. Decimal Fractions

1.41. Write the following common fractions in the form of decimals: 3/10; 21/100; 11/1000; 7/10 000; 101/10 000 000; $1\frac{9}{100}$; $23\frac{15}{100}$; $101\frac{27}{100\,000}$.

1.42. (Orally). Reduce the given pairs of decimals to a common denominator: 1.5 and 1.52; 0.5 and 0.49; 14.3 and 14.27; 43.04 and 43.1; 4.568 and 4.56; 0.52 and 0.5198.

1.43. (Orally). Reduce the following decimals: 2.100; 14.7000; 0.170010; 0.930000; 181.0100; 0.150; 23.400; 1.00100; 8.400700; 15.400000.

1.44. Find the sum of the given decimals:

(1) 2 + 0.43 + 7.24 + 34.1;

(2) 16.8 + 1.095 + 0.07 + 15.971;

(3) 252 + 327.63 + 400.507 + 3.7094;

(4) 29.06 + 71.904 + 11.37; (5) 4.05 + 3.2 + 8.99;

(6) 7.8 + 0.107 + 0.096 + 0.779999;

(7) 157.974 + 34.01 + 105.016;

(8) 15.8 + 21.45 + 30 + 40.01 + 3.015;

(9) 176 + 325.75 + 104.397 + 457.629;

(10) 1004.2 + 851.07 + 157.37;

(11) (16.4 + 13.2) + (10.6 + 4.8);

(12) (20.9 + 15.7 + 10.71) + (1.29 + 40.1 + 2.3).

1.45. (a) Find the difference: (1) 1 — 0.98765; (2) 68.2 — 44.763; (3) 45.327 — 17.043; (4) 24.73 — 16.8054; (5) 0.1 — 0.0308; —0.0102; —0.059; (6) 19.2 — 16.403 — 0.57 — 1.2; (7) 12.7 — 4.07 — 3.528.

(b) Perform the indicated operations:

(1) (4.48 — 1.02) — (2.27 — 1.14);

(2) (17.03 — 13.321) — (17.481 — 14.19);

(3) (15.75 — 13.2) — (8.92 — 7.54) + 0.01;

(4) 20.9 + 15.7 — (1.29 + 40.1 — 2.3).

1.46. Find x if:

(1) $x + 12.4 = 15.83$; (2) $21.7 + x = 23.04$;
(3) $x + 16.53 = 24.47$; (4) $28.4 - x = 27.93$;
(5) $x + (3.2 - 2.1) = 5.7$; (6) $(16 - 3.8) - x = 11.43$;
(7) $14.2 - (x + 3.4) = 10.8$; (8) $(11.4 - x) + 8.4 + 10.25$;
(9) $(101.5 - 0.92) - x = 2.66$.

1.47. Find the product:

(a) (1) $4.52 \cdot 5$; (2) $0.71 \cdot 3$; (3) $0.02 \cdot 7$; (4) $4.92 \cdot 20$;
(5) $0.154 \cdot 70$;

(b) (1) $5 \cdot 0.41$; (2) $12 \cdot 4.05$; (3) $17 \cdot 1.01$; (4) $40 \cdot 3.24$;
(5) $55 \cdot 0.011$;

(c) (1) $3.2 \cdot 0.25$; (2) $15.04 \cdot 0.7$; (3) $0.81 \cdot 1.12$;
(4) $0.064 : 27.2$; (5) $5.37 \cdot 7.12$.

1.48. Perform the indicated operations:

(1) $5.423 + 3.577 \cdot (5.423 - 3.577)$;
(2) $(9 - 0.4) \cdot (6.1 - 4.6) + (4.1 - 2.85) \cdot (3.2 - 3.12)$;
(3) $(2.743 + 12.257) \cdot 0.01 + 0.047 \cdot (10\,000 - 429.5)$;
(4) $5.872 \cdot 0.5 - (70.75 - 0.25 \cdot 283) \cdot 1.6 \cdot 100$;
(5) $6 - (23.265 + 4.735) \cdot 0.01 - 2.4 \cdot 0.1$;
(6) $34.8 \cdot 0.5 - (9.8 + 1.4) \cdot 0.2 + 0.6 \cdot (24.3 - 18.8)$;
(7) $41.5 \cdot 0.6 - 0.4 \cdot (15.8 - 12.3) + (13.4 + 15.4) \cdot 0.5$;
(8) $100 \cdot (5.423 - 4.908) + 0.1 \cdot (19.4 - 17.4) - 0.5 \times (14.7 - 11.6)$;
(9) $(0.1598 - 0.1379) \cdot 1000 + 0.1$;
(10) $(1.4 + 0.01 - 0.74) \cdot (53.2 - 5.06) \cdot 10 - 8 \cdot (4.2 - 2.7)$.

1.49. Find the quotient:

(a) (1) $1.44 \div 12$; (2) $0.9 \div 125$; (3) $2.35 \div 4$;
(4) $0.0153 \div 150$;

(b) (1) $3 \div 0.6$; (2) $1 \div 0.8$; (3) $36 \div 0.225$;
(4) $5 \div 0.002$;

(c) (1) $0.12 \div 0.4$; (2) $1.5 \div 0.03$; (3) $0.0121 \div 0.11$;
(4) $10.01 \div 0.1$;

(d) (1) $9 \div 0.032$; (2) $6.21 \div 3$; (3) $0.2205 \div 0.147$;

(e) (1) $(0.93 + 0.07) \div (0.93 - 0.805)$; (2) $(50\,000 - 1397.3) \div (1.98 + 2.02)$; (3) $(2779.6 + 8024.4) \div (3.98 + 4.02)$.

1.50. Compute:

(1) $\dfrac{57.24 \cdot 3.55 + 430.728}{2.7 \cdot 1.88 - 1.336} + \dfrac{127.18 \cdot 4.35 + 14.067}{18 + 2.1492 \div 3.582}$;

(2) $\dfrac{5.7 \cdot 16.2}{20.52} + \dfrac{127.68 \cdot 0.5}{4.56} + \dfrac{34.68 \cdot 15.4}{6.8 \cdot 3.57}$;

(3) $\left(\dfrac{0.3 \cdot (3.6 - 2.8)}{0.25 \cdot (0.94 + 1.06)} + \dfrac{(0.2 - 0.15) \div 0.001}{(4.7 - 3.9) \cdot 10}\right) \div 27.92$;

(4) $98 \div \left(\dfrac{6 \div (0.4 - 0.2)}{2.5 \cdot (0.8 + 1.2)} + \dfrac{(34.06 - 33.81)}{6.84 \div 28.57 - 25.15}\right) - 8$;

(5) $\dfrac{0.125 \div 0.25 + 1.5625 \div 2.5}{(10 - 22 \div 2.3) \cdot 0.46 + 1.6} + (0.85 + 1.9) \cdot 0.5$;

(6) $\dfrac{((6.625 - 0.25) \div 2.25 + 2.5 \div 1.25 \div 6.5) \cdot \dfrac{39}{86} + 1.364 \div 0.124}{(0.5 - 0.375) \div 0.125 + 0.25 \div (0.68 - 1.4796 \div 13.7)}$;

(7) $\dfrac{(21.85 \div 43.7 + 8.5 \div 3.4) \div 4.5 \div 1.4}{(1.4 + 3.5 \div 1.25) \div 2.4 + 3.4 \div 2.125 - 0.35} - 0.36$;

(8) $\left(\dfrac{3.75 + 2.5}{2.5 - 1.875} - \dfrac{2.75 + 1.5}{2.75 - 1.5}\right) + 0.5 \cdot \left(2 + \dfrac{12.5}{5.75 + 0.5}\right)$.

1.51. Compute:

(1) $(3.06 \div 7.5 + 3.4 \cdot 0.38) \cdot (20 - 2.38 \cdot 5.3)$;
(2) $(8.04 + 2.5 \cdot 0.24 - 0.5) \cdot (5.4 + 1.5 + 3.06)$;
(3) $7 \div 0.2625 - 3.6 \div (68.1 \div 7.5 - 7.85 + 1.02)$;
(4) $(0.125 \div 0.25 + 1.5625 \div 2.5) \div (10 - 22. \cdot 2.3) \times 0.46 + 0.923$;
(5) $((0.6 + 0.425 - 0.005) \div 0.1) \div (30.5 - 0.96 \div 0.48 + 22.5)$;
(6) $(0.4 + 8 \cdot (5 - 0.8 \cdot 0.625) - 5 \div 2.5) \div (15.4 + 1.6 \div 0.8 + 25.6)$.

1.52. Find x if:

(1) $2.6x = 40.54 + 50.46$; (2) $30.4x + 8.176 = 10$;
(3) $0.05x - 0.01 = 0.19$; (4) $(3.12 + 0.9) \cdot x = 2.412$;
(5) $\dfrac{x}{0.4} = 0.6 - 0.4$; (6) $\dfrac{x}{0.5} + 0.75 = 1.6$;

(7) $(86.9 + 667.6) \div (37.1 + x) = 15$;
(8) $(5000 - 1397.3) \div (x + 33.63) = 90$;
(9) $3.06 - 0.05 \cdot x + 66 \div 0.33 + 0.14 = 203$;
(10) $2.473 \cdot 0.05x + 0.1581 \div 0.06 = 15$.

1.53. Change the given common fractions to decimals: 9/15; 18/252; 21/28; 39/65; 30/75; 6/48; 8/5; 25/16; 47/32; 363/250; 312/125; 36/8; $1\frac{751}{625}$; $5\frac{2541}{2000}$; 6/31; $2\frac{13}{28}$; 29/35; $1\frac{53}{68}$; 22/147; 63/28; 48/11; $1\frac{5}{16}$.

1.54. Which of the following common fractions can be changed to finite decimals? 1/3; 1/4; $\frac{1}{6}$; $\frac{1}{12}$; $\frac{3}{32}$; $\frac{4}{21}$; $\frac{5}{54}$; $\frac{11}{90}$; $2\frac{7}{50}$; $\frac{3}{6}$; $\frac{15}{45}$; $\frac{9}{27}$.

1.55. Change the given common fractions to decimals:

(a) accurate to 0.01; 2/3; 5/9; 6/7; 13/60; 11/15; 17/24; $2\frac{5}{13}$; $\frac{5}{17}$; 4/36; 11/5;

(b) accurate to $\frac{5}{14}$; $\frac{7}{23}$; $\frac{13}{28}$; $\frac{19}{43}$; $\frac{16}{6}$; $5\frac{7}{24}$; $6\frac{11}{14}$; $\frac{8}{7}$; $\frac{4}{19}$.

1.56. Change the given decimal to common fractions:
(1) 0.3; 0.4; 0.14; 0.45; 0.125; 1.8; 2.24; 15.06; 0.2; 121.85; 4.625;

(2) 0.35; 0.36; 1.125; 5.004; 0.205; 7.305; 15.0075; 7.00625; 2.03.

§ 11. Repeating (or Periodic) Decimals

1.57. Read the following periodic decimals: 0.333...; 0.434343...; 5.727272...; 0.5222...; 0.21333...; 12.777...; 1.901901901...; 0.7; 0.301; 4.21; 1.145; 13.5232323...; 0.437; 15.4329; 0.123; 9.8999...; 0.3191919...; 2.708333...

1.58. Mention the period of each decimal: 8.555...; 86.346346346...; 0.730303...; 9.235423423...; 0.35737373...; 268.494949...; 8.5272727...; 34.444...; 58.1282282282...;

1.8121212...; 0.5777...; 0.4353535; 0.13888...; 2.6444...; 1.5896589658...; 12.121212...

1.59. Express in terms of repeating decimals:

$\frac{1}{3}$; $\frac{1}{9}$; $\frac{1}{11}$; $\frac{2}{7}$; $\frac{7}{99}$; $\frac{5}{6}$; $\frac{1}{14}$; $\frac{2}{15}$; $2\frac{1}{12}$; $4\frac{2}{3}$; $4\frac{2}{9}$; $\frac{12}{13}$; $2\frac{5}{6}$; $\frac{8}{11}$; $5\frac{5}{7}$; $\frac{4}{33}$; $\frac{17}{29}$; $\frac{7}{85}$; $1\frac{23}{49}$; $\frac{11}{18}$; $\frac{22}{53}$; $\frac{23}{30}$.

Determine the period of each fraction.

1.60. Change the following repeating decimals to common fractions: 0.(8); 0.2(35); 0.(5); 3.(27); 0.5(8); 28.10(01); 4.4(35); 0.42(6).

1.61. Perform the indicated operations:

(1) $\dfrac{\left(4.5\cdot 1\frac{2}{3}-6.75\right)\cdot 0.66\ldots}{\left(3.(3)\cdot 0.3+0.(2)+\frac{4}{9}\right)\div 2\frac{2}{3}}+\dfrac{1\frac{4}{11}\cdot 0.22\div 0.3-0.96}{\left(0.2-\frac{3}{40}\right)\cdot 1.6}$;

(2) $\dfrac{0.5+\frac{1}{4}+0.1666\ldots+0.125}{0.(3)+0.4+\frac{14}{15}}+\dfrac{0.1(6)+0.(3)}{0.(3)+1.1(6)}$;

(3) $\dfrac{\left(16\frac{1}{2}-13\frac{7}{9}\right)\cdot\frac{18}{33}+2.2\cdot(0.(24)-0.(09))+\frac{2}{11}}{(0.725+0.6+\frac{7}{40}+0.42(6)+0.12(3)}$;

(4) $\dfrac{0.8333\ldots-0.4(6)}{1\frac{5}{6}}\cdot\dfrac{1.25+1\frac{3}{4}-0.41(6)}{0.62}$;

(5) $\dfrac{\left(0.666\ldots+\frac{1}{3}\right)\div 0.25}{(0.12333\ldots)\div 0.0925}+12.5\cdot 0.064$;

(6) $\dfrac{\left(\frac{5}{8}+2.708333\ldots\right)\div 2.5}{\left(1.3+0.7(6)+0.(36)\frac{110}{401}\right)}\cdot 0.5$;

(7) $\dfrac{(7-6.35)\div 6.5+9.8999\ldots)\cdot\frac{1}{128}}{\left(1.2\div 36+1\frac{1}{5}\div 0.25-1.8(3)\right)\cdot 1\frac{1}{4}}\div 0.125$;

$$(8)\quad \frac{\left(2\frac{38}{45}-\frac{1}{15}\right)\div 13\frac{8}{9}+3\frac{3}{65}\cdot 0.(26)}{\left(18\frac{1}{2}-13.777\ldots\right)\cdot\frac{1}{85}}\cdot 0.5.$$

§ 12. Ratios of Numbers

1.62. Find the ratios:

(a) (1) 3.5 km : 14 km; (2) 1.5 km : 500 m; (3) 8.5 km : 170 m; (4) 80 cm : 5 m; (5) 3 m : 150 cm; (6) 5.6 sq.dm : 0.7 sq. cm; (7) 0.7 sq. cm : 5.6 sq. dm; (8) 60 kg : 3.5 t; (9) 4.5 cm : 32 m; (10) 1 cu. m : 1 cu. cm;

(b) (1) 72 to 216; (2) 216 to 72; (3) 56 to 154; (4) $\frac{1}{24}$ to $\frac{1}{6}$; (5) 3.5 to 14; (6) 560 to 0.7; (7) $\frac{48}{55}$ to $\frac{28}{99}$; (8) $5\frac{1}{4}$ to $12\frac{16}{17}$; (9) 4.36 to 0.4; (10) 1 to 0.08; (11) $2\frac{1}{3}$ to 3.5; (12) 60 to 3500;

(c) to within 0.01: (1) 8.412 to 49.07; (2) 2 to 7; (3) 5 to 3; (4) $\frac{1}{14}$ to $\frac{2}{15}$; (5) $\frac{21}{11}$ to $\frac{170}{44}$;

(d) to within 0.001: (1) 16 to 15; (2) $\frac{2}{9}$ to $\frac{1}{3}$; (3) 1.5 to 2.4; (4) $2\frac{1}{3}$ to $1\frac{1}{5}$.

1.63. Find x: (1) $1.5 \div x = 0.5$; (2) $18.24 \div x = 22.8$; (3) $x \div 6\frac{8}{9} = \frac{3}{4}$; (4) $x \div 1\frac{31}{35} = 9\frac{1}{3}$; (5) $72 \div 3x = 12$; (6) $24.4x \div 0.61 = 4$; (7) $48.96 \div 5.1x = 2.4$; (8) $5\frac{5}{6} \div 2\frac{1}{3}x = 2\frac{1}{4}$; (9) $3.06 \div 0.9x = 1.7$; (10) $2.25 \div 0.15x = 1.8$.

1.64. Simplify: (1) 56 ÷ 72; (2) 56 ÷ 84; (3) 280 ÷ 336; (4) 465 ÷ 375; (5) 288 ÷ 360 ÷ 2160; (6) 630 ÷ 714 ÷ 1386; (7) 825 ÷ 3630 ÷ 1155.

1.65. Replace the following ratios of fractions by ratios of integers:

(1) $\frac{3}{20} : \frac{7}{45}$; (2) $\frac{11}{12} : \frac{5}{24}$; (3) $4\frac{4}{5} : 3\frac{3}{7}$; (4) 0.18 : 0.47;

(5) $2.4 : 0.72$; (6) $6\frac{1}{2} : 5.2$; (7) $316 : 7.9$; (8) $\frac{1}{4} : \frac{5}{21} : \frac{7}{18}$;

(9) $2\frac{1}{3} : \frac{5}{6} : 1\frac{5}{24}$; (10) $0.32 : 0.06 : 1\frac{3}{5}$; (11) $\frac{7}{18} : 2\frac{5}{24} : \frac{7}{72}$.

§ 13. Proportions and Proportional Division

1.66. Check the following proportions:

(1) $4 : 14 = 14 : 49$; (2) $24 : 3 = 36 : 4$; (3) $25 : 15 = 15 : 9$;

(4) $42 : 14 = 75 : 25$; (5) $54 : 18 = 24 : 8$;

(6) $4\frac{1}{2} : 3\frac{1}{2} = 27 : 21$; (7) $3\frac{3}{4} : \frac{4}{5} = \frac{6}{13} : \frac{2}{13}$;

(8) $15\frac{3}{4} : \frac{4}{7} = \frac{3}{4} : \frac{1}{12}$; (9) $24\frac{3}{5} : 3 = \frac{5}{6} : \frac{5}{246}$;

(10) $10.2 : 0.66 = 0.85 : 0.055$; (11) $0.16 : 0.32 = 0.4 : 0.8$; (12) $2.25 : 9 = 0.3 : 2$; (13) $3.43 : 49 = 0.28 : 4$;

(14) $40.5 : 0.25 = 3.6 : \frac{1}{49}$; (15) $2.24 : 0.7 = 0.35 : 6.4$.

1.67. (a) Form proportions from the given equalities:

(1) $15 \cdot 42 = 35 \cdot 18$; (2) $54 \cdot 55 = 66 \cdot 45$;

(3) $2.5 \cdot 0.018 = 0.15 \cdot 3$; (4) $2\frac{1}{2} \cdot 1\frac{2}{7} = \frac{5}{7} \cdot 4\frac{1}{2}$;

(5) $5/8 \cdot 7/12 = 5.4 \cdot 4/24$; (6) $2\frac{1}{2} \cdot 0.9 = 4/15 : 8\frac{7}{16}$.

(b) Form proportions from given numbers:

(1) 0.16; 0.32; 0.4; and 0.8; (2) 44; 4; 11; and 16;

(3) 2/3; 5/6; 3/8; and 3/10; (4) 16; 24; 36; and 54;

(5) $3\frac{1}{2}$; 2; 7; and 4; (6) 57; 6; 14.4; 6.4; and 1.6;

(7) 7.5; $1\frac{9}{16}$; $5\frac{1}{4}$; and 25.2; (8) $7\frac{2}{3}$; 2.3; 0.8; and $2\frac{2}{3}$.

1.68. Find x from the proportion:

1) $x : 16 = 3 : 6$; (2) $24 : x = 8 : 5$; (3) $x : 15 = 8 : 24$; (4) $3 : x = 54 : 3$; (5) $1.04 : x = 4 : 5$; (6) $0.02 : 2.5 = x : 0.35$;

(7) $3\frac{1}{2} : 0.4 = x : 1\frac{1}{7}$; (8) $\frac{1}{6} : 2\frac{1}{3} = 3\frac{1}{4}x : 1.3$;

(9) $$\frac{1.2 \div 0.375 - 0.2}{6\frac{4}{25} \div 15\frac{2}{5} + 0.8} = \frac{0.016 \div 0.12 + 0.7}{x};$$

(10) $$\frac{0.125x}{\left(\frac{19}{24}-\frac{21}{40}\right)8\frac{7}{16}}=\frac{\left(1\frac{28}{63}-\frac{-17}{21}\right)0.7}{0.675\cdot 2.4-0.02};$$

(11) $$\frac{x}{10.5\cdot 0.24-15.15\div 7.5}-\frac{9\left(1\frac{11}{90}-0.945\div 0.9\right)}{1\frac{3}{40}-4\frac{3}{8}\div 7}.$$

1.69. (a) Divide the number 135 proportionally to the given numbers: (1) 2; 3; (2) 7; 8; (3) 1.3; 5; (4) $\frac{2}{3}$; 3; $5\frac{1}{3}$; (5) 6; $\frac{2}{3}$; 5.

(b) Divide the number 2400 into parts proportional to the given numbers: (1) $2\frac{2}{3}$; $\frac{2}{3}$; (2) $4\frac{2}{3}$; $\frac{2}{3}$; (3) 6; $\frac{2}{3}$; 5; (4) $11\frac{1}{5}$; 2; 3; $3\frac{4}{5}$; (5) $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{5}$; $\frac{3}{10}$.

1.70. (a) Divide the number 18.3 into parts inversely proportional to the given numbers: (1) 2; 3; 5; 1; (2) 1; 3; 4; 9; (3) 0.5; 0.3; 1.(3); $2\frac{1}{7}$.

(b) Divide the number 434 into parts inversely proportional to the given numbers: (1) 15; 16; (2) 2; 3; 5.

1.71. Divide the number 144 into three parts x, y, and z such that $x : y : z = 3 : 4 : 5$.

§ 14. Percent

1.72. Find:

(a) (1) 2% of 50; (2) 10% of 20; (3) 25% of 120; (4) 12% of 160; (5) 15% of 84; (6) 30% of 2000; (7) 25% of 340; (8) 13% of 360; (9) 26% of 150;

(b) (1) 32% of 12.5; (2) $4\frac{1}{2}$% of 1.44; (3) $6\frac{1}{4}$% of 64; (4) 12.5% of 8.88; (5) $7\frac{1}{2}$% of $1\frac{1}{225}$; (6) 8.5% of $1\frac{1}{170}$; (7) $11\frac{3}{4}$% of $5\frac{2}{47}$; (8) $22\frac{3}{5}$% of $\frac{15}{226}$; (9) 30.5% of $\frac{231}{610}$; (10) 42.5% of $\frac{257}{425}$.

1.73. Find the number if:

(1) 8% of it is 24; (2) 45% of it is 225; (3) 140% of it is 182; (4) 3.5% of it is 21; (5) 3% of it is 1.5; (6) 750% of it is 450.

1.74. Find the number x if:

(1) $7\%x = 182$; (2) $60\%x = 23$; (3) $1\frac{2}{3}\%x = 4.75$;

(4) $7.5\%x = 3.3$; (5) $2\frac{1}{2}\%x = 0.15$; (6) $\frac{2}{3}\%x = 4\frac{1}{5}$;

(7) $84\%x = 105$; (8) $16\frac{2}{3}\%x = 150$; (9) $8\%x = 1.84$;

(10) $500\%\ x = 1550$; (11) $120\%\ x = 24.4$; (12) $210\%x = 5.6$; (13) $11.5\%x = 0.116$; (14) $150\%x = 135$; (15) $0.17\%x = 0.51$.

1.75. Find the ratio in percent:

1 to 4; 12 to 25; $2\frac{1}{2}$ to 50; 14 to 20; 138 to 92; 11 to 440; 35 to 28; 26 to 208; 3.2 to 1.28; $\frac{3}{4}$ to $4\frac{1}{6}$; 5.5 to 0.1; 0.51 to 2.04; 0.65 to 2.6; 0.75 to $1\frac{1}{5}$; 0.35 to $\frac{7}{24}$; 1.25 to 6.25.

1.76. Find the percentage ratio of the given numbers:

(a) accurate to 0.1:

(1) 149 to 207; (2) 15 to 7; (3) 1407 to 1882; (4) 2.74 to 2.09; (5) 3.2 to 0.51;

(b) accurate to 0.01:

(1) 7 to 15; (2) 2. 76 to 20.91; (3) 1.9 to 3.7; (4) 0.03 to 0.51;

(c) accurate to 0.001:

(1) 1.73 to 4.8; (2) 0.418 to 4.6; (3) 27 to 480; (4) 1.3 to 19; (5) 0.11 to 52.

1.77. Find the ratio of A to B in percent if:

(1) $A = \left(5 \div 4\frac{1}{6} - 3\frac{9}{23} \cdot 3\right)\left(\frac{5}{18} + \frac{4}{15} + 0.35\right)$;

$B = \left(18.9 - 16\frac{13}{20}\right)\frac{8}{9} + \frac{13}{15} \cdot 30$;

(2) $A = \dfrac{152\frac{3}{4} - 148\frac{3}{8} \cdot 0.3}{0.75}$; $B = \dfrac{\left(6\frac{3}{5} - 3\frac{3}{14}\right) \cdot 5\frac{5}{6}}{(21 - 1.25)\,2.5}$;

(3) $A = 7\frac{1}{9} - 2\frac{14}{15} \div 2\frac{2}{3} + 1\frac{3}{5} - \frac{3}{4} - \frac{1}{20} \cdot \frac{5}{7} - \frac{15}{14}$;

$$B = \frac{45\frac{10}{63} - 44\frac{25}{84}}{2\frac{1}{3} + 1\frac{1}{9} \div 4 - \frac{3}{4}} \cdot \frac{1}{31};$$

(4) $$A = \frac{\left(1\frac{53}{520} - \frac{103}{312} - \frac{14}{195}\right) \div (0.14 - 0.12)}{20.35 - 19.602 \div 3.24 \cdot \frac{2}{55} - 0.12};$$

$$B = \frac{1\frac{5}{19} - \frac{8}{11} \cdot 5\frac{9}{40} + 7.2842 \div 0.605}{0.304 \cdot 24.5\frac{87}{875} + \frac{1}{70}};$$

(5) $$A = \frac{0.12 + 2\frac{11}{17} \cdot 6.342 \div 10.5 + 0.076}{2\frac{8}{125} - \frac{11}{45} - 1\frac{11}{36} \cdot 2.5 - \frac{1}{200}};$$

$$B = \frac{2\frac{5}{72} - 1\frac{71}{180} + \frac{6}{25} \cdot 1\frac{14}{61} - 0.125}{4\frac{1}{6} - 1\frac{29}{60} \div 0.9522 \div 0.46 - 1\frac{5}{8}};$$

(6) $$A = \frac{1\frac{32}{55} - \frac{71}{88} - \frac{3}{80} \div 0.59 + 0.19}{2\frac{22}{25} \div 25.344 \div 3.6 - 5.24 - 1.24};$$

$$B = \frac{4 + 1\frac{17}{33} \cdot 18.26 - 14.168 \div 2.8}{7\frac{9}{35} - 6\frac{29}{40} + \frac{9}{14} \cdot 3.2 - 2.8}.$$

CHAPTER 2

Algebra. Elementary Functions and Fundamentals of Analysis

§ 15. Positive and Negative Rational Numbers. Comparing Rational Numbers. Modulus (or Absolute Value) of a Number

2.1. Show on the number line the set of points corresponding to all integers x if:

(1) $10 < |x| < 18$; (2) $|x| < 10$;
(3) $10 < |x| < 28$; (4) $|x - 1| < 12$;
(5) $|x - 3| \leqslant 11$; (6) $|2x - 3| < 5$;
(7) $|x - 5| + 3 \leqslant 14$; (8) $|x - 8| < 20$;
(9) $|x + 2.2| \leqslant 6$.

2.2. Show on the number line the set of points if:

(1) $|x - 1| = 7$; (2) $|x + 3| = 5$; (3) $1 + |x| = 9$;
(4) $|3 - x| = 10$; (5) $|x - 2| - 4 = 3$;
(6) $|2 - x| - 7 = 2$.

2.3. (a) Show on the number line the set of points corresponding to the odd numbers whose modulus is less than 19.

(b) Show on the number line the points corresponding to the least positive two-digit number and to the least negative two-digit number.

(c) Show on the number line all the points corresponding to the even numbers whose modulus is greater than 7 but less than 15.

(d) Show on the number line the set of points corresponding to all the even numbers whose modulus is greater than 7 but less than 13.

2.4. (a) Arrange the following numbers in ascending order:

(1) -15; 3; -1; $-|-4|$; $|-2|$; 0; 1/2; 0.9; $|5|$; -4;
(2) -5; 2; $|-1|$; 0; $|-6|$; 7/2; -1; $-|-2|$; 4.5.

(b) Arrange the following numbers in descending order:

(1) -7; 3; $|-2|$; -1; $-|-3/4|$; -0.75; -0.8; 0.85;

(2) -11; 0.65; $|0.63|$; $-|+5.25|$; $-|-5.26|$; $-|-2.75|$; 7.4; $|0|$.

(c) Compare the following numbers:
(1) -27 and 8; (2) 4.25 and -16.2; (3) -0.75 and $-3/4$; (4) $|-5|$ and $|5|$; (5) $|-4.7|$ and $|-8|$; (6) -19.2 and -21.

2.5. (a) Can the expression $5 + |x|$ take on values: (1) less than zero; (2) equal to zero?

(b) find a and b if $|a| + |b| = 0$;

(c) find a if $|a| + |-a| = 6$;

(d) find m if $|m| = -n$.

2.6. (a) It is known that $|m| = |n|$. May we assert that $m = n$?

(b) It is known that $|m| > |n|$. May we assert that: (1) $m = n$; (2) $m = -n$; (3) $m > n$; (4) $|m| < n$?

§ 16. Operations on Rational Numbers

2.7. Find the sum:

(1) $(-20) + (-15) + (-40)$;
(2) $(+5.2) + (-0.6) + (3/5) + (-3.2)$;
(3) $(-5.2) + (+7.3) + (-6.8) + (-3.2)$;
(4) $(-15) + (-15) + (+30) + (-28)$;
(5) $(-11) + (-6) + (x + (-x)) + (+18)$;
(6) $(-5.4) + (+0.2) + (-0.6) + (+0.08)$;
(7) $(+0.65) + (-1.9) + (-0.1) + (-0.65)$;
(8) $\left(-2\frac{1}{2}\right) + (+5/6) + (-0.5) + \left(+1\frac{1}{6}\right)$;
(9) $(+0.25) + (-1/4) + \left(-3\frac{1}{8}\right) + \left(-5\frac{3}{4}\right)$;
(10) $(-0.1) + \left(+8\frac{1}{3}\right) + \left(+11\frac{2}{3}\right) + (+4.4)$;
(11) $|(-27) + (+5.2)| + (-0.21) + |(-30) + (-3.6)|$;
(12) $|(+62) + (-12)| + |(+72) + (-12)|$.

2.8. Find the result:

(1) $(-1) - (+1.2) - (+3.5)$;
(2) $(-22.5) - (+12.5) - (-9) - (-11)$;

(3) $(+25.6) - (-15.4) - (-10) - (-5.2)$;

(4) $(-33.8) - (-0.8) - (+2.3) - (-5.01)$;

(5) $(-24.7) - (-14.7) - (-20) - (+11.26)$;

(6) $(-18.2) + (-4.3) - |(-25) + (+4.5) - (-3.7) - (+2.9)|$;

(7) $\frac{|(-2)+(-8)|}{|5|} - \frac{|+3|-|-8|}{|-5|}$;

(8) $\frac{|(-6)+(-8)|}{|-14|} + \frac{|-3|-|-12|}{|-9|}$;

(9) $\frac{|-5+(-12)-(-2)|}{|-3|} - \frac{|-6|-|-8|+|-2|}{|-4|}$.

2.9. (a) Perform the indicated operations:

(1) $0.05 - (2 + ((-2.45) - (-0.55)))$;.

(2) $-1.9 - (((+1.125) + (-0.35)) - 1)$;

(3) $1.4 - (((-0.59) - (-0.39)) + (+0.37) + ((-0.7 - (-0.94)))$;

(4) $0.29 - (((-0.23) - (-0.06) + (+0.37)) - ((-0.47) - (-0.37)))$;

(5) $0.55 - \left(\left((-0.35) - \left(+\frac{1}{16}\right) - (-1.9)\right)\left(\left(-\frac{19}{24}\right) - \left(-1\frac{35}{48}\right)\right)\right)$;

(6) $\frac{1}{6} - \left(\left(\left(+\frac{37}{60}\right) - \left(-\frac{7}{25}\right) + (-0.85)\right) - ((-0.73) - (-0.97))\right)$;

(7) $(-15)+(-20)-(+11)-(-40)+(-3.5)+(+3.5)$.

(b) Compute:

(1) $a+b-(c-|d.|)$ if $a=-5.2$; $b=7.3$; $c=-6.8$; $d=-3.2$;

(2) $|2a-3b|+\left|\frac{1}{2}b-a\right|-7$ if $a=-4$; $b=-1$;

(3) $\left|7-\frac{a}{2}\right|-\left|\frac{3a}{2}+7\right|+|-4-a|+11$ if $a=-6$.

2.10. Evaluate the given expression:

(1) $3|m| + 2|m| + 4|m| - 4|-m| - |m|$ for $m = -2$;

(2) $4|x| - 2|a| + 6|x| - 3|a| + 4|a| - |-2| - |a| + 6|-x|$ for $a = 0.03$; $x = -0.15$;

(3) $|7a - 3b - 6a + 3b|$ if $a = -5$; $b = -1$;

(4) $\frac{|a| \cdot |b|^3}{c}$ if $a = -3$; $b = -2$; $c = 3$;

(5) $\frac{xy^2}{|z|}$ for $|x| = 3$; $|y| = 1$; $z = -3$.

2.11. Compare the following:

(1) a and $|a|$; a and $-|a|$; a and $|-a|$ if $a = 2$; -3; 0.25;

(2) x^2 and $|x|^2$ if $x = -4$; -1; 0.3; $3/4$; 3.

2.12. Verify the given equality:

(1) $|ab| = |a| \cdot |b|$ for $a = -5$; $b = 4$;

(2) $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$ for $a = 0$; $b = 0.75$.

2.13. Verify the following inequalities:

(a) $|a + b| \leqslant |a| + |b|$ if:

(1) $a = 10$; $b = -3$; (2) $a = -11$; $b = 4$;
(3) $a = 2.1$; $b = 0.9$; (4) $a = -1.2$; $b = -0.8$;
(5) $a = 0$; $b = -3$; (6) $a = 0.2$; $b = 0$; (7) $a = b = 0$;
(8) $a = 2.3$; $b = -2.3$;

(b) $|a - b| \geqslant |a| - |b|$ if:

(1) $a = b = 8$; (2) $a = -3$; $b = -7$; (3) $a = 0$; $b = 7$;
(4) $a = b = -7$.

2.14. What can be said about the numbers x and y if:

(1) $x^2 = y^2$; (2) $|x| = |y|$; (3) $|x| + |y| = 0$;
(4) $x^2 + y^2 = 0$; (5) $|x + y| = 0$?

2.15. Determine for what values of a and b the following equalities hold true:

(1) $\left|\frac{a-b}{b-a}\right| = 1$; (2) $\left|\frac{-a-b}{a+b}\right| = 1$.

2.16. Given $|a| \cdot |b| = 35$, find a, if $b = 7$.

2.17. Find the product:

(1) $(-2)\cdot(-4.5)\cdot(-5)$; (2) $(+1.25)\cdot\left(-4\frac{1}{20}\right)\cdot(-8)$;
(3) $(-8.24)\cdot(-10)\cdot(-0.1)$; (4) $(-5/6)\cdot(-2.4)\cdot(+3/5)$;
(5) $(-6)\cdot(+12.5)\cdot(-0.04)$; (6) $(-7.3)\cdot(-0.225)\cdot(-8)$;
(7) $(-8.24)\cdot(-210)\cdot(-0.1)$; (8) $(+16)\cdot(-18)\cdot(+0.25)$;
(9) $(-6)\cdot(+25)\cdot(+0.04)$; (10) $(17.3)\cdot(-0.125)\cdot(-3)$.

2.18. Perform the indicated operations:

(1) $(-5)\cdot(-4)+(+3)\cdot(-2)$; (2) $(+12)\cdot\left(+\frac{3}{4}\right)-(-15)\cdot\left(-1\frac{1}{5}\right)$;
(3) $(-3/8)\cdot(-16)+(+0.5)\cdot(-5)\cdot(-4)$;
(4) $(-1)-\left(-5\frac{1}{2}\right)\cdot(+4/11)$; (5) $((+10)-(-3))\cdot(-6)$
(6) $((-3)\cdot(-4)-(+5))\cdot((-8)-(+2)\cdot(-6))$.

2.19. Find the quotient:

(1) $(-8)\div((-3)+(+5))$; (2) $(-8)\div(|-3|+(+5))$;
(3) $\left(\left(-1\frac{1}{12}\right)+\left(-2\frac{1}{2}\right)\right)\div(-2)$; (4) $\left(\left(-1\frac{1}{2}\right)+\left(-2\frac{1}{2}\right)\right)\div(-2)$;
(5) $(-12)\div((+3)+(-15))\div(-5)$;
(6) $((-12)-(+3))\div((-2)-(+3))$;
(7) $((-25)+(-20))\div((-6)+(-2))$;
(8) $((+24)-(-3))\div((+6)-(-3))$;
(9) $((+29)-(-1))\div((+8)-(-2))$; (10) $\left(-3\frac{4}{5}-\left(+\frac{1}{5}\right)\right)\div(-5.5-(+14.5))$

2.20. (1) Can the sum of two negative numbers be greater than their quotient?
(2) Can the sum of two negative numbers be less than their quotient?

2.21. (1) May we say that if $a > b$, then always $1/a < 1/b$?

(2) May we say that if $a < 1$, then $1/a > 1$?

(3) Can the sum of two negative numbers be greater than their product?

2.22. Perform the indicated operations:

(1) $(((-1.75) - (-3.25)) \cdot 0.48 + (-0.3)) \div 0.028$;

(2) $(((+3.28) - (-1.52)) \div (-24) + (-1.3)) \cdot (-0.04)$;

(3) $((+0.45) + (-3.6) \cdot \left(\left(\frac{-3}{8} - \left(-\frac{7}{12}\right) + \left(-\frac{1}{18}\right)\right)\right) \div (-0.01)$;

(4) $\left(\left(\left(-\frac{7}{15}\right) - \frac{11}{18} - \left(-\frac{17}{45}\right)\right) \div (-0.015) + 18.5\right) \cdot (-1.2)$;

(5) $\left(1.6 - \left(+2\frac{1}{6}\right) - \left(-\frac{41}{90}\right)\right) \cdot \left(-3\frac{3}{5}\right) + 0.25 \div (-1.25)$;

(6) $-3.25 \div \left(-5\frac{1}{6}\right) + 6.75 \cdot \left(\frac{47}{60} - \left(+2\frac{17}{45}\right)\right) - (-1.65)$;

(7) $(-3.96) \div 5\frac{1}{2} - 2.4 \cdot \left(\left(\frac{5}{42}\right) - 1\frac{1}{28} - \left(-1\frac{19}{70}\right)\right)$;

(8) $(-0.96) \cdot (-15) - (-1.52) \div \left(\frac{47}{66} + \left(-\frac{7}{55}\right) - \left(+\frac{79}{110}\right)\right)$;

(9) $\left(+3\frac{1}{11}\right) \cdot \left(-1\frac{5}{17}\right) - \left((-3) \cdot \left(-5\frac{1}{3}\right) - (+1) \div \left(+\frac{1}{16}\right)\right) + (+5.12) \div (-1.28) \cdot (-8.3 + 0.35 - 2.05 + 7)$;

(10) $(-1) \cdot (24) \cdot \left(+\frac{3}{4}\right) - (-16) \cdot \left(-\frac{3}{11}\right) + (-2) - 1$;

(11) $(-36) \div ((-20) \div (-0.8) + (-2.5) \cdot (+14)) - \frac{1}{2} + \frac{1}{6} \cdot (6 + (-4) + (-8))$;

(12) $(-2.5)\cdot(-4)\cdot(-1)+(-2)^3\cdot(+5)^3\div(-1000)-(2.4)\cdot\left(+\frac{5}{12}\right)$.

2.23. Evaluate the given expression:

(1) $(x^2-4)\div(x+2)$ if $x=-0.3$;

(2) $\frac{(|a|+|b|)(b+c)(|c|+d)}{|d|+e}$ if $a=0$; $b=-1$; $c=-2$; $d=-3$; $e=4$;

(3) $(b+3)\div(a-2)\cdot(-4)$ if $a=-5$; $b=6$;

(4) $m-(m-n)\div(-2)\cdot(-5)$ if $m=-4$; $n=-6$;

(5) $(-1)\cdot(p-(-5)\cdot(-q))+(p+q)\cdot(-2)$ if $p=-3$, $q=7$;

(6) $x\div(y-1)\cdot(-4)-(xy+(-3))\div(-1)$ if $x=-5$, $y=-2$.

2.24. Verify the following equalities:

(1) $-\frac{1}{12}-\left(((+0.73)-(-0.37))-\left(\left(+\frac{5}{42}\right)-\left(+\frac{17}{35}\right)-\left(\frac{-11}{20}\right)\right)\right)=-1$;

(2) $\frac{1}{2}-\left(((+0.46)-(+0.16))+\left((+0.35)-\left(-\frac{5}{36}\right)-\left(+\frac{14}{45}\right)\right)\right)=\frac{1}{45}$;

(3) $(2.4-(0.3-3.21))\cdot 2+0.44\div(-2)\div\frac{2}{5}=20$;

(4) $3\div\left(-\frac{3}{5}\right)-\left(-\frac{4}{5}\right)\div 2+5\cdot\left(0.4-\frac{2}{5}\div(-2)+(-2)\div(-1)\right)=\frac{2}{5}$.

2.25. Find x if:

(1) $\left(-3\frac{3}{35}\right)\div\left(-\frac{2}{3}\right)\cdot x=7.2\div\left(-1\frac{5}{9}\right)$;

(2) $x\div\left(\left(-6\frac{2}{3}\right)\div(-2)\right)=(-3.18)\div 5.3$;

(3) $\left(-\left(-5\frac{7}{18}\right)-4\frac{23}{30}\right)\div(-1.62)\div\left(-1\frac{1}{9}\right)=x\div(3.2-(-0.8))\cdot(5.5-3.25)$.

§ 17. Involving (Raising to a Power) Rational Numbers

2.26. Involve the following numbers: $(3/4)^2$; $\left(3\frac{1}{3}\right)^2$; $\left(3\frac{3}{4}\right)^2$; $(-5/6)^2$; $(-2)^3$; $(-0.12)^2$; $(-1)^2$; $-(-1)^2$; $(-2/13)^2$; $(-2/3)^3$; $(-3)^4$; $(-3/4)^3$; $(-0.3)^3$; $(-0.2)^4$; $(-0.6)^4$; $(-0.4)^4$; $(1/2)^2$; $(-1/2)^3$; $(-1/2)^4$.

2.27. Arrange the given numbers in ascending order:

(1) $(-0.4)^3$; $(-1.5)^2$; $(+1/7)^3$; (2) $(+1.5)^2$; $(+0.8)^3$; $(-1.1)^2$;

(3) $\left(-1\frac{2}{5}\right)^2$; $(-3/8)^3$; $\left(-1\frac{1}{2}\right)^4$; $\left(-1\frac{2}{3}\right)^3$; $\left(-1\frac{3}{4}\right)^4$; $(-1.2)^2$;

(4) $\left(-1\frac{5}{8}\right)^2$; $(-2/9)^3$; $(-3/5)^3$; $\left(-\frac{3}{4}\right)^4$; $\left(-1\frac{1}{4}\right)^3$ $\left(-1\frac{1}{3}\right)^4$.

2.28. Arrange the given numbers in descending order:

(1) $\left(-1\frac{1}{3}\right)^3$; $(-1.8)^2$; $(-3/7)^3$; $\left(-1\frac{2}{9}\right)^2$; $(-0.9)^2$; $(-1.75)^3$;

(2) $(-3/4)^3$; $(-2/5)^2$; 0.3^2; 3) $(-3/2)^3$; $(-2/3)^3$; $(-2/3)^4$; $\left(-1\frac{2}{3}\right)^3$;

(4) $(-1.2)^3$; $(-5/9)^3$; $(-0.6)^2$; $\left(-1\frac{1}{3}\right)^2$; $(-0.25)^3$; $\left(-1\frac{2}{3}\right)^2$.

2.29. Represent the following fractions in the form of powers with a negative exponent:

(1) (Orally). 1/8; 1/27; 1/64; 1/625; 2/343; 4/243; 1/81; 1/225; 1/169; 1/729; 5/128; 3/125; 8/243; 9/361; 125/216; 256/625; 441/3364; 216/343; 1089/1024; 1225/1600;

(2) 0.1; 0.01; 0.001; 0.000001; 0.000000001; 0.125; 0.0625; 0.0081;

(3) $\frac{1}{x^2}$; $\frac{2}{x^5}$; $\frac{a^2}{3}$; $\frac{x^3}{y^2}$;

(4) $\frac{1}{a^3}$; $\frac{1}{a^n}$; $\frac{x^3}{y^5}$; $\frac{a^{2n}}{b^n}$; $\frac{a^{n^2-n}}{b^{2n}}$; $\frac{x^{n+2}}{y^{7n}}$ if $n>2$.

2.30. Perform the indicated operations:

(a) (1) $3^m \cdot 3$; (2) $2^n \cdot 8$; (3) $7^{m+1} \cdot 49$; (4) $a^{2m} \cdot a^{m-1}$;

(5) $a^m \cdot a^{3-m}$; (6) $a^{m+1} \cdot a^{1-m}$; (7) $\left(\left(\frac{m}{n}\right)^{-3}\right)^{-1}$;

(8) $\left(\left(\frac{2m}{3n}\right)^{-4}\right)^{-2}$;

(b) (1) $x^{2m} \div x^{m+n}$; (2) $a^{2n+1} \div a^{-3}$; (3) $a^{5+m} \div a^{m-1}$;

(4) $a^{m-3} \div a^{m-8}$; (5) $a^{m-4} \div a^{m+5}$; (6) $a^{2n-3} \div a^{2n-4}$;

(7) $x^{n^2-1} \div x^{n^2-5}$; (8) $y^{mn-1} \div y^{mn-6}$.

2.31. Evaluate the following expressions:

(a) (1) $2a^3 - 3a^2 - 1$ for $a = -2$;

(2) $a(2a - b)^3 - 3a(a - 2b)^2$ for $a = 2$, $b = -2/3$;

(3) $16x^4 - 4x^3 - 2x^2 - 4x^{-3}$ if $x = -1\frac{1}{2}$;

(4) $\frac{1/2 - a^{-1}}{4 - (1/a)^{-2}} \div \left(\frac{1}{2^{-2}(2+a)} - 2a^{-1} - 1\right)$ if $a = -1$;

(5) $(2/(-a)^3)^2 + (-(2/a)^2)^3 + (2/a^2)^3 - 2(-2/a^3) - ((1/2) \times (-2/a)^2)^3$ if $a = -2$;

(6) $\frac{1+a+a^2}{1+a-a^2}$ if $a = 1/2$;

(7) $2a^3 + 3a^2 - 5a + 6$ if $a = -1/2$;

(8) $\frac{a^2-2a+1}{a+1}$ if $a = -3$;

(b) (1) $\frac{2a+b^2-3}{2b-a}$ if $a = 2$, $b = 9$;

(2) $\frac{2x^4-3y^3}{1-x^2}$ if $x = -1/2$, $y = -1/3$;

(3) $3(a^2 - b^2) - 4(a + b)$ if $a = -0.5$, $b = 0.1$;

(4) $2(c-d)^2-3(c+d)(c-d)$ if $c=2\frac{1}{2}$, $d=-1\frac{1}{2}$;

(5) $5(t+u)^3-(t+u)^2+3(t+u)$ if $t=-1.2$, $u=2.5$;

(6) $\frac{2k^2-4k-1}{k^2+k+1}$ if $k=-3/4$;

(7) $\frac{3a^2-2ab-4b^2}{2a^3b^2-1}$ if $a=-\frac{2}{3}$, $b=1\frac{1}{2}$;

(8) $\frac{x^3+x^2-x-1}{x^2-1}$ if $x=-0.75$;

(c) (1) $\frac{3a^2-b^3}{3a-b(a+4b)}$ if $a=3$, $b=-2$;

(2) $\frac{(4b-x)b+x}{x^2+b^2}$ if $b=-3$, $x=5$;

(3) $\frac{(x-2y)x+y^3}{2x^2-3y^2}$ if $x=5$, $y=-4$;

(4) $\frac{a^3-b^2(a+b)}{b^3+a^2}$ if $a=-3$, $b=-2$;

(5) $\frac{(m-2n)\cdot m+n^2}{m^2-n(2m+n)+m^3}$ if $m=-1/2$, $n=1.5$.

2.32. Evaluate the following expressions:

(a) (1) 2^{-2}; (2) 3^{-2}; (3) 2^{-3}; (4) 8^{-1}; (5) $(1/2)^{-2}$;

(6) $(2/3)^{-1}$; (7) $(5^2\cdot 5^{-1}-8^0)/2^{-2}$; (8) $\frac{4^{-1}-3\cdot(2/3)^{-2}}{5-(1/2)^{-1}}$;

(9) $\frac{2^3-(3/4)^4\cdot(-1/2)^{-2}}{10^0+(-1/8)^0}$; (10) $\frac{2^{-3}-(3/4)^{-4}(-1/2)^2}{10^{-1}+(-4/5)^{-1}}$;

(b) (1) $((2/3)^{-1}-3/4)^{-1}$; (2) $\frac{(2/3)^{-2}+4^{-1}}{(1/2)^{-1}+5}$;

(3) $\frac{2^{-3}+(3/4)^{-4}\cdot(-1/2)^2}{(1\ 5)^0-12\cdot 3^{-3}}$;

(4) $\frac{(0.6)^0-(0.1)^{-1}}{(3/2^3)^{-1}\cdot(3/2)^3+(-1/3)^{-1}}$;

(5) $\frac{(1/2)^{-4}-5\cdot(-2)^{-2}+(2/3)^{-2}}{2^{-2}+1^0}$.

§ 18. Addition and Subtraction of Monomials and Polynomials

2.33. Collect like terms:

(1) $15ab + 4ab - 10ab$; (2) $-6xy - xy + 8xy$;

(3) $\left(-\frac{3}{4}\,ab\right) + \left(+\frac{2}{3}\,a^2b\right) + (+ab) - \left(+\frac{5}{6}\,a^2b\right) + \left(-\frac{1}{2}\,ab\right)$;

(4) $(-0.3ab) + (-0.2a^2) + (+1.4b) + (-5a^2) + (-2.3ab)$;

(5) $(-9.387m - 3.89n + 8.197m - 1.11n - 0.002m$;

(6) $5a^2 + (-2a^2) + (-4a^2)$; (7) $(-8xy) + (+10xy) + (-3xy)$;

(8) $3a^2b + (-a^2b) + (+2a^2b) + (-6a^2b)$;

(9) $(-7y^2) + (-4y) + (-y^2) + (+5y) + (-8y^2)$;

(10) $2d^2 - 1\frac{1}{2}\,d^2 - 3\frac{1}{2}\,d^2$;

(11) $\frac{3}{4}\,a^5 - \frac{1}{2}\,a^5 - \frac{5}{8}\,a^5$;

(12) $5ab - 4a^2b^2 - 8ab^2 + 3ab - ab^2 - 4a^2b^2$;

(13) $23a^2bc + 10abc^2 - 15a^2bc - abc^2 + 2a^2bc + abc^2$;

(14) $-1\frac{2}{3}\,ab^3 + 2a^3b - 4\frac{1}{2}\,a^2b - ab^3 - \frac{1}{2}\,a^2b - a^3b$;

(15) $\left(-\frac{1}{2}\,xy^2\right) + \left(-\frac{3}{8}\,x^2y\right) + \left(+\frac{3}{4}\,x^2y\right) + \left(-\frac{7}{8}\,xy^2\right) + \left(+\frac{1}{2}\,xy^2\right)$;

(16) $(+3pq) + (-4.2p^2) + (+0.3p^2) + (+2q) (-3q)$.

2.34. Perform the indicated operations:

(1) $(x^2 + 2xy + y^2) + (2xy - x^2 - y^2)$;
(2) $(5m^2 - 5m + 3) + (-4m^2 - 5m - 3)$;
(3) $(2y^2 - 4y - 1) + (-1 + 4y - 2y^2)$;
(4) $(10a - 6b + 5c - 4d) + (9a - 2b - 4c + 2d)$;

(5) $(2a^4 + 5a^3b + 2a^2b^2 - 6ab^3) + (3a^4 - 8a^3b + 2a^2b^2 - ab^3)$;
(6) $(+5a^n) + (-2a^n) + (-8a^{n+1}) + (+6a^n) + (-a^{n+1})$;
(7) $(3a^{n+1} + 10a^n - 7a) + (a - 9a^{n+1} - 10a^n)$;
(8) $12.5x^2 + y^2 - (8x^2 - 5y^2 - (-10x^2 + (5.5x^2 - 6y^2)))$;
(9) $1.7a^2 - 10b^2 - (a^2 - 3b^2 - (4.3a^2 - (2a^2 - 7b^2)))$;
(10) $0.6ab^2 + (2a^3 + b^3 - (3ab^2 - (a^3 + 2.4ab^2 + b^3)))$;
(11) $a^2 - b^2 - (3ab - 2b^2 - (a^2 + 2ab - (b^2 - ab)))$.

2.35. Find x and y:

(a) (1) $(17 - 3x) - (5 + 2x) = -3$;
(2) $31 + (25 - 7x) - (11 - 2x) = 0$;
(3) $(5x - 7) - (8 - 2x) = 6$;
(4) $(14 - 14x) - (8x - 13) = 16$;
(5) $(y^2 - 1.8y + 17.7) - (y^2 - 3.5y - 19.3) = 20$;
(6) $(1.6x + 31.7) - 0.9x + 26.3 = -0.3$;
(7) $23.7 - (0.6x - 12.2) - 0.8x = 12.9$;
(8) $(5x + 3) - (2x - 4) = (x - 2) - (x + 3)$;
(9) $(4 - 2y) - (5 - 3y) = (y - 1) - (y - 8)$;
(10) $8x - (2x - 9) = 4x - (8 + 4x)$;
(11) $(3x + 1) + (5x + 8) - (x - 4) = 48$;
(12) $\left(\frac{1}{2}x + \frac{1}{4}\right) - \left(-2x - \frac{1}{2}\right) - \left(\frac{2}{3}x - \frac{3}{4}\right) = \frac{5}{6}$;
(13) $\left(\frac{3}{4}x - \frac{2}{5}\right) + \left(\frac{2}{3}x + 0.6\right) - \left(\frac{7}{12}x - 0.3\right) = 5.8$;
(14) $(0.4x - 2) - (1.5x + 1) - (-4x - 0.8) = 3.6$;

(b) (1) $(3x - 4b) + (7b + 2x) = 13b$;
(2) $(13k + 10x) + (-8x - 9k) = 12k$;
(3) $(5x - 7a) + (-2x + a) = 3a$;
(4) $(2x - 4t) + (4x + 5t) = 19t$;
(5) $(5x - 3a) - (2x + 5a) = 4a$;
(6) $(x + 5b) - (3b - 2x) = 17b$;
(7) $4x - (3m - x) + (8x - 5m) = 5m$;
(8) $(x + a) + (x + 2a) - (x - 3a) = 8a$;
(9) $x^2 - (x + t) - (x^2 - 2x - 3t) = 0$;

(10) $(6x - 4m) - (2x^2 + x) + (2x^2 - m) = 0$;
(11) $(5x^2 + 2x - k) - (3k - 2x + 5x^2) = 0$;
(12) $(x - a - b) + (2x + 3a + b) = (2a - b) - (2a - 5b)$.

§ 19. Multiplication of Monomials and Polynomials. Division of Monomials

2.36. Perform the indicated operations:

(a) (1) $4a^2b^3 \cdot \frac{2}{3}b^4\ (-3b^3c^2)$; (2) $2ab^2 \cdot (-3a^2b)$;
(3) $4.4x^{n-2} \cdot 5ax^2$; (4) $(+3ab) \cdot (-2a^2b)$; (5) $(-8x^2y) \cdot (2xy^2)$;
(6) $\left(+\frac{2}{3}c^3d^2\right) \cdot \left(-\frac{3}{4}c^2d\right)$; (7) $(-m^2n^2) \cdot \left(+\frac{5}{6}m^3n\right)$;
(8) $(-0.6x^2y^3) \cdot (+0.5x^3y^3)$; (9) $(+2.4k^2b^4) \cdot (-0.5k^3)$;
(10) $(-8a^3b^2c) \cdot (-2ab^2c^3)$;
(11) $\left(-1\frac{1}{2}x^2y^3z\right) \cdot \left(-1\frac{1}{3}xy^2z^3\right)$;
(12) $\left(+1\frac{1}{4}a^2b^2c^3d\right) \cdot \left(-\frac{2}{5}a^3bc^2\right)$;
(13) $(-2.5m^3n^2p) \cdot (-3.4m^2n^3pq^2)$;
(14) $(+3a^n) \cdot (-4a)$; (15) $(-5x^{n+1}) \cdot (-2x^2)$;
(16) $(+4m^2n) \cdot (-6m^{k-1}n^{k+1})$;
(b) (1) $(-0.4a^nb^m) \cdot (-0.8a^{n+1}b^{2m})$;
(2) $\left(-\frac{2}{3}x^{k-1}y^2\right) \cdot \left(+\frac{3}{4}xy^{k+1}\right)$;
(3) $(-8a^mx^{n+1}y^n) \cdot \left(-\frac{1}{2}a^{3-m}x^{n-1}y^2\right)$;
(4) $2ab^2 \cdot (-3a^2b) - (-2.5a^{n-3}b^{n+1}) \cdot 6a^{6n}b^{2-n}$;
(5) $7a^3x\ (2a^3 - 3a^3x + 4ax^3) + 7a\ (3a^{2+n}x^2)$;
(6) $ay\ (2a - y) - (a^3 - (y^2\ (a - 3y) - a^2\ (a + 2y)))$;
(7) $a^2 \cdot (c - 3a) - (c^2\ (a + 3c) - (c\ (3c + ac - a^2) + 2a^2)))$;
(8) $2m\ (10m - 3n) - 5\ (n\ (5m + 3n) - (3n^2 - m\ (4m - 6n)))$;
(9) $-xy\ (x + y) - y\ ((3y^2 - x\ (5y + 2x)) - y\ (3y - 4x)$;
(10) $ab\ (a + b) - a\ (b\ (3b - 2a) - (a^2 - b\ (3a - 2b)))$;
(11) $a^2\ (c - 3a) - (c^2\ (a + 3c) - (c\ (3c + ac - a^2) + 2a^3))$.

2.37. Divide:

(a) (1) $a^5 \div a^3$; (2) $x^5 \div x^2$; (3) $m^7 \div m^4$;
(4) $c^5 \div c^4$; (5) $m^6 \div m^2$; (6) $p^8 \div p^4$; (7) $y^3 \div (-y)$;
(8) $-z^7 \div z^5$; (9) $t^{10} \div (-t^8)$; (10) $n^{10} \div n^9$;
(11) $b^8 \div b^3$; (12) $a^4 \div (-a^4)$; (13) $a^m \div a^n$; (14) $x^m \div x^m$;
(15) $-b^{2k} \div b^k$; (16) $a^{x-1} \div a$; (17) $a^{m+1} \div a^m$;
(18) $c^{2n+1} \div c^{n+1}$; (19) $x^{n+1} \div x^{n-1}$; (20) $y^{2n+2} \div y^{n+2}$;
(21) $8a^2b \div 2ab$; (22) $-9ab^2 \div 3ab$; (23) $16x^3y^2 \div 4x^2y$;
(24) $20m^4n^3 \div 5m^2n^3$; (25) $4a^2b^2c \div (-5abc)$;
(26) $-6a^3b^2c \div (-2a^2bc)$;

(b) (1) $\left(-\frac{2}{5}\, a^4x^3y^2\right) \div \left(-\frac{1}{2}\, a^3xy^2\right)$;

(2) $\left(-\frac{3}{4}\, a^5b^3c\right) \div \left(1\,\frac{1}{2}\, a^2b^2c\right)$;

(3) $0.4x^7y^5z^4 \div (-0.5x^3y^5z^2)$; (4) $(-1.2a^8b^3c^4) \div (-0.3a^5bc^4)$;

(5) $0.5a^mb^nc^3 \div \left(-\frac{2}{3}\, a^2bc\right)$; (6) $1.5x^{m+1}y^{n-1} \div 3x^{m-1}y^{n-2}$;

(7) $0.12x^{n-2}y^{n-1} \div (-0.6x^{n-3}y^{n-2})$;

(8) $1.8a^{n+3}b^{n+2}c^{n+1} \div 0.9a^{n+1}b^{n-1}c^n$.

2.38. Simplify:

(1) $(7 + 3a^2 - 3a)\cdot(-2a + 5 - a^2)$;
(2) $(5ab^2 - 3a^3 + 2a^2b)\cdot(-ab + 2a^2 - 4b^2)$;
(3) $(3m^2 + 4n^2 - 2mn)\cdot(-mn - n^2 + 5m^2)$;
(4) $(2a^2 - 5b^2 - 3ab)\cdot(ab^2 + 3a^3 - 2a^2b)$;
(5) $(5m^2 - 3m^3 + 4m - 1)\cdot(3 - 2m^2 - 6m)$;
(6) $(-2 + 4x^3 - 5x - 3x^2)\cdot(-1 + 6x^2 + x)$;
(7) $(2a - 3b^3 + 9b\,(4a^2 + 3b^2)) \div 8a - 6\frac{3}{4}b$;
(8) $[5x\,(25x^2 + 12y^2) - (125x^3 - 150x^2y + 60xy^2 - 8y^3)] \div 1\frac{1}{3}y^2$;
(9) $(8a^3b^3 - 12a^2b^4 + 6ab^5 - b^6) + (9a^2b^4 - 6ab^5 + b^6) \div 2a^2b^3 + 1.5b$;
(10) $(64a^6 + 240a^5b + 300a^4b^2 - 125a^3b^3) - (64a^6 - 240a^5b + 225a^4b^2) \div 25a^3b^2 + 5b$.

2.39. Find the product:

(1) $(ac + bc)\cdot(c + 2d)$; (2) $(x^2 + 2xy - 5y^2)\cdot(2x^2 - 3y)$;
(3) $(a^2 - 5ab + 3b^2)\cdot(a - 2ab)$;
(4) $(2b^2 - 2 - 5b + 6b^3)\cdot(3 + 3b^2 - b)$;
(5) $(x - a)\cdot(x - b)\cdot(x - c)$;
(6) $(x^2 + x + 1)\cdot(x^2 - x + 1)\cdot(x^2 - 1)$;
(7) $(-a + b + c)\cdot(a - b + c)\cdot(a + b - c)$;
(8) $(a^3 + a^2b + ab^2 + b^3)\cdot(a - b)$;
(9) $(a^4 + a^3b + a^2b^2 + ab^3 + b^4)\cdot(a - b)$;
(10) $(a^4 - a^3b + a^2b^2 - ab^3 + b^4)\cdot(a + b)$.

2.40. Simplify the given expression and evaluate it:

(1) $bc\,(11c - 7b) - ((b - 2c)\cdot(b^2 - 5bc + c^2) + c^3)$ for $b = -1/2$, $c = 3/2$;
(2) $(p - 5n)\cdot(3p^2 + 2pn - 7n^2) - (13n\,(3n^2 - p^2) - 17pn^2)$ for $p = -3$, $n = -2$;
(3) $(p - 4q)\cdot(3p^2 + 2pq - 5q^2) - (12pq^2 - 10qp^2 - 2q^2)$ for $p = 2$, $q = -3/5$;
(4) $(-5x^2 + 4xy - 3y^2)\cdot(2x - y) - x\,(xy - 10\,(x^2 + y^2))$ for $x = -1/6$, $y = 3$;
(5) $(3k - 2n)\cdot(3n^2 + 2kn + 7k^2) - k\,(5\,(n^2 + 4k^2) + k^2)$ for $k = -2.5$, $n = -2$;
(6) $(a^2 - 3ab + 4b^2)\cdot(2a - 3b) - 6b^2\,(3a - 2b) - ab^2$ for $a = 1/2$, $b = -1/9$;
(7) $(2z^2 - 5yz - 3y^2)\cdot(z + 2y) - (2y^3 - y^2z - yz\,(z + 12y))$ for $z = -1/3$, $y = -1/6$.

2.41. Find x and y:

(a) (1) $(3x - 1)\cdot(2x + 7) - (x + 1)\cdot(6x - 5) = 16$;
(2) $3x^2 - (3x + 2)\cdot(x - 1) = 8$;
(3) $(3x - 2)\cdot(2x + 3) - (6x^2 - 85) = 99$;
(4) $(3y + 1)\cdot(4y - 5) - (6y - 11)\cdot(2y - 7) = 24$;
(5) $(x + 1)\cdot(x + 2) - (x - 3)\cdot(x + 4) = 6$;
(6) $(2x - 3)\cdot(3x - 1) - (6x - 2)\cdot(x - 5) = 25$;
(7) $3\,(x + 1)\cdot(x + 2) - (3x - 4)\cdot(x + 2) = 36$;

(8) $2(3x-1)\cdot(2x+5)-6(2x-1)\cdot(x+2)=48$;
(9) $3(-4x+1)\cdot(x-1)+2(6x-4)\cdot(x+3)=50$;
(10) $5(2x+3)\cdot(x+2)-2(5x-4)\cdot(x-1)=32$;

(b) (1) $7(2x-5)-(5(7x-2)-2(5x-7))=-72$;
(2) $(2x-25)-3x+(8x+5(6-x))=7$;
(3) $3(2x-5)-2((3x+4)-(4x-5))+(2(x-1)-3(2x-3))=2$;
(4) $10x-(6x-2(3x-4(1-x))-(9x+8))=27$;
(5) $2x+2(-(-x-3(x-3)))=2$;
(6) $\frac{3}{8}x-\frac{7}{8}-\frac{1}{6}(1-x)=\frac{5}{3}$;
(7) $\frac{1}{6}x-\frac{1}{\quad}-\frac{12}{9}x-1\frac{1}{9}=-2$.

§ 20. Involution of Monomials

2.42. Perform the indicated operations:

(a) (1) $(-6a^3x^2)^2$; (2) $\left(-\frac{1}{3}a^2x^2\right)^3$;
(3) $(-x^{n-1})\cdot(x^5)^{13-n}$;
(4) $(-4m^3n^5)^2\cdot\left(-\frac{1}{2}m^2n\right)^4$; (5) $(-p^{11-n})^3\cdot(p^3)^{n-2}$;
(6) $\left(-\frac{1}{9}a^2c^4\right)^2\cdot(3a^5c^2)^5$;
(7) $(22a^3b^{-1})^2\cdot\left(\frac{1}{11}a^{-2}b\right)^3$; (8) $(m^2)^4$; (9) $(-2x^3)^2$;
(10) $(-4a^2)^3$;
(11) $(-3y^4)^2$; (12) $\left(\frac{1}{2}b^2\right)^2$; (13) $(-0.3x^3)^3$;

(b) (1) $(ab^2c^3)^2$; (2) $5(x^2y)^3$; (3) $(-2a^2bc)^3$;
(4) $-3(ab^3c^2)^2$; (5) $3(-ab^2c^2)^2$; (6) $\left(-2\frac{1}{2}x^4y^3\right)^2$;
(7) $-\left(-1\frac{1}{2}a^2b^3c^4\right)^2$; (8) $-\left(-\frac{2}{3}x^3y^4\right)^3$;
(9) $-\left(-1\frac{1}{3}a^5b^6c^3\right)^2$; (10) $(-2.5m^2n^3)^2$;

(c) (1) $5\,(-a^3b^2c)^3$; (2) $-3\,(2a^2b^3)^2$; (3) $2\,(-3x^4y^3)^3$; (4) $-\frac{1}{2}(-5a^3b^4c^2)^2$; (5) $(a^k)^3$; (6) $(x^{n+1})^2$; (7) $(c^{2n})^3$; (8) $(a^{n-1})^3$; (9) $((-x^n)^2)$; (10) $(-x^n)^{12}$.

2.43. (a) Are the given numbers equal to each other: (1) $-x^2$ and $(-x)^2$; (2) $-x^3$ and $(-x)^3$; (3) $-(2a)^4$ and $(-2a)^4$; (4) $-(2a)^5$ and $(-2a)^5$?

(b) May we assert that for $a > 0$, $a^5 > a^2$, $a < a^3$?

(c) For what values of x are the following inequalities satisfied: (1) $x > x^2$; (2) $x^2 < x^3$?

§ 21. Formulas for Abridged Multiplication

The Square of a Sum of Two Expressions

2.44. Represent the square of a binomial in the form of a polynomial:

(a) (1) $(x + 10)^2$; (2) $(y^2 + 1)^2$; (3) $(8a^3 + 5a^2b)^2$; (4) $(a^2 + 1)^2$; (5) $(a^2 + 0.1)^2$; (6) $(x + bx^3)^2$; (7) $(3x + 2y)^2$; (8) $(m^2 + n^2)^2$; (9) $(b + 1/3)^2$; (10) $(a/4 + b/3)^2$; (11) $\left(2\frac{1}{3}m + 1\frac{1}{2}n\right)^2$;

(b) (1) $(0.3a^2 + 4b)^2$; (2) $(1.3m^2 + 2.5m^2)^2$; (3) $\left(1\frac{1}{2}x^2y^2 + \frac{5}{6}x^3y^3\right)^2$; (4) $\left(1\frac{3}{4}p^4q^2 + 1\frac{2}{3}p^3q^3\right)^2$; (5) $\left(1\frac{2}{5}a^5b^3 + 0.4a^3b^2\right)^2$; (6) $\left(\frac{4}{5}a^3b + 1\frac{1}{4}b^2\right)^2$;

(c) (1) $(a^m + b^n)^2$; (2) $(2x^m + 3y^n)^2$; (3) $\left(a^{n+1} + \frac{1}{2}b^2\right)^2$; (4) $\left(\frac{1}{2}a^{n-1}b^2 + a^{n+1}\right)^2$; (5) $\left(\frac{3}{5}a^{2n+1}b^2 + \frac{2}{3}a^{n-1}b^3\right)^2$; (6) $\left(\frac{3}{5}a^{n-1}b^2 + a^{n+1}\right)^2$; (7) $\left(\frac{2}{3}a^{2n+1}b^2 + \frac{2}{3}a^{n-1}b^3\right)^2$.

2.45. Represent the following polynomials in the form of a square of a binomial:

(1) $x^2 + 2xy + y^2$; (2) $a^2 + 6ab + 9b^2$; (3) $25b^2 + 20b + 4$;
(4) $a^4 + 18a^2b + 81b^2$; (5) $49a^2 + 84ab + 36b^2$;
(6) $a^2 + a + 1/4$; (7) $a^2 + \frac{2}{3}a + \frac{1}{9}$;
(8) $25a^2 + 20ab + 4b^2$; (9) $\frac{1}{4}a^2 + a + 1$;
(10) $\frac{1}{25}x^2 + \frac{2}{15}xy + \frac{1}{9}y^2$;
(11) $0.36x^2 + 0.72x + 1$; (12) $0.16x^2 + 0.24xy + 0.09y^2$.

2.46. Represent the following expressions in the form of the sum of squares of two expressions:

(1) $x^2 + b^2 + 2b + 1$; (2) $a^2 + 2a + 10$;
(3) $a^2 + 10a + 29$; (4) $b^2 + 6b + 10$; (5) $5x^2 + xy + \frac{1}{4}y^2$;
(6) $4a^2 + 12a + 10$; (7) $49a^2 + 42a + 25$;
(8) $\frac{1}{4}a^2 + a + 5$; (9) $0.25a^2 + 3a + 10$;
(10) $0.81a^2 + 5.4ab + 25b^2$; (11) $0.01a^2 + ab + 26b^2$;
(12) $0.09a^2 + 3ab + 34b^2$.

2.47. Replace the question-marks "?" by monomials so as to get a true equality:

(1) $(6a^5 + ?)^2 = ? + ? + 25x^2$;
(2) $(100m^5 + ?)^2 = ? + ? + 36m^4n^6$;
(3) $(5b^2 + ?)^2 = ? + ? + 49c^4$;
(4) $(? + ?)^2 = ? + 70b^3c + 49c^2$;
(5) $(3a^3b + ?)^2 = ? + ? + 25b^4$;
(6) $(? + ?)^2 = ? + 84ab + 49b^2$;
(7) $\left(-\frac{1}{2}a + ?\right)^2 = ? + \frac{1}{3}a + ?$;
(8) $(0.5x + ?)^2 = ? + 3xy + ?$;
(9) $(0.49a + ?)^2 = ? + 7ab + ?$; (10) $(? + 3a)^2 = ? + 12ab + ?$;
(11) $(? + ?)^2 = \frac{1}{25}x^2 + ? + \frac{1}{9}y^2$;

(12) $(?+?)^2=0.36x^2+?+\frac{1}{4}y^2$;

(13) $(?+0.2a)^2=?+ab+?$.

The Square of the Difference of Two Expressions

2.48. Represent the square of binomial in the form of a polynomial:

(a) (1) $(9-b)^2$; (2) $\left(1-\frac{1}{2}z\right)^2$; (3) $(3-5n)^2$;

(4) $(2xy-x^2)^2$; (5) $(a^3-b^2)^2$; (6) $\left(\frac{3}{5}x-\frac{5}{6}y\right)^2$;

(7) $(10x^2-3xy)^2$; (8) $(x^3-1)^2$; (9) $(x^3-y^3)^2$;

(10) $\left(a-\frac{1}{2}\right)^2$; (11) $\left(\frac{x}{2}-\frac{y}{3}\right)^2$; (12) $\left(3\frac{3}{8}k-1\frac{2}{3}t\right)^2$;

(13) $(0.2x^2-5y)^2$; (14) $\left(\frac{3}{4}a^2-0.5b^3\right)^2$;

(15) $\left(\frac{5}{6}a^2b^3-\frac{2}{3}a^3b^4\right)^2$; (16) $(1.2x^2y-0.5x^3y^2)^2$;

(b) (1) $(a^m-b^n)^2$; (2) $(2x^m-3y^n)^2$; (3) $(5x^2-2y^{n-1})^2$;

(4) $\left(\frac{2}{3}x^{m-2}-\frac{3}{4}x^{2n-1}\right)^2$; (5) $\left(\frac{5}{6}x^{m-1}y^n-\frac{3}{5}x^{n+1}y^2\right)^2$.

2.49. Represent the following expressions in the form of a square of a binomial:

(1) $c^2-2cd+d^2$; (2) $4a^2-28ab+49b^2$;

(3) $\frac{1}{4}c^2-cy+y^2$; (4) $a^4-4a^3+4a^2$;

(5) $x^4-6x^2y^3+9y^6$; (6) $a^6-26a^3b^2+169b^4$;

(7) $\frac{1}{100}a^2-0.2ab+b^2$; (8) $\frac{1}{36}x^2-\frac{1}{9}xy+\frac{1}{9}y^2$;

(9) $0.64x^2-8xy+25y^2$; (10) $0.49a^2-7ab+25b^2$;

(11) $0.81a^2-10.8ab+36b^2$; (12) $2.25a^2-21ab+49b^2$.

2.50. Represent the given expressions in the form of the sum of squares of two expressions:

(1) $4y^2-4y+1+p^2$; (2) $b^2-6b+13$;

(3) $a^4-6a^2b^2+25b^4$; (4) $x^4-2x^2y+5y^2$;

(5) $a^4 - 14a^2b + 53b^2$; (6) $a^8 - 10a^4b^2 + 29b^4$;
(7) $0.16a^2 - 4ab + 34b^2$; (8) $0.25x^2 - 3xy + 10y^2$;
(9) $0.64x^4 - 8x^2y^2 + 29y^4$; (10) $0.09a^2 - 6ab + 104b^2$;
(11) $0.49x^4 - 5.6x^2y^2 + 17y^4$; (12) $3.24a^2 - 18ab + 74b^2$;
(13) $1.21x^2 - 6.6xy^2 + 10y^4$.

2.51. Replace the question-marks "?" by monomials so as to get a true equality:

(1) $(? - ?)^2 = 81x^2 - ? + 100x^4y^6$;
(2) $(? - 4x^7)^2 = 25x^4y^2 - ? + ?$;
(3) $(8a^3 - ?)^2 = ? - ? + 49a^8b^6$;
(4) $(? - 2b)^2 = ? - 12ab + ?$;
(5) $(? - 5b)^2 = ? - 20ab + ?$;
(6) $(7c - ?)^2 = ? - ? + 4b^4$;
(7) $\left(\frac{1}{2}a - ?\right)^2 = ? - ? + \frac{1}{9}b^4$;
(8) $\left(\frac{1}{5}x^2 - ?\right)^2 = ? - ? + \frac{1}{4}y^2$;
(9) $\left(\frac{1}{8}x^3 - ?\right)^2 = ? - ? + 16y^6$;
(10) $\left(? - \frac{1}{7}y^2\right)^2 = \frac{36}{49}x^2 - ? + ?$;
(11) $\left(? - \frac{3}{4}b^3\right)^2 = \frac{1}{25}a^6 - ? + ?$;
(12) $(? - ?)^2 = ? - 7ab + 26b^2$;
(13) $\left(? - \frac{5}{6}a^n\right)^2 = 25a^{4n} - ? + ?$;
(14) $(? - ?)^2 = 0.09x^4 - 3x^2y^4 + ?$;
(15) $(? - ?)^2 = ? - 5.4a^3b^3 + 0.81b^6$;
(16) $(? - ?)^2 = 0.36a^8 - 6a^4b^4 + ?$.

The Difference of Two Squares

2.52. Represent the following products in the form of the difference of two squares:

(1) $(2a + 5)\cdot(2a - 5)$; (2) $(0.3x - 1)\cdot(0.3x + 1)$;
(3) $(8a^3 + 3b^3)\cdot(3b^3 - 8a^3)$; (4) $(9 - 2xy)\cdot(2xy + 9)$;

(5) $(a + b/3)\cdot(b/3 - a)$; (6) $(x/10 + xy/3)\cdot(x/10 - xy/3)$;
(7) $(a + 2b/3)\cdot(2b/3 - a)$; (8) $(2 + a)\cdot(2 - a)\cdot(4 + a^2)$;
(9) $(x + y)\cdot(x - y)\cdot(x^2 + y^2)$;
(10) $(a^2 - 4)\cdot(a^2 + 4)\cdot(a + 16)$;
(11) $(0.25a^4 + b^6)\cdot(0.5a^2 - b^4)\cdot(0.5a^2 + b^4)$;
(12) $(1.5a^3 + b^3/3)\cdot(2.25a^6 + b^6/9)\cdot(1.5a^3 - b^3/3)$.

2.53. Perform the indicated operations:

(1) $(3 + a)\cdot(3 - a)\cdot(3 - a)\cdot(3 + a)$;
(2) $(a + 2)^2\cdot(a - 2)^2$; (3) $(a + b + c)\cdot(a + b - c)$;
(4) $(x - y + z)\cdot(x - y - z)$; (5) $(a - b + c)\cdot(a + b - c)$;
(6) $(a + b + c)\cdot(a - b - c)$;
(7) $(a + b + c + d)\cdot(a + b - c - d)$;
(8) $(a - b + c + d)\cdot(a - b - c - d)$;
(9) $(x + 2y + 3z)\cdot(x - 2y + 3z)$;
(10) $(a + 2b + 4c)\cdot(a - 2b - 4c)$;
(11) $(x^2 + 2)^2 - (x - 2)\cdot(x + 2)\cdot(x^2 + 4)$;
(12) $2(m - n)^2 - 2(m + n)^2 - 4(m + n)\cdot(m - n)$.

2.54. Replace the question-marks "?" by monomials so as to get a true equality:

(1) $(?-15a)\cdot(?+?)=4c^2-?$; (2) $(?-10z^5)\cdot(?+?)=$
$0.49x^6-?$; (3) $(?+?)\cdot(7p^6-?)=?-\frac{16}{121}q^4$;
(4) $(?+?)\cdot(5a^4-?)=?-9b^6$;
(5) $\left(\frac{1}{3}a^3+?\right)\cdot(?-?)=?-36b^4$;
(6) $(?+?)\cdot(6a^8-?)=?-49b^8$;
(7) $\left(\frac{1}{5}a^5+?\right)\cdot\left(?-\frac{2}{5}b^3\right)=?-?$;
(8) $(?+0.1a^2b^2)\cdot(?-?)=0.25a^6-?$;
(9) $\left(\frac{a^n}{b^m}+?\right)\cdot\left(?-\frac{1}{a^n}\right)=?-?$;
(10) $\left(?+\frac{2}{a^{n-1}}\right)\cdot(?-?)=\frac{1}{25}-?$.

2.55. Solve the following equations:

(1) $(x+5)^2-(x-1)^2=48$;
(2) $3(x+2)^2+(2x-1)^2-7(x+3)\cdot(x-3)=28$;
(3) $(x+3)\cdot(x-3)-x(x-2)=1$;
(4) $(2x+5)\cdot(2x-5)-(2x-1)^2=4$;
(5) $(5-3x)\cdot(5+3x)+9(x+1)\cdot(x-1)+7x=-5$;
(6) $2(x+3)^2-2(x+5)\cdot(x-5)=4$;
(7) $(3x-7)\cdot(3x+7)-(3x-2)^2-11x=20$;
(8) $(0.5x-3)\cdot(0.5+3)-(x-1)\,x/4+5x=12$.

2.56. Represent the given products in the form of the difference of two squares:

(1) $a^2(x-y)\cdot(x+y)$; (2) $49(a+1)\cdot(1-a)$;
(3) $a(b-c)\cdot(ab+ac)$; (4) $(a+b+1)\cdot(a+b-1)$;
(5) $(1+x+y)\cdot(1-x-y)$;
(6) $225(a^2-b^2)\cdot(a^2+b^2)$;
(7) $324(2x^4-3y^4)\cdot(2x^4+3y^4)$;
(8) $121(0.5x^3-2y^2)\cdot(0.5x^3+2y^2)$.

2.57. Reduce the given fractions:

(1) $\frac{53^2-27^2}{79^2-51^2}$; (2) $\frac{53^2-32^2}{61^2-44^2}$; (3) $\frac{38^2-17^2}{47^2-19^2}$;

(4) $\frac{101^2-31^2}{139^2-29^2}$; (5) $\frac{137^2-23^2}{91^2-11^2}$; (6) $\frac{105^2-35^2}{161^2-21^2}$.

The Cube of a Sum of Two Expressions

2.58. Represent the following expressions in the form of a polynomial:

(1) $(x+y)^3$; (2) $(a+1)^3$; (3) $(x^2+5)^3$; (4) $(x^2+y^2)^3$;

(5) $\left(\frac{1}{3}ab^2+a^2\right)^3$; (6) $\left(b^3+\frac{2}{3}bc^2\right)^3$;

(7) $\left(\frac{4}{3}ab^4+a^2\right)^3$; (8) $(4x^3+5y^2)^3$;

(9) $(7p^3+9p^4)^3$; (10) $(0.5x+0.1y)^3$;

(11) $(0.2a+0.5b)^3$; (12) $\left(10x^3+\frac{1}{3}y^2\right)^3$;

(13) $\left(2a+\frac{1}{2}ab\right)^3$; (14) $(1.5m^3+0.3m^4)^3$;

(15) $(a^{m}+2)^3$; (16) $(0.3a^5+0.5a)^3$;

(17) $(a^n+a^{n-1})^3$; (18) $(x^{n+1}+x^{n+2})^3$.

2.59. Represent the given expressions in the form of a cube of a binomial:

(1) $m^3+n^3+3m^2n+3mn^2$;

(2) $x^3+6x^2y^4+12xy^8+8y^{12}$;

(3) $1000+300a+30a^2+a^3$;

(4) $x^6+15x^4+75x^2+125$;

(5) x^3+3x^2+3x+1; (6) $-b^3-12b^2-48b-64$.

2.60. Replace the question-marks "?" by monomials so as to get a true equality:

(1) $(?+2x)^3=z^6+?+?+?$;

(2) $(4x^5+?)^3=?+?+?+125x^6$;

(3) $(?+4a)^3=?+?+240a^5+?$;

(4) $(3+?)^3=?+108y^3+?+?$;

(5) $(5+?)^3=?+150a^4+?+?$;

(6) $64b^3+?+?+125c^3=(?+?)^3$;

(7) $8a^3+12a^2b+?+?=(?+?)^3$.

2.61. Represent the following expressions in the form of a cube of a binomial:

(1) $x^6(a+b)^3$; (2) $8c^9(a+b)^3$; (3) $x^3(2a+x)^3$;

(4) $a^6(b+c^2)^3$; (5) $8b^6(1+a^2)^3$; (6) $125a^{3n}(2+a)^3$;

(7) $64y^6(2x+3y)^3$; (8) $216a^6b^9(a^2+b^3)^3$;

(9) $\frac{8}{125}a^{12}\left(\frac{5}{a}+\frac{1}{2a}\right)^3$.

The Cube of a Difference of Two Expressions

2.62. Represent the following expressions in the form of a polynomial:

(1) $(2z-3)^3$; (2) $\left(3a^2-\frac{1}{3}a\right)^3$;

(3) $\left(\frac{2}{3}a-1\frac{1}{3}\right)^3$; (4) $\left(0.6ab^2-\frac{2}{3}a^6\right)^3$;

(5) $(n^2-0.4m^3n)^3$; (6) $\left(\frac{3}{4}a^2-\frac{2}{3}b^2\right)^3$;

(7) $(2m^2-3n^2)^3$; (8) $(2ab^2-0.3c)^3$; (9) $\left(\frac{2}{3}-x-3y\right)^3$;

(10) $(10a^4-6b^2)^3$; (11) $\left(\frac{1}{2}a-\frac{1}{3}b^2\right)^3$;

(12) $(x^2-y^2)^3$; (13) $(0.2m-0.1n)^3$; (14) $(3a^2-6a)^3$;

(15) $\left(0.1a^4-\frac{1}{2}a^3\right)^3$; (16) $(0.3x^m-0.2y^n)^3$;

(17) $(a^{2n}-b^m)^3$; (18) $(a^{m+n}-b^{n-m})^3$.

2.63. Represent the given expressions in the form of a cube of a binomial:

(1) $p^3-3p^2q+3pq^2-q^3$; (2) $a^3-12a^2+48a-64$;
(3) $x^3-6x^2y^4+12xy^8-8y^{12}$;
(4) $8a^3-36a^2b+54ab^2-27b^3$;
(5) $27a^3-27a^2b+9ab^2-b^3$;
(6) $125x^3-300x^2y+240xy^2-64y^3$;
(7) $27a^3-13.5a^2b+2.25ab^2-0.125b^3$;
(8) $\frac{1}{27}a^6-a^4b^2-3a^2b^4-27b^6$.

2.64. Replace the question-marks "?" by monomials to get a true equality:

(1) $(3-?)^3=?-108y^3+?-?$;
(2) $(?-4c)^3=?-144c+?-?$;
(3) $1000x^3-?+?-?=(?-3b)^3$;
(4) $?-?+60y^2-8y^3=(?-?)^3$.

2.65. Represent the given expressions in the form of a cube of a binomial:

(1) $8c^9(a-b)^3$; (2) $(a+x)^3\cdot(a-x)^3$;
(3) $(2a+1)^3\cdot(2a-1)^3$; (4) $125a^9b^6(a^2-b^2)^3$;
(5) $343a^3b^6(x^2-2y^3)^3$; (6) $64a^{6m}b^{3n}(a^n-3b^n)^3$.

The Sum and Difference of a Cube of Two Expressions

2.66. Represent the given expressions in the form of a product:

(1) $8c^3 + p^3$; (2) $27a^3 + 1$; (3) $a^6 + b^6$;
(4) $(a + b)^3 + c^3$; (5) $(x - 3)^3 + 64$;
(6) $(b - c)^3 - a^3$; (7) $(a - b)^3 - (b - c)^3$;
(8) $(5x + y)^3 - (3x - 2y)^3$.

2.67. Prove the following equalities:

(1) $(-a - b)^3 = -(a + b)^3$; (2) $(a - b)^3 = -(b - a)^3$;
(3) $(ax + ay)^3 = a^3 (x + y)^3$;
(4) $x^3 + y^3 = (x + y)^3 - 3xy (x + y)$;
(5) $(x + y) (x^2 + y^2) - x^3 - y^3 = xy (x + y)$;
(6) $a^6 - b^6 = (a - b) (a + b) (a^2 - ab + b^2) (a^2 + ab + b^2)$;
(7) $(a + b)^3 = a (a - 3b)^2 + b (b - 3a)^2$;
(8) $(x^3 + y^3)^2 - (x^2 + y^2)^3 + 3x^2y^2 (x + y)^2 = (2xy)^3$.

2.68. Multiply:

(1) $(p + 5) \cdot (p^2 - 5p + 25)$; (2) $(2b - 1) \cdot (4b^2 + 2b + 1)$;
(3) $(3a^3 - 4) \cdot (9a^6 + 12a^3 + 16)$;
(4) $(1 + m^2) \cdot (1 - m^2 + m^4)$;
(5) $(10x - 3y) \cdot (100x^2 + 30xy + 9y^2)$;
(6) $(a + 0.5) \cdot (a^2 - 0.5a + 0.25)$;
(7) $(4a + 5b) \cdot (16a^2 - 20ab + 25b^2)$;
(8) $(0.5a - 2b) \cdot (0.25a^2 + ab + 4b^2)$;
(9) $(7x^2 + 2) \cdot (-49x^4 + 14x^2 - 4)$;
(10) $(a^3 - 0.2) \cdot (a^6 + 0.2a^3 + 0.04)$.

2.69. Solve the following equations:

(1) $(x + 2) (x^2 - 2x + 4) - x (x - 3) (x + 3) = 26$;
(2) $6 (x + 1)^2 + 2 (x - 1) (x^2 + x + 1) - 2 (x + 1)^3 = 32$;
(3) $x^3 - (x - 4) (x^2 + 3x + 12) - x (x + 3) - 64 = 5$;
(4) $x^3 + (x + 3) (4 - x^2 - 12) - x (x + 2) + 27 = 6$.

§ 22. Factoring a Polynomial

Taking Out a Common Factor

2.70. Factor:

(a) (1) $13a^2 - 52b + 39$; (2) $12a^3b - 18ab^2 - 30ab^3$;
(3) $a^2bc + ab^2c - abc^2$; (4) $135a^{12}b^8 + 9a^{10}b^{11} - 36a^8b^{16}$;
(5) $72a^5b^4 - 54a^3b^5 + 36a^2b^6$;
(6) $-56c^7x^{10} + 42c^5x^{15} - 70c^4x^{20}$;
(7) $132x^9z^7 + 165x^8z^5 - 99x^5z^4$;
(8) $195p^6q^5 - 91p^5q^6 + 221p^3q^{10}$;
(9) $288c^{13}x^5 - 126c^{10}x^8 - 198c^7x^{10}$;
(10) $399b^4c^{10} - 114b^3c^{15} + 95b^2c^{20}$;
(11) $621x^9a^{12} + 135a^{10}x^{12} + 108a^8x^{15}$;
(12) $85m^5n^4 - 34m^3n^3 + 17m^2n^2$;
(13) $21x^2y^3 - 42x^3y^2 + 14xy$;
(14) $5a^{6n} + a^{18n-1}$; (15) $a^{2n-1}b^3 + a^{n+3}b^{12}$;
(16) $x^{2n+1}y^3 + x^{n-1}y^{15}$; (17) $8^{3n+1} - 8^{2n-1}$;
(18) $9^{2n+1} - 9^{n-1}$;

(b) (1) $5a(2b - 3) - 3b(2b - 3) + (3 - 2b)$;
(2) $5a(x + c) + b(x + c) - 8(x + c)$;
(3) $x(y - 1) - 3(y - 1) - (y - 1)^2$;
(4) $12a(b - a) - 18b(a - b) + 24(b - a)^2$;
(5) $15a(a^2 + b^2) - 25a^2b(a^2 + b^2) + 10ab(a^2 + b^2)$;
(6) $12(c - y) + 6y(y - c) - 2y^2(c - y)$;
(7) $-35p(p + 8) - 42(p + 8) + 11p(-p - 8)$;
(8) $6x^2(x - z) + z(z - x)^2$;
(9) $2x(5x - 2) - xy(5x - 2) + y(2 - 5x)$;
(10) $n(3n - 4)^2 + (3n - 4)^3$;
(11) $(x + 7)^3 - 8x^2(x + 7)$.

2.71. Prove the following identities:

(1) $b - c + b(c - 1) = c(b - 1)$;
(2) $2bx - a(x - b) = b(a + x) - x(a - b)$;
(3) $(-5a)^3 \cdot (2b)^4 = -(10ab)^3 \cdot 2b$;

(4) $(x - 2b)(x^2 - 5bx + b^2) + (2b - x)(x^2 - 6bx + b^2) = bx(x - 2b)$;
(5) $(a - 3c)(2a^2 - 7ac - c^2) - (3c - a)(c^2 + 7ac - a^2) = a^2(a - 3c)$.

2.72. Find x if:

(1) $x(x - 7) + 3(x - 7) = 0$;
(2) $5x(x - 4) - 20(4 - x) = 0$;
(3) $x(x - 9) - 5(9 - x) = 0$;
(4) $(x + 1)(x - 2) - 5(x - 2) = 0$;
(5) $21(x - 6) + (x + 5)(6 - x) = 0$;
(6) $(x - 4)(x - 3) + (4 - x)(5 - 2x) = 0$.

2.73. Compute:

(1) $0.756^2 - 0.241 \cdot 0.756 - 0.415 \cdot 0.756$;
(2) $0.25^2 \cdot 2.4 + 0.25 \cdot 2.4^2 - 0.25 \cdot 0.65 \cdot 2.4$;
(3) $0.16 \cdot 6.41 \cdot 1.25 - 0.16 \cdot 1.25^2 - 0.16^2 \cdot 1.25$;
(4) $3.24 \cdot 26.3 + 3.24 \cdot 7.6 - 1.62 \cdot 47.8$.

The Grouping Method

2.74. Factor:

(1) $ax + ay + 2x + 2y$; (2) $ac + bc + a + b$;
(3) $56x^2 - 45y - 40xy + 63x$;
(4) $a^3 + a^2b + a^2c - abc$;
(5) $14a^2c + 25b^2d - 10abd - 35abc$;
(6) $21a^2b - 4b - 12a + 7ab^2$;
(7) $20a^2c + 9c - 15a - 12ac^2$;
(8) $11x + 11y - x^2 - xy$;
(9) $18x^2z - 10dxy + 20d^2y - 36dxz$;
(10) $x^2 - ax - a^2y - axy$;
(11) $30a^2b^2 + 5ac^3 - 25bc^2 - 6a^3bc$;
(12) $5a^3c + 10a^2 - 6bc - 3abc^2$;
(13) $8xy^3 - 24y^2 - 7axy + 21a$;
(14) $3x^3 - 2y^3 - 6x^2y^2 + xy$;
(15) $an^2 + cn^2 - ap + ap^2 - cp + cp^2$;
(16) $p^2q^2 + pq - q^3 - p^3$;

(17) $ax^2 + ay^2 - bx^2 - by^2 + b - a$;
(18) $2a + ac^2 - a^2c - 2c$;
(19) $xy^2 - by^2 - ax + ab + y^2 - a$;
(20) $ac^2 - ad - bc^2 + cd + bd - c^3$.

2.75. Prove the following identities:

(1) $5x^3 - 2x^2 + 5x - 2 = (x^2 + 1)(5x - 2)$;
(2) $(2ab - 3c)(3ac - 2b) = 6a^2bc - 9ac^2 - 4ab^2 + 6bc$;
(3) $5(a + b)^2 - 4a^2 - 4ab = (a + b)(a + 5b)$;
(4) $3(x - 7)^2 + 8x - 56 = (x - 7)(3x - 13)$.

2.76. Find x if:

(1) $2x^2 - 3x + 6x - 9 = 0$;
(2) $5x^2 - 2x + 4 - 10x = 0$;
(3) $6x^2 - x - 12x + 2 = 0$;
(4) $12x^2 - 3x - 8x + 2 = 0$;
(5) $7x^2 - 3x - 21x + 9 = 0$;
(6) $9x^2 - 2x - 18x + 4 = 0$.

2.77. Compute:

(1) $23 \cdot 17.8 - 3 \cdot 7.2 + 23 \cdot 7.2 - 17.8 \cdot 3$;
(2) $77.3 \cdot 13 + 8 \cdot 37.3 - 77.3 \cdot 8 - 13 \cdot 37.3$;
(3) $56.2 \cdot 29 + 60.3 \cdot 41 + 43.8 \cdot 29 + 39.7 \cdot 41$;
(4) $109 \cdot 9.17 - 5.37 \cdot 72 - 37 \cdot 9.17 + 1.2 \cdot 72$;
(5) $123 \cdot 1.32 - 28 \cdot 0.148 - 123 \cdot 0.468 + 151 \cdot 0.148$;
(6) $68.7 \cdot 1.1 + 48 \cdot 1.25 - 16.7 \cdot 1.1 - 48 \cdot 0.15$;
(7) $24.3 \cdot 6.78 + 45.7 \cdot 11.7 + 30 \cdot 6.78 - 4.92 \cdot 45.7$;
(8) $\frac{59^3 - 41^3}{18} + 59 \cdot 41$; (9) $\frac{67^3 + 52^3}{119} - 67 \cdot 52$.

Making Use of Formulas for Abridged Multiplication

2.78. Factor:

(1) $x^2 - y^2$; (2) $4 - a^2$; (3) $a^2b^2 - c^2$; (4) $100 - b^4$;
(5) $121a^4 - 49b^4$; (6) $81a^2b^2 - 100c^4$;
(7) $0.25 - 0.64a^6$; (8) $2.25a^{12} - 0.16b^8$;
(9) $0.1a^4 - 4/9$; (10) $1/a^{10} - 100/b^4$;

(11) $a^{2n} - b^{4n}$; (12) $a^2b - b^3$; (13) $a^4 - 0.01$;
(14) $9a^3 - 9a^2b - a + b$; (15) $a^3 - 5b^2 - 16b + 80$.

2.79. Find x:

(1) $25x^2 - 49 = 0$; (2) $64x^2 - 361 = 0$;
(3) $225x^2 - 729 = 0$; (4) $324x^2 - 961 = 0$;
(5) $64x^3 - 36x = 0$; (6) $49x^3 - 81x = 0$;
(7) $121x^3 - 25x = 0$; (8) $256x^3 - 841x = 0$;
(9) $0.01x^4 - x^3 = 0$;
(10) $81x^4 - 25x^2 = 0$.

2.80. Compute:

(a) (1) $58 \cdot 62$; (2) $102 \cdot 98$; (3) $56 \cdot 64$; (4) $108 \cdot 92$;
(5) $95 \cdot 105$; (6) $109 \cdot 91$; (7) $196 \cdot 204$; (8) $79 \cdot 121$;

(b) (1) $47^2 - 37^2$; (2) $53^2 - 63^2$; (3) $87^2 - 13^2$;
(4) $126^2 - 74^2$; (5) $21.3^2 - 21.2^2$; (6) $50.7^2 - 50.6^2$;
(7) $\left(5\frac{2}{3}\right)^2 - \left(4\frac{1}{3}\right)^2$; (8) $\left(7\frac{6}{11}\right)^2 - \left(2\frac{5}{11}\right)^2$;
(9) $94^2 - 36^2$; (10) $108^2 - 62^2$; (11) $361^2 - 121^2$;
(12) $961^2 - 41^2$.

2.81. Factor:

(1) $64a^2 - 25^4$; (2) $a^2 + 4ab + 4b^2 - 1$;
(3) $4(a + b)^2 - 9(a - b)^2$; (4) $a^4 - a^2(b^2 + 1) + b^2$;
(5) $a^2 + ax + 2x^2 + a + 2x$; (6) $a^4 + 4$;
(7) $x^4 - x^2 + 16$; (8) $x^4 + 6x^2 + 25$.

2.82. Find the result:

(1) $(16 - 25x^4) \div (4 + 5x^2)$;
(2) $((2a - 3b)^2 - 1) \div (2a + 3b + 1)$;
(3) $(2.25a^4 - 1.44b^4) \div (1.5a^2 - 1.2b^2)$;
(4) $(3.24a^8 - 1.21b^8) \div (1.8a^4 + 1.1b^4)$;
(5) $(2^{4m} - 2^{4n}) \div (2^{2m} - 2^{2n})$;
(6) $(5^{6m} - 4^{8n}) \div (5^{3m} - 4^{4n})$.

2.83. Factor:

(a) (1) $a^{3n} + b^{3n}$; (2) $8a^9 + b^{3/n}$; (3) $125a^3 - b^{3n}$;
(4) $n^3 + 1000m^3$; (5) $x^3 + 125y^{6n}$; (6) $27x^3 + 125$;
(7) $(1/27)a^9 + 216$; (8) $343a^6 + 216b^6$;

(b) (1) $(2a - 3)^3 + 1000$; (2) $(a - 2)^3 + 27$;
(3) $(a + b)^3 + 8c^3$; (4) $(a - 3)^3 + 64$;
(5) $(a + b)^3 + (a - b)^3$; (6) $(2a + 1)^3 - (2a - 1)^3$.

2.84. Find the result:

(a) (1) $(a + 5)(a^2 - 5a + 25)$;
(2) $(x + y)(x^2 - xy + y^2) - 2y^2$;
(3) $(a/b + 1)(a^2/b^2 - a/b + 1)$;
(4) $(a + 2)(a^2 - 2a + 4) - 16$;
(5) $(2b + c)(4b^2 - 2bc + c^2) - 8b^3$;
(6) $(7x^2 + 2)(-49x^4 + 14x^2 - 4) + (7x^2 - 2)^3$;
(7) $(10x + 3y)(100x^2 + 60xy + 9y^2) + (10x - 3y)^3$;

(b) (1) $(39^3 + 19^3)/58 - 39 \cdot 19$;
(2) $(67^3 + 52^3)/119 + 67 \cdot 52 \cdot 3$;
(3) $(89^3 + 9^3)/98 - 89 \cdot 9$;
(4) $(79^3 + 59^3)/138 - 79 \cdot 59$;
(5) $(36.5^2 - 27.5)^2 \div \left(\frac{57^3 + 33^3}{90} - 57 \cdot 33\right)$;
(6) $(94.5^2 - 30.5^2) \div \left(\frac{69^3 + 29^3}{98} - 69 \cdot 29\right)$.

2.85. Factor:

(a) (1) $27c^3 - 8$; (2) $125b^6 - 216$; (3) $343a^6 - 64b^3$;
(4) $(1/64)x^3 - 1/27$; (5) $0.008x^6 - 0.027y^6$;
(6) $(1/125)x^6y^3 - (1/64)b^6$;

(b) (1) $8 - (3 - a)^3$; (2) $8x^3 - (5x - 3)^3$;
(3) $(2a + b)^3 - (2b - a)^3$;
(4) $(3x + 2y)^3 - (2y - 3x)^3$;
(5) $(3a + 2)^3 - (2 - 3a)^3$;
(6) $(5c + 3)^3 - (5c - 3)^3$.

2.86. Compute:

(1) $\frac{101^3 - 1}{100} + 100$; (2) $\frac{49^3 - 23^3}{20} + 49 \cdot 29$;
(3) $\frac{59^3 - 41^3}{18} - 59 \cdot 3 \cdot 41$;

(4) $\frac{79^3-59^3}{2}+79\cdot 59$; (5) $\frac{89^3-49^3}{40}+89\cdot 49$;

(6) $\frac{69^3-29^3}{40}+69\cdot 29$.

2.87. Factor:

(1) $(7n+8.5)^2-(4n+2.5)^2$;
(2) $(7n+6.5)^2-(2n+11.5)^2$;
(3) $(6n+17)^2-(n-3)^2$; (4) $(5n+2)^2-(2n+5)^2$;
(5) $a^2-a=12$; (6) a^3-3a+2; (7) $m^4+m^2n^2+n^4$;
(8) $a^3-4a^2+20a-125$; (9) $27b^3-3b^2+2b-8$;
(10) $81x^2-9x-10y-100y^2$;
(11) $8y^3-26y^2-13y+1$;
(12) $16v^2-20v+35t-49t^2$;
(13) $18a^2-9a-15b-50b^2$;
(14) $a^4-6a^3+54a-81$.

2.88.

(1) $(a^2-a-3ab+3b)^2-(3ab-a+3b-9b^2)^2$;
(2) $(2a^3-3a^2x+3ax^2-2x^3)^2-9a^2x^2(a-x)^2$;
(3) $4b^2-(x^2-b^2-1)^2$;
(4) $(2a^3-4a^2-a+1)^2-(3a-1)^2$;
(5) x^3+5x^2+3x-9;
(6) $(x+y+p)^3-x^3-y^3-p^3$;
(7) $a^3+8a^2+17a+10$; (8) $a^4+5a^3+15a-9$;
(9) $a^4+a^3+6a^2+5a+5$; (10) $a^5+a^4+a^3+a+1$;
(11) $2x^3+10x+12$; (12) a^3+3a^2-4.

2.89.

(1) a^4+a^2+1; (2) x^8+x^4+1;
(3) $a^4-51a^2b^2+b^4$; (4) $a^4-123a^2b^2+b^4$;
(5) n^4+n^3-n-1; (6) $(x+y)^4-(x-y)^4$;
(7) $a^2-b^2-c^2+2bc$; (8) $1-x^2-2xy-y^2$;
(9) a^4+a^2-2; (10) $x^3+3x^2-4x-12$;
(11) $11x-3x^2+70$; (12) $15x^3+x^2-2x$;
(13) $x^5+2x^4+4x^3+2+x$.

2.90. Prove the following identities:

(1) $(a + 1/2)^2 = a(a + 1) + 1/4$;

(2) $(4x^2 + 4ax + a^2) \div (2x + a) - (2x + a)^3 \div (4x^2 + 4ax + a^2) = 0$;

(3) $\left(\frac{x-3}{2}\right)^3 - \left(\frac{x+3}{2}\right)^3 = \frac{9(x^2+3)}{-4}$;

(4) $\left(\frac{x-3}{3}\right)^3 + \left(\frac{x+3}{3}\right)^3 = \frac{2x(x^2+27)}{27}$;

(5) $x^2(y - z) + y^2(z - x) + z^2(x - y) = (x - z) \times (z - y)(y - x)$;

(6) $a^3 + b^3 + c^3 - 3abc = (a + b + c) \times (a^2 + b^2 + c^2 - ab - ac - bc)$;

(7) $a(b + c)^2 + b(c + a)^2 + c(a + b)^2 - 4abc = (b + c)(c + a)(a + b)$.

§ 23. Algebraic Fractions. The Fundamental Property of the Fraction. Reducing Fractions

2.91. Put the minus sign before the fraction leaving its value unchanged:

(a) (1) $\frac{1-x}{a}$; (2) $\frac{m}{1-m}$; (3) $\frac{a-b}{c+d}$; (4) $\frac{x-5}{x-2}$;

(5) $\frac{a-x}{b-x}$; (6) $\frac{-a-b}{c+d}$;

(b) (1) $\frac{1}{(b-a)(a-c)}$; (2) $\frac{a+b}{6x(5y-x)}$;

(3) $\frac{72}{(a-b)(3-a)}$; (4) $\frac{x^3-y^3}{(5-c)(x+y)}$.

2.92. For what values of a does the given fraction equal zero:

(1) $\frac{(5a-1)^2-36}{a^2+7}$; (2) $\frac{4a^2-64}{3a+5}$; (3) $\frac{(a-3)(a-5)(a^2+11)}{a}$;

(4) $\frac{a(a+4)}{a^2-5a}$; (5) $\frac{2|a|-9}{11}$; (6) $\frac{|a|-a}{a^2}$; (7) $\frac{|a-2|-3}{a^2+1}$;

(8) $\frac{|a-2| \cdot (a^3+1)}{15}$?

2.93. For what values of x does the given fraction have no meaning:

(1) $\frac{x^2-49}{x^2-36}$; (2) $\frac{x^2-6x+9}{x^2+8x+16}$; (3) $\frac{x^2+1}{(x-7)(x^2-9)}$;

(4) $\frac{x+15}{|x|+9}$; (5) $\frac{25a^2+c^2}{|x-2|-7}$; (6) $\frac{5+x^2}{|x|+x}$?

2.94. Reduce the following fractions:

(a) (1) $\frac{5a^4b^{3n}}{10b^n}$; (2) $\frac{6x^{2n}b^{2.5m}}{2xy^{n/2}}$; (3) $\frac{18a^4b^3c}{12ab^4c}$;

(4) $\frac{225a^{n+1}b^{2n}}{45a^{n-2}b^{2n-1}}$; (5) $\frac{135a^{5-n}b^{m-4}}{27a^{3-n}b^{m-5}}$; (6) $\frac{27^5+27^4}{9^8+9^7+9^6}$;

(7) $\frac{16^7-16^6}{8^{10}-8^9+8^8}$;

(b) (1) $\frac{8^{11}-8^{10}-8^9}{4^{15}-4^{14}-4^{13}}$; (2) $\frac{9^{23}+9^{22}+9^{21}}{27^{14}-27^{13}}$;

(3) $\frac{(8^{n-2}+8^{n-3})^2}{(4^{n-1}-4^{n-2})^3}$; (4) $\frac{(9^{n-1}+9^n)^3}{(3\cdot 27^{n-2}+27^{n-1})^2}$;

(5) $\frac{(27^n-3\cdot 27^{n-1})^2}{(9^{n+1}-9^n)^3}$; (6) $\frac{a^2-ab+bc-c^2}{b^2-a^2-2ac-c^2}$;

(7) $\frac{25-a^2-2ab-b^2}{a^2+ab+5b-25}$; (8) $\frac{b^2-18b-c^2+81}{b^2+2bc+c^2-9b-9c}$;

(c) (1) $\frac{a^2-25b^2+10bc-c^2}{a^2-5ab+5bc-c^2}$; (2) $\frac{36x^2+6xy+yz-z^2}{36x^2-y^2+2yz-z^2}$;

(3) $\frac{2a(b^2+9)+3b(a^2+4)}{2a(b^2-9)+3b(a^2-4)}$; (4) $\frac{(2a+b)(3m+1)}{(b^2-4a^2)(1-9m^2)}$;

(5) $\frac{(a-b)(c-d)}{(b^2-a^2)(c^2-d^2)}$; (6) $\frac{(a-2)(x+a)(b-x)}{(4-a^2)(a^2-x^2)(x^2-b^2)}$;

(d) (1) $\frac{x^3-2x^2+5x+26}{x^3-5x^2+17x+13}$; (2) $\frac{2a^4+a^3+4a^2+a+2}{2a^3-a^2+a-2}$;

(3) $\frac{a^2-4-|a-2|}{a^3+2a^2-5a-6}$; (4) $\frac{x|x-3|+x^2-9}{2x^3-3x^2-9x}$.

§ 24. Reducing Fractions to a Common Denominator. Addition and Subtraction of Fractions

2.95. Reduce the fractions to a common denominator:

(1) $\frac{n}{6a^3c^2}$; $\frac{p}{9a^4b^2}$; (2) $\frac{m}{4a^5}$; $\frac{5}{12a^3b^2}$; $\frac{3}{7a^2c}$; $\frac{17}{21a^4c^4}$;

(3) $\frac{1}{24a^6b^{n-1}}$; $\frac{1}{60a^3b^{n-2}}$; $\frac{1}{40a^{24}x^{11-n}}$;

(4) $\frac{1}{35b^9x^{10-n}}$; $\frac{1}{56b^{18}x^{12-n}}$; $\frac{1}{40b^6x^{11-n}}$;

(5) $\frac{1}{252a^{12}(5x-1)^2}$; $\frac{1}{98a^8(25x^2-1)}$; $\frac{1}{56a^7(5x+1)^2}$;

(6) $\frac{a}{a-b}$; $\frac{4a}{b^3-a^3}$; $\frac{2}{a^2+ab+b^2}$;

(7) $\frac{1}{(2a-b)^2}$; $\frac{2}{4a^2-b^2}$; $\frac{1}{(2a+b)^3}$;

(8) $\frac{1}{(c+2)^2}$; $\frac{4}{(c-2)^2}$; $\frac{7}{c^2-4}$.

2.96. Perform the indicated operations:

(a) (1) $\frac{a+b}{c}+\frac{c-2b}{c}$; (2) $\frac{m+n}{a+b}+\frac{m-n}{a+b}$; (3) $\frac{a}{a+b}+\frac{b}{a+b}$;

(4) $\frac{a-b}{b+c}-\frac{a}{b+c}$; (5) $\frac{1}{ab}+\frac{1}{bc}+\frac{1}{ac}$; (6) $\frac{c}{m}+\frac{ab}{mn}+\frac{b}{n}$;

(b) (1) $\frac{6}{4c-c^2}-\frac{2}{c^2+4c}-\frac{c}{c^2-16}$;

(2) $\frac{x-2}{x^2+2x}+\frac{x+2}{x^2-2x}-\frac{4x}{x^2-4}$;

(3) $\frac{x+2}{2x-4}+\frac{2-x}{6+3x}+\frac{5x^3+8}{24-6x^2}$;

(4) $\frac{12c}{4c^2-9}+\frac{2c-3}{4c+6}+\frac{2c+3}{6c-9}$;

(5) $\frac{1}{a^2-2ax}+\frac{1}{2ax+4x^2}+\frac{2}{4x^2-a^2}$;

(6) $\frac{1}{2z^2+5}-\frac{2}{25-10z}-\frac{4}{4z^2-25}$;

(7) $\frac{1}{x^2+3xy}+\frac{2}{9y^2-x^2}+\frac{1}{2x-6y}$;

(8) $\frac{25}{a^2+5a+25}-\frac{2a}{5-a}-\frac{a^3+25a^2}{a^3-125}$;

(9) $\frac{a}{a-b}+\frac{4a^2b-ab^2}{b^2-a^2}+\frac{b^2}{a^2+ab+b^2}$.

§ 25. Multiplication, Division, and Involution of Fractions

2.97. Multiply:

(1) $\frac{x^2+xy}{x^2-xy}\cdot\frac{x-y}{x^2+2xy+y^2}$; (2) $\frac{5m-5n}{4m+4n}\cdot\frac{8m+8n}{15m-15n}$;

(3) $\frac{a^2-b^2}{a^2}\cdot\frac{a^4}{(a+b)^2}$; (4) $\frac{a^2-an+3n-9}{an+3n}\cdot\frac{a^2+3a}{(a-n)^2-9}$;

(5) $\dfrac{9-4p^2}{1-2p-t}\cdot\dfrac{t^2+2pt+2p-1}{4p^2-12p+9}$;

(6) $\dfrac{9m^2-6mk+2k-1}{9k^2-16m^2}\cdot\dfrac{16m^2-24mk+9k^2}{2k-3m-1}$;

(7) $\dfrac{(a^2-b^2-c^2+2bc)\cdot(a+b+c)}{a+b-c}$;

(8) $\dfrac{2}{x^2+x-2ax-2a}\cdot\dfrac{x^2+4x+3}{x+3}$;

(9) $\dfrac{2x}{2x^2-6x-ax-3a}\cdot\dfrac{2x^2-ax+10x-5a}{2x^2+2x}$.

2.98. Divide:

(1) $\dfrac{1}{x}\div y$; (2) $a\div\dfrac{1}{a}$; (3) $10x^3y^3\div\left(-\dfrac{20x^2y^3}{3ab}\right)$;

(4) $-\dfrac{18a^4b^3}{5x^2y^2}\div\dfrac{27x^2y^2}{10ab^2}$; (5) $\dfrac{a+b}{m}\div\dfrac{a+b}{n}$;

(6) $\dfrac{a^2-b^2}{ab}\div\dfrac{a+b}{ab\,(a-b)}$; (7) $\dfrac{2a+2b}{3a-3b}\div\dfrac{6a+6b}{5a-5b}$;

(8) $\dfrac{am^2-an^2}{m^2+2mn+n^2}\div\dfrac{am^2-2amn+an^2}{3m+3n}$;

(9) $\dfrac{16-(a+x)^2}{a^2+ax}\div\dfrac{a^2+4a-x^2-4x}{4a+4x}$;

(10) $\dfrac{n^2-10n+25-t^2}{n^2-t^2}\div\dfrac{t^2-5t+5n-n^2}{n^2+nt}$;

(11) $\dfrac{4x^2-10ax-5a-1}{1-25a^2}\div\dfrac{1-2x+5a}{25a^2-10a+1}$;

(12) $\dfrac{25y^3-9z^2}{2z-5y+2}\div\dfrac{9z^2-30zy+25y^2}{25y^2-10yz+4z^2-4}$;

(13) $\dfrac{x^2+2x-3}{x^2+3x-10}\div\dfrac{x^2+7x+12}{x^2-9x+14}$;

(14) $\dfrac{5x^2-10xy+5y^2}{2x^2-2xy+2y^2}\div\dfrac{8x-8y}{10x^3+10y^3}$;

(15) $\dfrac{(a-b)^2+ab}{(a+b)^2-ab}\div\dfrac{a^5+b^5+a^2b^3+a^3b^2}{(a^3+b^3+a^2b+ab^2)\,(a^3-b^3)}$.

2.99. Perform the indicated operations:

(1) $\left(-\dfrac{5k^5}{3c^4}\right)^5\div\left(-\dfrac{10k^3}{9c^7}\right)^3$; (2) $-\left(-\dfrac{6a^4}{5b^3}\right)^4\cdot\left(-\dfrac{25b^4}{9a^5}\right)^3$;

(3) $\left(-\dfrac{10a^4}{9q^6}\right)^4\div\left(-\dfrac{5a^5}{27q^3}\right)^3$; (4) $\left(-\dfrac{15p^7}{8q^4}\right)^3\cdot\left(\dfrac{4q^3}{5p^5}\right)^4$;

(5) $\left(\dfrac{81a^{4n-1}}{25b^5}\right)^3\cdot\left(\dfrac{-5b^3}{27a^{3n-1}}\right)^4$; (6) $\left(-\dfrac{16x^{4n+7}}{9y^3}\right)^3\div\left(\dfrac{8x^{3n+5}}{3y^2}\right)^4$;

(7) $\left(-\frac{32a^4}{25b^{3n-2}}\right)^2 \div \left(-\frac{16a^3}{5b^{2n-1}}\right)^3$;

(8) $\frac{a^{2n}-b^{2n}}{a^{3n}+b^{3n}} \div \frac{a^n-b^n}{a^{2n}-a^nb^n+b^{2n}}$;

(9) $\frac{a^{3n}-b^{3n}}{a^{2n}-b^{2n}} \div \frac{(a^n+b^n)(a^{2n}+a^nb^n+b^{2n})}{a^{2n}+b^{2n}}$;

(10) $\frac{a^{4n}-b^{4n}}{a^{3n}-b^{3n}} \cdot \frac{(a^{2n}+b^{2n})(a^{3n}+a^nb^n+b^{3n})}{a^n+b^n}$.

2.100. Perform the indicated operations:

(1) $\left(\frac{c+5}{5c-1}+\frac{c+5}{c+1}\right) \div \frac{c^2+5c}{1-5c}+\frac{c^2+5}{c+1}$;

(2) $\left(\frac{y-3}{7y-4}-\frac{y-3}{y-4}\right) \cdot \left(\frac{7y-4}{9y-3y^2}+\frac{y^2-14}{4-y}\right)$;

(3) $\left(\frac{1}{3p-q}+\frac{3pq-4}{27p^3-q^3}\right) \div \left(\frac{1}{9p^2+3pq+q^2}+\frac{2-2q}{q^3-27p^3}\right)$;

(4) $\left(\frac{1+6ac}{a^3-8c^3}-\frac{1}{a-2c}\right) \div \left(\frac{1}{a^3-8c^3}-\frac{1}{a^2+2ac+4c^2}\right)$;

(5) $\left(\frac{1}{c^2-4c}+\frac{2}{16-c^2}+\frac{1}{4c+16}\right) \div \left(\frac{c-4}{2c+8}\right)^2+\frac{4c+16}{4c^2-c^3}$;

(6) $\frac{9y-27}{3y^2-y^3}+\left(\frac{3y+9}{y-3}\right)^2 \cdot \left(\frac{1}{3y-9}+\frac{2}{9-y^2}-\frac{1}{y^2+3y}\right)$.

2.101. Simplify:

(1) $\frac{11-3x-3y}{(x+y)^2-8(x+y)+15} \cdot \frac{1}{x+y-3}+\frac{1}{x+y-5}$;

(2) $\frac{a+3}{2a-1}-\frac{a^2-5}{4a^2-4a+1}-\frac{2a^3-a(1-5a)-1}{8a^3-12a^2+6a-1}$;

(3) $\frac{x}{ax-2a^2}-\frac{2}{x^2+x-2ax-2a} \cdot \left(1+\frac{3x+x^2}{3+x}\right)$;

(4) $\left(\frac{a}{a-2}-\frac{a}{a+2}\right) \div \frac{2a}{\frac{1}{2}a^4-a^3+4a-8}$;

(5) $\left[\frac{4}{3a}+\frac{1}{3a-6b} \cdot \left(2b-a-\frac{2b-a}{a}\right)\right] \div \frac{5/a^2-3/a}{3/a}$;

(6) $\left(a+\frac{2}{1+0.5a}\right) \div \frac{a^3-8}{a+2}+\frac{2}{2a-a^2}$;

(7) $\left(\frac{1}{2+4m}-\frac{1-m}{8m^3+1} \div \frac{1-2m}{4m^2-2m+1}\right) \cdot \frac{4m+2}{2m-1}$;

(8) $\left(\frac{3x+2}{2x+3}-\frac{4x-1}{2x+3}-\frac{2x^2+3x}{4x^2+12x+9}\right) \div \frac{3-2x}{2x+3}$;

(9) $\left(\frac{1}{a^3-9} \div \frac{x-a}{3a^2+9a} - \frac{3a}{9-3x-3a+ax}\right) \cdot \frac{x^3-27}{3a}$;

(10) $\frac{a-c}{a^2+ac+c^2} \cdot \frac{a^3-c^3}{a^2b-bc^2} \cdot \left(1 + \frac{c}{a-c} + \frac{1+c}{c}\right) \div \frac{c(1+c)-a}{bc}$.

2.102. Prove the given identities. Determine the permissible values of the parameters:

(1) $\frac{2-a}{5} + \left(\frac{1}{1-2a}\right)^2 \div \left(\frac{a+2}{4a^3-4a^2+a} - \frac{2-a}{1-8^3} \frac{4a^2+2a+1}{2a^2+a}\right) = \frac{1}{2}$;

(2) $\left(\frac{x^2-2x+4}{4x^2-1} \cdot \frac{2x^2+x}{x^2+8} - \frac{x+2}{2x^2+x}\right) \div \frac{4}{x^2+2x} - \frac{x+4}{3-6x} = -\frac{1}{3}$;

(3) $\left[\left(1 - \frac{2}{1-3a}\right) \cdot \left(1 - \frac{9a-9a^2}{3a+1}\right)\right] \div [2(1-9a^2)] = -\frac{1}{2(3a+1)}$;

(4) $\frac{a-c}{a^2+ac+c^2} \cdot \frac{a^3-c^3}{a^2b-bc^2} \cdot \left(1 + \frac{c}{a-c} - \frac{1+c}{c}\right) \div \frac{c(1+c)-a}{bc} = \frac{1}{a+c}$;

(5) $\frac{\frac{1}{a} - \frac{1}{b+c}}{\frac{1}{a} + \frac{1}{b+c}} \cdot \left(1 + \frac{b^2+c^2-a^2}{2bc}\right) \div \frac{a-b-c}{abc} = \frac{(a-b-c)a}{2}$;

(6) $\frac{2b+a-\frac{4a^2-b^2}{a}}{b^3+2ab^2-3a^2b} \cdot \frac{a^3b+2a^2b^2+ab^3-4a^2b^2}{a^2-b^2} = \frac{a-b}{a+b}$;

(7) $\left(\frac{1}{3p-q} + \frac{3pq-4}{27p^3-q^3}\right) \div \left(\frac{1}{9p^2+3pq+q^2} + \frac{2-2q}{q^3-27p^3}\right) = 3p+q+2$;

(8) $\frac{a^3}{(a-b)(a-c)} + \frac{b^3}{(b-c)(b-a)} + \frac{c^3}{(c-a)(c-b)} = a+b+c$;

(9) $\frac{\frac{b^2-c^2}{a} + \frac{c^2-a^2}{b} + \frac{a^2-b^2}{c}}{\frac{b-c}{a} + \frac{c-a}{b} + \frac{a-b}{c}} = a+b+c$;

(10) $\frac{3n+14}{n+4} - \left(\frac{n-4}{n+6}\right)^2 \cdot \left(\frac{n+21}{16-8n+n^2} - \frac{n+3}{16-n^2}\right) = 3$;

(11) $\left(\frac{x+5}{x^2-81} + \frac{x+7}{x^2-18x+81}\right) \div \left(\frac{x+3}{x-9}\right)^2 - \frac{1+x}{9+x} = -1$;

(12) $\left[\left(\frac{a+1}{a-1}\right)^2 + 3\right] \div \left[\left(\frac{a-1}{a+1}\right)^2 + 3\right] \div \frac{a^3+1}{a^3-1} - \frac{2a}{a-1} = -1$.

2.103. Perform the indicated operations:

(1) $\left(\frac{1-2z}{3+6z^2}+2z-1\right)\cdot\frac{1+2z}{1-2z}+\frac{6z^2-1}{3z}$;

(2) $\left[\frac{3+a}{(3-a)^2}-\frac{6}{9-a^2}+\frac{3-a}{(a+3)^2}\right]\div\frac{24a^2}{81-a^4}+\frac{2a^2}{a^2-9}$;

(3) $\left[\frac{x^2+24}{(2x-5)^2}+\frac{8}{5-2x}\right]\div\frac{1}{(2x-5)^2}-\frac{2}{2x^2+x-15}+\frac{1}{(x+3)^2}$.

§ 26. Equality, Identity, and Equation. Equivalent Equations. Solving First-degree Equations in One Variable

2.104. Determine which of the below equalities are identities:

(1) $a+3=3+a$; (2) $xy=yx$; (3) $b+3=5$;
(4) $3(a+b)=3a+3$; (5) $4a=20$; (6) $3=a/4$;
(7) $1+m=9$; (8) $(a+b)^2=a^2+2ab+b^2$;
(9) $(a+b)(a^2-ab+b^2)=a^3+b^3$;
(10) $(3-a)(9+3a+a^2)=27-a^3$;
(11) $(a+b)^2(a-b)^2=(a^2-b^2)^2$;
(12) $(1+a)(1-a)(1+a^2)=1-a^4$;
(13) $a^3+3ab(a+b)+b^3=(a+b)^3$;
(14) $a^3-3ab(a-b)-b^3=(a-b)^3$;
(15) $(a+b)^3(a-b)^3=(a^2-b^2)^3$;
(16) $(2-a)(2+a)(4+a^2)=16-a^4$;
(17) $(a^2-b^2)(a^4+a^2b^2+b^4)=a^6-b^6$;
(18) $(a^2-4)(a^2+2a+4)=(a+2)(a^3-8)$;
(19) $(a^2-9)(a^2-3a+9)=(a-3)(a^3+27)$;
(20) $(a+b)^2-4ab=(a-b)^2$;
(21) $x^2+xy+y^2=x+2y$.

2.105. For what values of m do the given equations have no solution:

(1) $2x=5-mx$; (2) $mx-6=3x$;
(3) $mx=4x+12$; (4) $2mx+3=2m-x$;
(5) $mx-2x=m^2-4$; (6) $mx+4=4x+m^2$;
(7) $mx-1=m^2+2m-x$;

(8) $m^2x - 29 = m^2 + 6m + 9x$;
(9) $m^2x - 29 = m^4 - 12x$; (10) $m^4x - 3 = m^2 - 5x$;
(11) $m^2x - 35 = m(mx + 2m)$; (12) $\frac{m}{x+2/m} = \frac{m}{x+1/m}$?

2.106. Determine which of the following equations have no solution and which equations have infinitely many solutions:

(1) $2x+1=2x+3$; (2) $5x-15-3x-6=2x-25$;
(3) $4x-3-3x-4=x-7$; (4) $5(x-2)=5x-10$;
(5) $\frac{x-5}{2}+\frac{x-1}{8}=\frac{2.5x-3}{4}$; (6) $\frac{x+6}{2}+\frac{5-1.5x}{3}=\frac{14}{3}$;
(7) $\frac{9x+7}{2}-4.5x=3.5$; (8) $\frac{5x-1}{7}+\frac{4x-3}{2}=2\frac{5}{7}x+\frac{1}{7}$;
(9) $x-\frac{8x+7}{6}+\frac{1}{3}x=-1\frac{1}{6}$; (10) $2x-\frac{5-x}{3}-2\frac{1}{3}x=2$.

2.107. Determine whether or not the two given equations are equivalent:

(1) $5x+1=2$ and $10x+2=4$; (2) $5x+4=9$ and $2x-1=1$;
(3) $2x-1=4$ and $x+5=7$; (4) $x+1=5$ and $4x-3=2$;
(5) $5x+3=18$ and $\frac{5x+3}{3}=6$; (6) $x+1=4$ and $\frac{x+1}{2}=\frac{4}{3}$;
(7) $1-\frac{2x-5}{6}=\frac{3-x}{4}$ and $\frac{x+17}{5}-\frac{3x-7}{4}=-2$;
(8) $\frac{1}{2}(x+4)+\frac{1}{3}(x+2)=3-\frac{1}{4}(x+3)$ and
$\frac{2x-5}{6}+\frac{x+2}{4}=\frac{5-2x}{3}-\frac{6-7x}{4}-x$;
(9) $\frac{2}{3}y-\frac{5}{6}=13$ and $\frac{3}{4}(y-1)-\frac{2y-7}{7}=0$;
(10) $\frac{x-4}{5}=9-\frac{2x-41}{9}$ and $2x+\frac{3x-1}{2}-\frac{5x-2}{3}=2$;
(11) $\frac{1}{4x-6}+\frac{2x-5}{18-8x^2}-\frac{1}{2x^2+3x}=0$ and $\frac{3x-1}{6x-3}-\frac{1}{1-4x^2}=\frac{x}{2x+1}$;
(12) $\frac{2x-2}{x^2-36}-\frac{x-2}{x^2-6x}-\frac{x-1}{x^2+6x}=0$ and $\frac{11x-1}{x}+\frac{30}{x-1}=11$.

2.108. Solve the following equations:

(1) $x+1\frac{1}{2}x+9=\frac{2}{3}x+4+\frac{5}{6}x-\frac{6}{5}x+\frac{1}{5}$;

(2) $2\frac{1}{2}x-3\frac{1}{2}x+1=x-5\frac{1}{3}x+3\frac{1}{5}x$;

(3) $\frac{4}{5}x-2\frac{1}{2}x-2=-2\frac{1}{3}x-\frac{1}{6}-\frac{1}{5}$;

(4) $15(x+2)=6(2x+7)$; (5) $8(9+2x)=5(2-3x)$;

(6) $(x-3)(x+4)-2(3x-2)=(x-4)^2$;

(7) $(x+5)(x+2)-3(4x-3)=(x-5)^2$;

(8) $(2x-1)^2-5(2x+1)^2+(6x-3)(2x+1)=-4(x-1)^2$;

(9) $(x+1)^3-(x-1)^3=6(x^2+x+1)$;

(10) $x(x+2)-(x+3)(x-3)=13$;

(11) $4x(x-1)-(2x+5)(2x-5)=1$;

(12) $3x-5(x+1)(x-1)+5(x+2)(x-2)=6$;

(13) $3(2x-1)(2x+1)-4(3x-2)(3x+2)+6x(4x+1)=31$;

(14) $(x+2)(x^2-2x+4)-x(x+3)(x-3)=26$;

(15) $6(x+1)^2-2(x-1)(x^2+x+1)-2(x+1)^3=32$;

(16) $5x(x-3)^2-5(x-1)^3+15(x+2)(x-2)=5$;

(17) $(x+2)^3-x(3x+1)^2+(2x+1)(4x^2-2x+1)=42$;

(18) $6x+(4x^3-12x^2)\div 2x^2=10$;

(19) $6x-(14x^3-21x^2)\div 7x^2+15=14$.

2.109. Solve the following equations:

(1) $5x-10a=15b$; (2) $(a+b)x=ac+bc$;

(3) $bx-abx=b^2-ab^2$; (4) $ax-bx=a^2-b^2$;

(5) $a^2x-b^2x=a^2+2ab+b^2$;

(6) $a^2x-2abx=2a^3-2b^3$;

(7) $3mx+3nx=6m^2-6n^2$;

(8) $a^2x-b^2x=5a^3+5b^3$; (9) $ax+x=a^2+2a+1$;

(10) $m^2x+2mnx+n^2x=3m^2-3n^2$.

2.110. Solve the following equations:

(1) $\frac{3x-1}{5}-\frac{5x+1}{6}=\frac{x+1}{8}-3$;

(2) $\frac{3y+1}{3}-\frac{16-y}{6}=\frac{9y+1}{7}+3$;

(3) $\frac{8x-1}{5}-\frac{50-2x}{9}=\frac{3x+3}{4}+1$;

(4) $\frac{1-7x}{8}-\frac{x+30}{3}-\frac{x-1}{5}=3$;

(5) $\frac{5x+1}{3}-\frac{17-x}{2}=\frac{3x+1}{8}+15$;

(6) $\frac{4x+1}{3}-\frac{3x-1}{5}=15-\frac{25-x}{4}$;

(7) $y(3-y)^2=20-(2-y)^3$; (8) $4-x(x-6)^2=(4-x)^3$;

(9) $(x-2)^3=(x-5)^2(x+4)$;

(10) $(1-x)^3+x(x-1.5)^2=4$;

(11) $(2y+1)^3=(2y-1)^2(2y+5)$;

(12) $(x+2)^3=x(x+3)^2+23$.

2.111. Solve the following equations:

(1) $\frac{11}{x^2-5x}+\frac{6}{x^2+4x}+\frac{4}{(x+4)(5-x)}=0$;

(2) $\frac{2x+5}{6}+\frac{10}{x-3}=\frac{2x-3}{6}$;

(3) $\frac{2}{3x}-\frac{2}{x+5}\left(\frac{x+5}{3x}-x-5\right)\div\frac{x-15}{x}=1$;

(4) $\frac{5}{(6x-1)^3}-\frac{7}{(1-6x)^2}+\frac{7}{36x^2-1}=0$;

(5) $\frac{1}{49x^3-4}+\frac{2}{(2-7x)^2}-\frac{21x}{(7x-2)^3}=0$;

(6) $\frac{3}{x^3-3x^2+6x-8}+\frac{2}{x^2-x+4}=\frac{2}{4-4x+x^2}$;

(7) $\frac{3x-1}{6x-3}-\frac{1}{1-4x^2}=\frac{x}{2x+1}$;

(8) $\frac{x+6}{x^2-7x}-\frac{4}{(7-x)^2}=\frac{1}{x-7}$;

(9) $\frac{1}{1-8x+16x^2}-\frac{1}{1+8x+16x^2}=\frac{2}{64x^3-16x^2-4x+1}$.

2.112. Solve the following equations:

(1) $\frac{2}{x+3}-\frac{3}{2x-6}=\frac{2x+3}{4x^2-36}$;

(2) $\frac{74-9x}{x^3-2x^2-4x+8}-\frac{8}{x^2-4x+4}=\frac{7}{x^2-4}$;

(3) $\frac{3x+1}{2}+4=\frac{x+4}{3}-\left(3x-\frac{5x-1}{6}\right)$;

(4) $\frac{b+1}{5by+2b^2}-\frac{b-1}{2b^2-5by}=\frac{8y}{25y^2-4b^2}$;

(5) $\frac{5}{b+3}-\frac{x}{3-b}=\frac{13x}{b^2-9}$;

(6) $\frac{x}{a^2-a+1}-\frac{1}{2a+2}=\frac{2x-1}{2a^2-2a+2}+\frac{a-ax}{a^3+1}$;

(7) $(a+1)^2\cdot x=1+x-(1-a)(1+a)$;

(8) $\frac{4}{x^2-3ax}+\frac{1}{x^2+3ax}+\frac{14}{9a^2-x^2}=0$;

(9) $\frac{a^2-1}{a^2+1}=\frac{a(x-1)+a^2-1}{a(x-1)-(a^2-1)}$;

(10) $\frac{a^3-1}{a^3+1}=\frac{a(x-1)+a^2-x}{a(x-1)-a^2+x}$;

(11) $((n-1)^2+n)\div\left(\frac{(n+1)^2}{n}-n\right)=x\div\left(\frac{(n-1)^2}{4n}+1\right)$.

2.113. Solve the following equations:

(1) $|x-8|=x-8$; (2) $|x-4|=4-x$;

(3) $\frac{|x-6|}{x-6}-1$; (4) $\frac{|x-9|}{x-9}=-1$;

(5) $|5x-30|=30-5x$; (6) $\frac{|3-2x|}{2x-3}=1$;

(7) $\frac{|x-7|}{x-7}=1$; (8) $\frac{|1.5-x|}{x-1.5}=-1$.

2.114. Solve the following equations:

(1) $|x|-1=5$; (2) $3|x|+1=|x|+7$;

(3) $|x+4|=4$; (4) $|2x-3|=5$;

(5) $|x+5|=x+5$; (6) $|x-2|=x-2$;

(7) $|x+5|=8$; (8) $2-x=|3x-1|$;

(9) $1-|x|=0.5$; (10) $1+|x|=a$;

(11) $|1-x|=0.5$; (12) $|1-x|=a$;
(13) $|x+3|=3+2x$; (14) $|7x-1|=21-9x$;
(15) $|5-x|=|x+4|$; (16) $|1-3x|=|3-2x|$;
(17) $|x|+|x-1|=1$; (18) $|x+1|+|x+2|=1$;
(19) $|x+3|+|2x-1|=5$;
(20) $|5x+3|-|0.5x-1|=3$;
(21) $|x+3|-|x-1|=4$; (22) $x=|ax+b|$;
(23) $|2x+b|=3$; (24) $|ax+b|=c$.

2.115. Solve the following equations:

(1) $|x+1|+|x-2|=3$;
(2) $|x-1|+|x+1|=2$;
(3) $|5-2x|+|x+3|=|2-3x|$;
(4) $|x|+|x+2|+|2-x|=x+1$;
(5) $|1-x|-|x+3|=|x+2|$;
(6) $|6-2x|-|x-3|=|x-5|$;
(7) $\left|\frac{4-x}{3}-1\right|=7$; (8) $\left|\frac{x-3}{2}+1\right|=2$.

2.116. For $y=4$ the fraction $\frac{p+19}{17-3y}$ is equal to 4. For what value of y will the given fraction be equal to 1?

2.117. The fraction $\frac{x-4}{5x-2b+1}$ is equal to 0 for $x=4$. For what value of x will this fraction have no meaning?

2.118. The fraction $\frac{5x-7}{x+3a}$ is equal to 1 for $x=10$. For what value of x will this fraction be equal to 3?

2.119. Does the equation $a+x=b$, where a and b are natural numbers, have a solution:
(1) in the set of all natural numbers;
(2) in the set of all whole numbers;
(3) in the set of all positive rational numbers?

2.120. Does the equation $ax=b$ (a and b integers) have a solution:
(1) in the set of all whole numbers;
(2) in the set of all rational numbers?

§ 27. Determinants

2.121. Compute:

(1) $\begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix}$; (2) $\begin{vmatrix} -2 & -2 \\ 4 & 0 \end{vmatrix}$; (3) $\begin{vmatrix} 2 & 3 \\ 3 & 4.5 \end{vmatrix}$;

(4) $\begin{vmatrix} -3/4 & 1/3 \\ -1 & -8/9 \end{vmatrix}$; (5) $\begin{vmatrix} a & b \\ b & a \end{vmatrix}$; (6) $\begin{vmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{vmatrix}$;

(7) $\begin{vmatrix} 1 & 1 & 1 \\ 3 & 1 & 4 \\ 8 & 9 & 5 \end{vmatrix}$; (8) $\begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & -1 & 1 \end{vmatrix}$; (9) $\begin{vmatrix} 2 & -1 & 1 \\ 1 & -2 & 1 \\ 1 & -1 & 2 \end{vmatrix}$;

(10) $\begin{vmatrix} 1 & 2 & 19 \\ 1 & 4 & 0 \\ 4 & 5 & -8 \end{vmatrix}$; (11) $\begin{vmatrix} 7 & 1 & 1 \\ 8 & 2 & 1 \\ -9 & -1 & -2 \end{vmatrix}$; (12) $\begin{vmatrix} 1 & 3 & 3 \\ 3 & 7 & 2 \\ 2 & 2 & 1 \end{vmatrix}$;

(13) $\begin{vmatrix} 1 & -a & a^2 \\ 1 & -b & b^2 \\ 1 & -c & c^2 \end{vmatrix}$; (14) $\begin{vmatrix} 2 & -4 & 1 \\ 2 & -5 & 3 \\ -1 & -1 & 1 \end{vmatrix}$; (15) $\begin{vmatrix} 0 & a & 0 \\ b & 0 & c \\ 0 & d & 0 \end{vmatrix}$;

(16) $\begin{vmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 1 \end{vmatrix}$.

2.122. Prove the given equalities:

(1) $\begin{vmatrix} a_1 & k & b_1 \\ a_2 & k & b_2 \end{vmatrix} = k \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$;

(2) $\begin{vmatrix} 1 & a & a \\ 1 & b & b \\ 1 & c & c \end{vmatrix} = (b-a)(c-a)(c-b)$;

(3) $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = abc(a-b)(b-c)(c-a)$.

2.123. Perform the indicated operations:

(1) $\dfrac{\begin{vmatrix} 2a & b \\ b & 2a \end{vmatrix} + \begin{vmatrix} b & 2b \\ -2a & 2b \end{vmatrix}}{\begin{vmatrix} 2 & b \\ -1 & a \end{vmatrix}}$; (2) $\dfrac{\begin{vmatrix} a & -2 \\ b & 3 \end{vmatrix}^2 - \begin{vmatrix} (3a+b) & 2 \\ -2 & 4b \end{vmatrix}}{3a-2}$.

2.124. Solve the following equations:

(1) $\begin{vmatrix} 2x-3x \\ 5x-3 \end{vmatrix} = \begin{vmatrix} 2x+1 & -4 \\ 4 & x \end{vmatrix}$;

(2) $\begin{vmatrix} 3y-1 & \frac{1}{2}(3+3y) \\ 4y & y \end{vmatrix} - \begin{vmatrix} -y & -1 \\ 1 & 3y \end{vmatrix} = 0.$

2.125. (1) For what value of x is the determinant $\begin{vmatrix} 2x & 1 \\ -3 & 2 \end{vmatrix}$ equal to the determinant $\begin{vmatrix} 2 & 1 \\ -1 & 3 \end{vmatrix}$?

(2) For what value of m does the equation $\begin{vmatrix} 2x-1 & 2 \\ 3m & -1 \end{vmatrix} = 0$ have the root $-5/2$?

2.126. For what value of x is the fraction D_1/D_2:

(a) equal to $2/5$;

(b) meaningless if $D_1 = \begin{vmatrix} 1 & -2 \\ x & 3 \end{vmatrix}$; $D_2 = \begin{vmatrix} -1 & x \\ 2 & -3 \end{vmatrix}$.

2.127. Find x from the proportion $D_1/D_2 = D_3/D_4$, where

$$D_1 = \begin{vmatrix} 2x & -5 \\ 2 & 3 \end{vmatrix}; \quad D_2 = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix};$$

$$D_3 = \begin{vmatrix} 2 & 1 \\ -3 & 2x \end{vmatrix}; \quad D_4 = \begin{vmatrix} -3 & 1 \\ 2 & -1 \end{vmatrix}.$$

2.128. Write the following expressions in the form of determinants:

(1) x^2+2x+1; (2) $4a^2-9a-9$; (3) $x^2+y^2+z^2+3xyz$.

2.129. Compute the determinant $\begin{vmatrix} a & b \\ a & b \end{vmatrix}$. Give some examples of such determinants and formulate the relevant property of the determinant.

2.130. Compute the determinant $\begin{vmatrix} a & ka \\ a & ka \end{vmatrix}$. Give some examples of such determinants and formulate the relevant property of the determinant.

2.131. Prove that $\begin{vmatrix} a_1 & kb_1 \\ a_2 & kb_2 \end{vmatrix} = k \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$. Give some examples of such determinants and formulate the relevant property of the determinant.

§ 28. Systems of First-degree Equations in Two Variables and Systems Reducible to Them

2.132. Solve the following systems by the substitution method:

(a) (1) $\begin{cases} 2x+y=11, \\ 3x-y=9; \end{cases}$ (2) $\begin{cases} 3x+4y=253, \\ y=5x; \end{cases}$

(3) $\begin{cases} 9x-4y=98, \\ x=\frac{3}{5}y; \end{cases}$ (4) $\begin{cases} 3x-8x=15, \\ 7y-2y=0; \end{cases}$

(5) $\begin{cases} x+y=a+2b, \\ x-y=a; \end{cases}$ (6) $\begin{cases} ax+y=1, \\ ay-b^2x=b; \end{cases}$

(b) (1) $\begin{cases} \frac{7x-3y}{5}=\frac{5x-y}{3}-\frac{x+y}{2}, \\ 5(y+1)=3(x-1); \end{cases}$

(2) $\begin{cases} 1-0.3y+0.6=\frac{x+1}{5}, \\ \frac{y-3}{4}=\frac{4x+9}{20}-1.5; \end{cases}$

(3) $\begin{cases} 7+\frac{x-3y}{4}=2x-\frac{y+5}{3}, \\ \frac{10(x-y)-4(1-x)}{3}=y; \end{cases}$

(4) $\begin{cases} \frac{x+2y}{3}-\frac{1}{5}=\frac{x+5}{15}, \\ x-3y-12.5=-0.25y. \end{cases}$

2.133. Solve the following systems using the method of algebraic addition:

(a) (1) $\begin{cases} x+y=13, \\ 2x-y=12.5; \end{cases}$ (2) $\begin{cases} 16x-27y=-6, \\ 5x-18y=-21; \end{cases}$

(3) $\begin{cases} 2x+5y=15, \\ -x+3y=-2; \end{cases}$ (4) $\begin{cases} 18x-21y=2, \\ 24x-15y=7; \end{cases}$

(5) $\begin{cases} 64x+51y=90, \\ 25x+34y=7; \end{cases}$ (6) $\begin{cases} x-7y=3, \\ 5x-21y=11; \end{cases}$

(b) (1) $\begin{cases} x+y=a, \\ 2x+4y=3a-b; \end{cases}$ (2) $\begin{cases} ax+2y=3, \\ 3x-ay=4; \end{cases}$

(3) $\begin{cases} 4ax-3by=1, \\ ax+2by=25; \end{cases}$ (4) $\begin{cases} (a+b)x-(a-b)y=4ab, \\ \frac{x}{a+b}+\frac{y}{a-b}=2; \end{cases}$

(5) $\begin{cases} \frac{x}{a}+\frac{y}{b}=1, \\ \frac{x}{b}-\frac{y}{a}=1; \end{cases}$ (6) $\begin{cases} |x-1|+|y-5|=1, \\ y-5=|x-1|. \end{cases}$

2.134. Solve the following systems by introducing an auxiliary variable:

(1) $\begin{cases} \frac{1}{x+y}+\frac{1}{x-y}=2, \\ \frac{3}{x+y}+\frac{4}{x-y}=7; \end{cases}$ (4) $\begin{cases} \frac{6}{2x+y-1}-\frac{2}{2x-y+3}=\frac{5}{2}, \\ \frac{4}{2x+y-1}+\frac{4}{2x-y+3}=3; \end{cases}$

(2) $\begin{cases} \frac{3}{2x+y}+\frac{7}{x-y}=1.9, \\ \frac{5}{x-y}-\frac{2}{2x+y}=1.15; \end{cases}$ (5) $\begin{cases} \frac{3}{4x+3y}+\frac{2}{4x-3y}=\frac{37}{55}, \\ \frac{5}{4x-3y}-\frac{1}{4x-3y}=\frac{14}{55}; \end{cases}$

(3) $\begin{cases} \frac{11}{2x-3y}+\frac{18}{3x-2y}=13, \\ \frac{27}{3x-2y}-\frac{2}{2x-3y}=1; \end{cases}$ (6) $\begin{cases} \frac{5}{2x+y}+\frac{4}{2x-y}=\frac{7}{3}, \\ \frac{15}{2x+y}+\frac{2}{2x-y}=\frac{11}{3}. \end{cases}$

2.135. Solve the following systems using the determinant method:

(a) (1) $\begin{cases} 3x+5y=4, \\ 7x-3y=2; \end{cases}$ (2) $\begin{cases} 7x-3y=15, \\ 5x+6y=27; \end{cases}$

(3) $\begin{cases} 7x-3y+1=0, \\ 4x-5y+17=0; \end{cases}$ (4) $\begin{cases} 15x+23y+10=0, \\ 3x+4y+2=0; \end{cases}$

(b) (1) $\begin{cases} 8x+2ay=1, \\ 5x+4ay=2; \end{cases}$ (2) $\begin{cases} ax+4y=9, \\ 9x+ay=1; \end{cases}$

(3) $\begin{cases} ax-y=-a^2, \\ 3ax+y=5a; \end{cases}$ (4) $\begin{cases} -x+ay=0, \\ ay+x=a; \end{cases}$

(5) $\begin{cases} 2x+ay=-6, \\ ax+8y=12; \end{cases}$ (6) $\begin{cases} 3x-ay=6-a, \\ -ax+3y=3-2a; \end{cases}$

(7) $\begin{cases} (a-1)x-y=1, \\ x+ay=a; \end{cases}$ (8) $\begin{cases} ax-(a-1)y=0.5, \\ (a-1)x-ay=a; \end{cases}$

(c) (1) $\begin{cases} x+y+z=6, \\ x+y=5, \\ y+z=0; \end{cases}$ (2) $\begin{cases} 2x+y-4z=7, \\ 3x-2y+2z=22, \\ x-3y+3z=12; \end{cases}$

(3) $\begin{cases} 2x-4y-z=9, \\ 3x+y-2z=8, \\ 4x+y+z=0; \end{cases}$ (4) $\begin{cases} x+y+z=3, \\ x+2y-z=2, \\ x-y+7z=3; \end{cases}$

(5) $\begin{cases} x+y-z=0, \\ x-y+z=2, \\ -x+y+z=4. \end{cases}$

2.136. (a) Without solving the following systems of equations, determine whether the given system will have: (1) one solution; (2) infinitely many solutions; (3) no solution:

(1) $\begin{cases} 2x+y=11, \\ x+3y=18; \end{cases}$ (2) $\begin{cases} 2x+3y=4, \\ 4x+6y=8; \end{cases}$ (3) $\begin{cases} 5x+3y=15, \\ 10x-6y=9; \end{cases}$

(4) $\begin{cases} 3x+10y=16, \\ 6x+20y=32; \end{cases}$ (5) $\begin{cases} 6x-9y=4, \\ 4x-6y=9; \end{cases}$

(6) $\begin{cases} 4x+10y=12, \\ 6x+15y=18. \end{cases}$

(b) Determine the values of a and b for which the given systems of equations have infinitely many solutions:

(1) $\begin{cases} ax-by=15, \\ 4x+by=2; \end{cases}$ (2) $\begin{cases} ax-y=b, \\ 4x+3y=10; \end{cases}$

(3) $\begin{cases} ax+4y=a+1, \\ 2x+by=b; \end{cases}$ (4) $\begin{cases} 3x+8y=10, \\ x-ay=7; \end{cases}$

(5) $\begin{cases} 3x+2ay=1, \\ (3a-1)x-ay=1; \end{cases}$ (6) $\begin{cases} x+ay=6a, \\ bx+2y=b. \end{cases}$

2.137. Solve the following systems of equations:

(1) $\begin{cases} \frac{1}{2}\left(y+\frac{x}{2}\right)-\frac{1}{5}(x+2)=1.1, \\ x-2y+4=\frac{1}{4}\left[2x+3\left(y-\frac{1}{2}\right)\right]; \end{cases}$

(2) $\begin{cases} \frac{x+3}{y-4}-\frac{x-1}{y+4}+\frac{16}{y^2-16}=0, \\ -3y=1-11x; \end{cases}$

(3) $\begin{cases} \frac{5y-2y^2}{x^2-2xy+6y-9}=\frac{y}{x-3}-\frac{y-4}{x-2y+3}, \\ 7x+3y=11; \end{cases}$

(4) $\begin{cases} \frac{3y+1}{z-5}-\frac{z-5}{3y+1}=\frac{9y^2-z^2-21}{3yz-15y+z-5}, \\ -15y+16z=13; \end{cases}$

(5) $\begin{cases} \frac{2x^2-5}{16-x^2}+\frac{x-y}{x-4}+\frac{x+y}{x+4}=0, \\ 7x-10y=3; \end{cases}$

(6) $\begin{cases} \frac{2}{x^2-9}+\frac{1}{(y+3)(x-3)}=\frac{48}{x^2y+3x^2-9y-27}, \\ x-2y=1; \end{cases}$

(7) $\begin{cases} \frac{1}{(x-y)(y+5)}-\frac{2}{y^2-25}+\frac{48}{xy^2-25x+25y-y^3}=0, \\ 2x+y=3. \end{cases}$

2.138. Solve the following systems of equations:

(1) $\begin{cases} x+y+z=-2, \\ x-y+2z=-7, \\ 2x+3y-z=1; \end{cases}$ (2) $\begin{cases} x+2y-z=7, \\ 2x-y+z=2, \\ 3x-5y+2z=-7; \end{cases}$

(3) $\begin{cases} x-2y+3z=-1, \\ 2x+y-5z=9, \\ 4x-3y+z=7; \end{cases}$ (4) $\begin{cases} x+2y-z=0, \\ 3x+5y+z=-10, \\ x+y+3z=-10; \end{cases}$

(5) $\begin{cases} x-y-z=5, \\ 2x+y+3z=3, \\ x-4y-6z=7; \end{cases}$ (6) $\begin{cases} x-3y+z=7, \\ 3x+y-2z=3, \\ x+7y-4z=0; \end{cases}$

(7) $\begin{cases} x+y+z+t=2, \\ 2x-y-z+2t=7, \\ 3x+2y-5z+t=3, \\ x2-y+3z+t=5; \end{cases}$ (8) $\begin{cases} x+2y+3z+4t=30, \\ 2x-3y+5z-2t=3, \\ 3x+4y-2z-t=1, \\ 4x-y+6z-3t=8. \end{cases}$

§ 29. Definition of the Root. The Arithmetic Root. Finding the Square Root with the Help of an Algorithm and Tables

2.139. (Orally). Compute:

$\sqrt[3]{-1}$; $\sqrt[3]{-8}$; $\sqrt[5]{32}$; $\sqrt[5]{-32}$; $\sqrt[3]{-8/27}$; $\sqrt[4]{(-5)^4}$; $\sqrt[3]{0.008}$; $\sqrt[3]{-0.064}$; $\sqrt[3]{-64/125}$; $\sqrt[5]{-243/1024}$; $\sqrt[3]{3215/32}$.

2.140. Find the arithmetic root:

$\sqrt{25}$; $\sqrt{16}$; $\sqrt[3]{27}$; $\sqrt[3]{64}$; $\sqrt{(-3)^2}$; $\sqrt[4]{(-3)^4}$; $\sqrt[3]{125}$; $\sqrt{(1-\sqrt{3})^2}$; $\sqrt{(5-\sqrt{2})^2}$; $\sqrt[3]{(3-\sqrt{2})^3}$; $\sqrt[4]{(2-\sqrt{5})^4}$; $\sqrt[4]{(\sqrt{2}-\sqrt{3})^4}$; $\sqrt[6]{(\sqrt{5}-\sqrt{7})^2}$; $\sqrt[8]{(\sqrt[4]{2}-\sqrt[4]{3})^8}$.

2.141. Compute:

(1) $\sqrt{(5-a)^2}$ for $a \leqslant 5$; (2) $\sqrt{(5-a)^2}$ for $a \geqslant 5$;
(3) $\sqrt[4]{(x-3)^4}$ for $x \geqslant 3$; (4) $\sqrt[4]{(x-3)^4}$ for $x \leqslant 3$;
(5) $\sqrt{(x-y)^2}$ for $x < y$; (6) $\sqrt[4]{(a-b)^4}$ for $a > b$, for $a < b$;
(7) $\sqrt[6]{(m-n)^6}$ for $m < n$, for $m > n$;
(8) $\sqrt[5]{(a-b)^5}$ for $a < b$, for $a > b$,

and show that $\sqrt{a^2} = |a|$ for any a.

2.142. Prove that

(1) $2a + \sqrt{(a-3)^2} = \begin{cases} 3a-3 & \text{if } a \geqslant 3, \\ a+3 & \text{if } a < 3; \end{cases}$

(2) $m + n + \sqrt{(m-n)^2} = \begin{cases} 2m & \text{if } m \geqslant n, \\ 2n & \text{if } m < n. \end{cases}$

2.143. Find the arithmetic values of the given roots:

(1) $\sqrt{(2x^2-3x+1)^2}$; (2) $\sqrt{(x+2)^2}$;
(3) $\sqrt{9a^2-6\sqrt{2a}+2}$; (4) $\sqrt{(-3x^2+x-1)^2}$;
(5) $\sqrt[4]{(a^2-2a+3)^4}$; (6) $\sqrt{a^2-4a\sqrt{2}+8}$.

2.144. Find x:

(a) (1) $\sqrt{x}=5$; (2) $\sqrt[3]{x}=2$; (3) $\sqrt[4]{x}=3$; (4) $\sqrt[5]{x}=2$;
(5) $3+\sqrt{x}=5$; (6) $\sqrt{x}-4=3$; (7) $7-\sqrt{x}=4$;
(8) $\sqrt{x+2}=8$; (9) $\sqrt[4]{x-1}=0.5$;

(b) (1) $\sqrt{4x-20}+\sqrt{x-5}-(1/3)\sqrt{9x-45}=4$;
(2) $\sqrt{16x+16}-\sqrt{9x+9}+\sqrt{4x+4}=16-\sqrt{x+1}$.

2.145. Find the permissible values of x:

(1) $\sqrt{x+1}$; (2) $\sqrt{1-x}$; (3) $\sqrt{3-2x}$; (4) $\sqrt{x(x-1)}$;
(5) $\sqrt{-x^2}$; (6) $\sqrt{-(-x)^2}$; (7) $\sqrt[4]{5-x}$;
(8) $\sqrt[4]{(7-2x)^2-4x^2}$; (9) $\sqrt[8]{(2-x)^2+(3-x)^2-2x^2}$;
(10) $\sqrt[6]{(x-1)^2-16}$; (11) $\sqrt[4]{|x|-3}$; (12) $\sqrt[4]{x^2+x+1}$.

2.146. Find the square root of the following numbers: 841; 784; 1 225; 1 849; 7 921; 5 329; 4 624; 3 136; 2 401; 7 225; 57 600; 32 400; 14 400; 28 900; 54 756; 17 424; 56 169; 42 849; 94 864; 64 009; 831 744; 687 241; 259 081; 879 844; 725 904; 488 601; 501 264; 700 569; 632 025; 613 089; 225 625; 5 616 900; 3 587 236; 2 105 401; 3 426 201; 2 934 369.

2.147. Take the square root of the following fractions:

(1) $\sqrt{\frac{64}{81}}$; $\sqrt{\frac{121}{324}}$; $\sqrt{\frac{256}{289}}$; $\sqrt{\frac{361}{1841}}$; $\sqrt{2\frac{7}{9}}$;
$\sqrt{5\frac{1}{16}}$; $\sqrt{552\frac{1}{4}}$; $\sqrt{5\frac{31}{225}}$; $\sqrt{\frac{144}{961}}$; $\sqrt{7\frac{21}{25}}$;
$\sqrt{11\frac{7}{81}}$; $\sqrt{6\frac{3}{121}}$; $\sqrt{14\frac{25}{36}}$; $\sqrt{12\frac{37}{49}}$; $\sqrt{2\frac{218}{256}}$;
$\sqrt{15\frac{1}{64}}$; $\sqrt{23\frac{13}{36}}$;
(2) $\sqrt{0.9801}$; $\sqrt{0.0625}$; $\sqrt{0.0484}$; $\sqrt{0.8649}$; $\sqrt{0.2116}$;
(3) $\sqrt{0.00001225}$; $\sqrt{0.00005329}$; $\sqrt{19.0969}$; $\sqrt{83.1744}$;
$\sqrt{19.9809}$; $\sqrt{5.6169}$; $\sqrt{70.0569}$; $\sqrt{5.4756}$.

§ 30. Approximate Roots

2.148. Take the roots of the following numbers to within 0.1:

(a) $\sqrt{5}$; $\sqrt{45}$; $\sqrt{152}$; $\sqrt{1000}$; $\sqrt{2346}$; $\sqrt{566}$; $\sqrt{6270}$;

(b) $\sqrt{95.3}$; $\sqrt{10.9}$; $\sqrt{2.24}$; $\sqrt{12.27}$; $\sqrt{27.4}$; $\sqrt{75.68}$.

2.149. Find the root to within 0.01:

(1) $\sqrt{2}$; $\sqrt{3}$; $\sqrt{5}$; $\sqrt{10}$; $\sqrt{12}$; $\sqrt{6.4}$; $\sqrt{23.4}$; $\sqrt{152.4}$; $\sqrt{3.2}$;

(2) $\sqrt{\frac{7}{25}}$; $\sqrt{1\frac{2}{9}}$; $\sqrt{5\frac{7}{25}}$; $\sqrt{8\frac{11}{16}}$; $\sqrt{3\frac{5}{36}}$; $\sqrt{7\frac{2}{25}}$.

2.150. Take the root:

$\sqrt[6]{2^{12}}$; $\sqrt[4]{3^8}$; $\sqrt[3]{-a^6}$; $\sqrt[5]{-10^{10}}$; $\sqrt[3]{8\cdot 3^3}$; $\sqrt[5]{32\cdot 10^5}$; $\sqrt[6]{27^4}$; $\sqrt[4]{16\cdot 81}$; $\sqrt[-3]{27}$; $\sqrt{3\frac{1}{16}b^4}$; $\sqrt[3]{\frac{8}{125}b^6}$; $\sqrt[3]{-2\frac{10}{27}a^6b^6}$; $\sqrt[4]{\frac{16}{81}x^4y^8}$; $\sqrt[3]{-\frac{a^3}{64}}$; $\sqrt[4]{a^{16}b^8c^4}$; $\sqrt{2^4a^{-6}b^{12}}$.

2.151. Take the root:

(1) $\sqrt[3n]{a^{6m+9mn}}$; $\sqrt[n+2]{a^{3n+6}}$; $\sqrt[3+n]{a^{15+5n}}$;

(2) $\sqrt{\frac{a^4}{9}}$; $\sqrt[3]{-\frac{a^{3n}}{64^{-n}}}$; $\sqrt[5]{-\frac{a^{10}}{b^{14}}}$; $\sqrt[5]{\frac{16}{81}a^{8n}b^{16}}$;

(3) $\sqrt[3]{\frac{125}{64}a^{6n}c^{15}}$; $\sqrt[p+3]{a^{(p-3)^2+12p}}$.

2.152. Remove a factor from the radicand:

(1) $\sqrt{75}$; $\sqrt[3]{500}$; $\sqrt[3]{-108}$; $\sqrt[3]{-72}$; $\sqrt[4]{162}$; $\sqrt[4]{1250}$; $\sqrt[4]{112}$; $\sqrt[5]{486}$; $\sqrt[5]{-224}$;

(2) $\sqrt[3]{x^{10}y^7}$; $\sqrt[6]{a^5b^6}$; $\sqrt[4]{a^{10}b^7}$; $a\sqrt{\frac{0.54z}{a^2x^2}}$; $a^2\sqrt{\frac{-0.54z}{a^6x^6}}$; $\sqrt{\frac{(a^2-2ab+b^2)y}{25}}$; $\sqrt{\frac{50z}{(a^2+2ab+b^2)}}$;

(3) $\sqrt{\frac{a}{b^2}-\frac{1}{b}}$; $\sqrt{\frac{x^3}{a^6}-\frac{1}{a}}$; $\sqrt{\frac{(a^2-2ab+b^2)^3\,y^5}{250}}$;
$\sqrt[3]{\frac{(y^2-x^2)^4}{8(x+y)}}$; $\sqrt[5]{\frac{(x^2-y^2)^6}{32(y-x)}}$; $\sqrt[3]{\frac{b^3}{a^4}-\frac{b^5}{a^6}}$;

(4) $\frac{3}{2a}\sqrt{4a^2-\frac{8a^2b^2}{9}}$; $\sqrt[m]{2^{m+1}\cdot a^{5m}\cdot b^{m+n}\cdot c^{mp+1}}$;
$\frac{ac}{b}\sqrt{3^{n+2}\cdot a^{n+5}\cdot b^{2n-1}\cdot c^{1-3n}}$;
$\sqrt[m+n]{a^{2m+n}b^{m+2n}c^{m^2-n^2}}$; $yz^2\sqrt[2p]{x^{4p+1}y^{6p+2}z^5}$.

2.153. Introduce a factor into the radicand:

(1) $6\sqrt{5}$; $4\sqrt{3}$; $3\sqrt[3]{2}$; $5\sqrt[3]{3}$; $7\sqrt[3]{3}$; $a^3\sqrt{2ab}$:

(2) $m^2\sqrt[3]{mn}$; $2n\sqrt[3]{m^2n}$; $m\sqrt[5]{1-\frac{1}{m^5}}$;
$\frac{1}{m}\sqrt[5]{m^5-1}$; $2ac^3\sqrt[3]{3abc^2}$; $\frac{4a}{3b}\sqrt[5]{\frac{127b^3}{16a^4}}$;

(3) $3a^nb\sqrt[m]{3a^2\cdot b}$; $2ab^m\sqrt[n]{3a\cdot b^2}$;

(4) $(2-a)\sqrt{\frac{2a}{a-2}}$, $a>2$;

$(x-5)\sqrt{\frac{x}{25-x^2}}$, $0<x<5$;

$(a-b)\sqrt{\frac{3a^2}{b^2-a^2}}$, $0<a<b$;

$\frac{2}{x-y}\sqrt{\frac{x^2-y^2}{2}}$, $0<x<y$, $x>y$;

$(a-3)\sqrt{\frac{3a}{a-3}}$, $a>3$.

2.154. Reduce both the indices of the given roots and exponents in the radicands:

(1) $\sqrt[3]{a^6}$; $\sqrt[8]{a^{10}\cdot b^{12}}$; $\sqrt[10]{a^{15}\cdot b^{25}}$; $\sqrt[3n]{a^{2n}\cdot b^{3n}}$;
$\sqrt[5n]{a^{10}\cdot b^{5n}}$; $\sqrt[mn]{a^m\cdot b^{2m}}$;

(2) $\sqrt[mn]{a^{2mn}\cdot b^{3n}}$; $\sqrt[6]{9a^4\cdot b^6}$; $\sqrt[4]{4a^8\cdot b^2}$; $\sqrt[13]{81a^{16}\cdot b^{4n}}$;
$\sqrt[6n]{\frac{16a^{10}\cdot b^6}{9c^{18}}}$; $\sqrt[6n]{\frac{27a^9\cdot b^{12}}{8c^{15}}}$; $\sqrt[12]{\frac{1000a^{-6}}{729b^9c^{-3}}}$;
$\sqrt[15]{\frac{32a^{-25}c^{15}}{3125b^{30}}}$; $\sqrt[18]{\frac{729a^{12}b^{-24}}{64c^{-36}}}$;

(3) $\sqrt[6]{(1-\sqrt{3})^2}$; $\sqrt[4]{(1-\sqrt{2})^2}$; $\sqrt{(2-\sqrt{5})^2}$;
$\sqrt[6]{(2-\sqrt{6})^2}$; $\sqrt[8]{(3-\sqrt{10})^4}$; $\sqrt[12]{(5-\sqrt{26})^{30}}$;
(4) $\sqrt[4]{(x-1)^2}$, $x<1$; $\sqrt[6]{(2-x)^2}$, $x>2$; $\sqrt[8]{(3-x)^4}$, $x>3$.

2.155. Reduce the radicals to a common index:
(1) $\sqrt[6]{a^5}$ and $\sqrt[5]{a^3}$; (2) $\sqrt[9]{a^4}$ and $\sqrt[6]{a^5}$;
(3) $\sqrt[6]{a^2}$, $\sqrt[15]{a^3b^4}$, and $\sqrt[50]{a^{10}b^{20}}$;
(4) $\sqrt[12]{3a^5/b^3}$ and $\sqrt[9]{10b^2/a}$;
(5) $\sqrt{5a/b}$ and $\sqrt{3a/b}$; (6) $\sqrt[4]{(x-1)/(x+1)}$,
$\sqrt[2n]{(x+1)/(x-1)}$, and $\sqrt[n]{x/y}$;
(7) $\sqrt[n^2]{3a^2/(bc^3)}$ and $\sqrt[mn]{2ab^2/c^3}$;
(8) $\sqrt[2n]{(a+b)/x}$, $\sqrt[6]{a/(a+y)}$, and $\sqrt[3n]{a/b}$;
(9) $\sqrt[mn]{2b^3/(ac^4)}$, $\sqrt[n^2]{3a^2c/b^2}$, and $\sqrt[m]{5a^nc^{n+1}/b^{3m+1}}$.

2.156. Get rid of fractions in the radicand:
(1) $\sqrt{5/12}$; $\sqrt[3]{1/4}$; $\sqrt[3]{2/9}$; $\sqrt[4]{3/8}$;
(2) $x\sqrt{x/y}$; $n\sqrt[3]{m/n}$; $b\sqrt[4]{a/b^3}$; $y\sqrt[5]{x^2/y^3}$;
(3) $(x-y)^2\sqrt{xy/(x-y)^2}$; $\sqrt[5]{a/b^3+c/b^2}$;
$(m-n)\sqrt[3]{(m+n)/(m-n)^2}$; $\sqrt[n]{2b^5c^3/a^7}$.

2.157. Reduce the given roots to the simplest form:

(1) $\frac{3xy^2}{2}\sqrt[3]{\frac{8}{xy}}$; $\frac{2x}{3y^2}\sqrt{\frac{8y^3}{x^5}}$; $a^2\sqrt{\frac{2ab^3}{3c^2d}}$;

(2) $\frac{2ab^2}{c}\sqrt[3]{\frac{5a}{16b^2c^3}}$; $\frac{a+b}{a}\sqrt[3]{\frac{a^{13}-a^{12}b}{(a-b)^2}}$;

(3) $\frac{a}{c}\sqrt{\frac{a^3b-4a^2b^2+4ab^3}{c^2}}$; $3x^2y\sqrt{\frac{12}{xy}}$;

(4) $\frac{5a^2}{7b}\sqrt{\frac{49b^3}{5a}}$; $\frac{3a^3b}{2c}\sqrt{\frac{4c^3}{9a^5b}}$; $\frac{x}{y}\sqrt[n]{\frac{y^{3n}+2}{x^{n-2}}}$;

(5) $\frac{2xy^2}{3ab}\sqrt{\frac{9a^3b^4}{8xy^3}}$; $2ab\sqrt{\frac{b^2}{8a}}$; $\frac{4}{ab}\sqrt[n]{a^{n+1}b^{2n+2}}$;

(6) $\frac{x^2}{y}\sqrt[3]{\frac{3y}{2x^2}}$; $\sqrt{25m^3-50n^2}$; $\sqrt{4x^6y^2+12x^4y^3}$;

(7) $2m\sqrt[4]{\frac{1}{m^3}+\frac{1}{m^4}}$; $\frac{2a^2}{3b}\sqrt[3]{\frac{b^3}{a^4}-\frac{b^5}{a^6}}$;

(8) $\frac{5x}{3y^2}\sqrt{\frac{y^5}{x^4}+\frac{y^4}{x^5}}$; $\frac{x}{x+1}\sqrt[4]{(1+2x+x^2)(2x+2)(x^4-x^2)}$.

2.158. Which is greater:

(1) $2\sqrt{3}$ or $3\sqrt{2}$; (2) $5\sqrt{7}$ or $8\sqrt{3}$;
(3) $2\sqrt[8]{3}$ or $3\sqrt[3]{2}$; (4) $3\sqrt[4]{4}$ or $4\sqrt[3]{2}$;
(5) $\sqrt{2}$ or $\sqrt[3]{3}$; (6) $\sqrt{8}$ or $\sqrt[3]{19}$;
(7) $\sqrt[3]{12}$ or $\sqrt{5}$; (8) $\sqrt[22]{2}$ or $\sqrt[15]{3}$;
(9) $\sqrt[15]{8}$ or $\sqrt[45]{3}$; (10) $\sqrt[18]{5}$ or $\sqrt[54]{4}$?

2.159. Prove that the roots are similar:

(1) $\sqrt[3]{54}$; $\sqrt[3]{16}$; $\sqrt[3]{432}$ and $\sqrt[3]{686}$;
(2) $(1/4)\sqrt{0.2}$; $(1/5)\sqrt{5}$; $\sqrt{20/49}$ and $\sqrt{80/121}$;
(3) $\sqrt[3]{8/3}$ and $\sqrt[3]{9/8}$; $\sqrt[3]{72/125}$ and $\sqrt[3]{243/64}$;
(4) $\sqrt{a-1/a^2}$ and $\sqrt{(a^3-1)/a^4}$;
(5) $\sqrt{1/b-a}$ and $\sqrt{(bd^2-ab^2d^2)/c^2}$;
(6) $\sqrt[3]{8a^5-16a^3b^2}$; $ab\sqrt[3]{1/a-2b^2/a^3}$ and $\sqrt[3]{2/(a^3b)-1/(ab)^3}$;
(7) $\sqrt[3]{\frac{x+y}{(x-y)^2}}$ and $\sqrt[3]{1/y-x^2/y^3}$;
(8) $\sqrt{1/(mn-nq)}$ and $\sqrt{n^3/(m-q)}$.

§ 31. Addition and Subtraction of Radicals

2.160. Perform the indicated operations:

(1) $(5\sqrt{2}-4\sqrt[3]{3})+(3\sqrt{2}+6\sqrt[3]{3})$;
(2) $(7\sqrt[3]{4}-2\sqrt{5})-(5\sqrt[3]{4}-4\sqrt{5})$;
(3) $(10\sqrt[4]{7}+\sqrt[5]{3})-(5\sqrt[3]{3}+2\sqrt[4]{7})$;
(4) $(2\sqrt[3]{11}-8\sqrt[5]{7})+(7\sqrt[5]{7}-\sqrt[3]{11})$;
(5) $(2+3\sqrt{32})+((1/2)\sqrt{128}-6\sqrt{18})$;
(6) $\sqrt{75}-\sqrt{147}+\sqrt{48}-(1/5)\sqrt{300}$;
(7) $20\sqrt{245}-\sqrt{5}+\sqrt{125}-2\frac{1}{2}\sqrt{180}$;

(8) $\sqrt[4]{32\sqrt[3]{4}}+\sqrt[4]{64\sqrt[3]{1/2}}-3\sqrt[3]{2\sqrt[4]{2}}$;

(9) $5\sqrt{48\sqrt[3]{2/3}}+\sqrt{32\sqrt[3]{9/4}}-11\sqrt[3]{12\sqrt{8}}$;

(10) $2\sqrt{40\sqrt{12}}+3\sqrt{5\sqrt{48}}-2\sqrt[4]{75}-4\sqrt{15\sqrt{27}}$;

(11) $5\sqrt[3]{6\sqrt{32}}-3\sqrt[3]{9\sqrt{162}}-11\sqrt[6]{18}+2\sqrt[3]{75\sqrt{50}}$.

2.161. Perform the indicated operations:

(1) $\frac{2}{\sqrt{3}-1}+\frac{3}{\sqrt{3}-2}+\frac{15}{3-\sqrt{3}}$;

(2) $\frac{5}{4-\sqrt{11}}+\frac{1}{3+\sqrt{7}}-\frac{6}{\sqrt{7}-2}-\frac{\sqrt{7}-5}{2}$;

(3) $\frac{4}{\sqrt{5}-\sqrt{2}}+\frac{3}{\sqrt{5}-2}-\frac{2}{\sqrt{3}-2}+\frac{\sqrt{3}-1}{6}$;

(4) $\frac{\sqrt{2}+1}{\sqrt{2}-1}+\frac{\sqrt{2}-1}{\sqrt{2}+1}-\frac{\sqrt{2}+3}{\sqrt{2}}$;

(5) $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}+\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}-\frac{\sqrt{5}+1}{\sqrt{5}-1}$.

2.162. Perform the indicated operations:

(a) (1) $\sqrt{a^3}+b\sqrt{a}-\sqrt{9a}$;

(2) $\sqrt[3]{a^2}+\sqrt[3]{a^5/8}-3a\sqrt[3]{1/a}$;

(3) $\sqrt[3]{27a^4}-3\sqrt[3]{8a}+3\sqrt[3]{125a^7}$;

(4) $\sqrt[5]{a^5b}-\sqrt[5]{32b^6}+3a\sqrt[5]{b}$;

(5) $3\sqrt{125a^3b^2}+b\sqrt{20a^3}-\sqrt{500a^3b^2}$;

(6) $2\sqrt[3]{a^6b}-3a^2\sqrt[3]{64b}+2a^2\sqrt[3]{125b^4}$;

(7) $(5\sqrt{4x}+4\sqrt{x}-6\sqrt{9x}-8\sqrt{2x})+\left(8\sqrt{\frac{1}{4}x}+4\sqrt{8x}+1\right)$;

(8) $\left(3\sqrt{8x}-\sqrt{18x}-5\sqrt{\frac{1}{2}x}\right)+\left(\sqrt{4\frac{1}{2}x}+\sqrt{50x}-\sqrt{32x}+\sqrt{72x}\right)$;

(b) (1) $5\sqrt[3]{x^2y^5}+4y^2\sqrt[3]{x^2/y}+(4y/x^2)\sqrt[3]{-x^6y^2}-6xy\sqrt[3]{y^2/x}-(3/2)\,xy\sqrt[3]{-8/(xy)}$;

(2) $\sqrt[3]{xy}+6xy\sqrt[3]{\frac{1}{x^2y^2}}-4x^2y^2\sqrt[3]{-\frac{1}{x^5y^5}}+\frac{1}{2y}\sqrt[3]{\frac{x}{y^2}}-\frac{3}{2x}\sqrt[3]{-x^4y}$;

(3) $\sqrt{m^3-m^2n}-\sqrt{(m+n)(m^2-n^2)}-\sqrt{mn^2-n^3}$;

(4) $\frac{\sqrt{a}}{\sqrt{a}-\sqrt{b}}-\frac{\sqrt{b}}{\sqrt{a}+\sqrt{b}}$;

(5) $\frac{1}{a+a\sqrt{b}}-\frac{1}{a-a\sqrt{b}}$;

(6) $\frac{a}{\sqrt{ac}+c}+\frac{c}{\sqrt{ac}-a}-\frac{a+c}{\sqrt{ac}}$;

(7) $\frac{1}{a+\sqrt{a^2-b^2}}+\frac{1}{a-\sqrt{a^2-b^2}}$.

§ 32. Multiplication and Division of Radicals

2.163. Multiply:

(a) (1) $\sqrt{2}\cdot\sqrt{6}$; (2) $\sqrt{10}\cdot\sqrt{20}$; (3) $3\sqrt{7}\cdot2\sqrt{14}$; (4) $(3/4)\sqrt{24}\cdot(2/3)\sqrt{6}$; (5) $\sqrt[3]{3}\cdot\sqrt[3]{4}$; (6) $5\sqrt[3]{48}\cdot2\sqrt[3]{4}$; (7) $\sqrt[4]{3}\cdot\sqrt[4]{27}$; (8) $\sqrt[6]{4}\cdot\sqrt[6]{16}$;

(b) (1) $\sqrt{a}\cdot\sqrt{ab}$; (2) $\sqrt{3m}\cdot\sqrt{3}$; (3) $\sqrt[4]{a^2}\cdot\sqrt[4]{a^3}$; (4) $5\sqrt[4]{2a}\cdot\sqrt[4]{8a^3}$; (5) $0.5\sqrt{a^2b^3}\cdot2\sqrt{a^3b^2}$; (6) $6\sqrt[3]{12a^7b^4}\cdot\sqrt[3]{32a^9b^7}$; (7) $\sqrt[5]{4a^3b^3}\cdot\sqrt[5]{8a^2b^3}$;

(c) (1) $(\sqrt{21}+\sqrt{14}-2\sqrt{35})\cdot(1/7)\sqrt{7}+\sqrt{20}$; (2) $1-0.1\sqrt{5}\cdot(\sqrt{15}+\sqrt{20}-\sqrt{55})$; (3) $(2\sqrt{6}-3\sqrt{5})\cdot\sqrt{3}+2\sqrt{2}$; (4) $\sqrt[9]{9/4}\cdot\sqrt[4]{2/3}\cdot\sqrt[6]{2}\cdot\sqrt[12]{3}$; (5) $(3\sqrt{10}-2\sqrt[3]{4}+6\sqrt[6]{25})\cdot\sqrt[4]{2}$; (6) $(2\sqrt{6}+3\sqrt[3]{15}-\sqrt[5]{10})\cdot\sqrt[4]{12}$;

(d) (1) $\frac{x}{a}\sqrt[3]{\frac{a^2}{x}}\cdot\frac{1}{4}\,a\sqrt[3]{\frac{8a}{x^4}}$;

(2) $5\sqrt[3]{\frac{2a^4}{24x^5}}\cdot\frac{3}{4}\sqrt[3]{\frac{4a^5}{5x^2}}$;

(3) $(\sqrt{a}+\sqrt{ab}-\sqrt{a/b})\cdot\sqrt{a/b}$;

(4) $(\sqrt[3]{a^2b}+\sqrt[3]{b^2/a^2}-\sqrt[3]{ab^2})\cdot\sqrt[3]{a^2/b^2}$;

(5) $(\sqrt{a}+\sqrt{b/a})\cdot(\sqrt{ab}-\sqrt{a/b})$;

(6) $(\sqrt[3]{a^2}-2\sqrt[4]{b^2}-a\sqrt[6]{a^5})\,a^2\sqrt{ab}$;

(7) $(\sqrt[5]{a^2}-4\sqrt[4]{a^4}+a\sqrt{a^3})\cdot(-2a\sqrt[3]{a^2})$;

(8) $(\sqrt{a}-\sqrt[3]{a^2}+\sqrt[5]{a^4})\cdot(\sqrt[3]{a}-\sqrt[15]{a^4})$.

2.164. Perform the indicated operations:

(1) $\left(\frac{\sqrt{3}+1}{1+\sqrt{3}+\sqrt{a}}+\frac{\sqrt{3}-1}{1-\sqrt{3}+\sqrt{a}}\right)\cdot\left(\sqrt{a}-\frac{2}{\sqrt{a}}+2\right)$;

(2) $\left(\frac{1}{a+\sqrt{2}}-\frac{a^2+4}{a^3+2\sqrt{2}}\right)\cdot\left(\frac{a}{2}-\frac{1}{\sqrt{2}}+\frac{1}{a}\right)$;

(3) $\left(\frac{(a-1)^{-1}}{a^{-3}}-(1-a)^{-1}\right)\cdot\frac{1+a\,(a-2)}{a^2-a+1}\cdot\sqrt{\frac{1}{(a+1)^2}}$.

2.165. Divide:

(a) (1) $\sqrt[3]{6a^4}\div\sqrt[3]{2a}$; (2) $\sqrt[3]{x^7}\div\sqrt[3]{x^4}$;

(3) $\sqrt[4]{32}\div\sqrt[4]{16}$; (4) $\sqrt[4]{a^5}\div\sqrt[4]{a}$; (5) $\sqrt[4]{9a^3}\div\sqrt[4]{a/9}$;

(6) $\sqrt[3]{a}\div\sqrt[3]{a^2}$; (7) $\sqrt{a}\div\sqrt[4]{a}$; (8) $\sqrt[6]{x^5}\div\sqrt[3]{x^2}$;

(9) $\sqrt[5]{a^4}\div\sqrt[15]{a^2}$; (10) $\sqrt{a}\div\sqrt[5]{a^2}$; (11) $\sqrt[5]{a^3}\div\sqrt[3]{a^2}$;

(12) $\sqrt[3]{4a^2}\div\sqrt[6]{2a^3}$; (13) $\sqrt[10]{2a^4}\div\sqrt[5]{2a^2}$;

(14) $5x^2y\div\sqrt[3]{25xy^4}$;

(15) $2x^2y^3\div\sqrt[4]{8x^3y^2}$, $(x+y)\div\frac{1}{3}\sqrt{x^2-y^2}$;

(16) $(x^2-y^2)\div\frac{a}{x}\sqrt[3]{\frac{2a}{(x+y)^2}}$;

(b) (1) $(5\sqrt[3]{4}-6\sqrt{10}+15\sqrt[3]{16})\div 3\sqrt[3]{1/2}$;

(2) $(3\sqrt[3]{6}+2\sqrt[3]{18}-4\sqrt[3]{12})\div 2\sqrt[3]{2/3}$;

(3) $\left(\frac{3x}{2}\sqrt{\frac{x}{y}}-\frac{2}{5}\sqrt{\frac{3}{xy}}+\frac{1}{3}\sqrt{\frac{xy}{2}}\right)\div\frac{4}{15}\sqrt{\frac{3y}{2x}}$;

(4) $\left(\frac{a}{2}\sqrt[3]{a^2b}+\frac{b}{3a^2}\sqrt[3]{\frac{15a}{b^2}}-\frac{4a}{5b}\sqrt[3]{\frac{b}{3a^2}}\right)\div\frac{2a^3}{15b^2}\sqrt[3]{\frac{5a^2}{2b}}$;

(5) $(ab^2\sqrt{x}-x\sqrt{b})\div\sqrt{bx}$; (6) $(2ab\sqrt[3]{x^2}-x\sqrt[3]{b})\div\sqrt{bx}$;

(7) $(\sqrt[3]{a^2}-\sqrt[3]{b^2}) \div (\sqrt[3]{a}+\sqrt[3]{b})$;

(8) $(\sqrt[4]{8a^3}-b\sqrt[4]{27b^2}) \div (\sqrt[4]{2a}-\sqrt[4]{3b^2})$;

(c) (1) $(3\sqrt{2}-12\sqrt[3]{12}+10\sqrt[4]{2}) \div \frac{2}{3}\sqrt[4]{2}$;

(2) $(\sqrt[4]{8a^6b^9}-ab\sqrt{8a^4b^5}+ab^2\sqrt[4]{2a^4b}) \div \sqrt[4]{2b}$;

(3) $(\sqrt[6]{9+4\sqrt{5}}+\sqrt[3]{2+\sqrt{5}}) \div \sqrt[3]{2-\sqrt{5}}$.

2.166. Perform the indicated operations:

(1) $(\sqrt[3]{4m^2}-\sqrt[3]{9n^2}) \div (\sqrt[3]{2m}+\sqrt[3]{3n})$;

(2) $(\sqrt[3]{25a^2}-\sqrt[3]{16b^2}) \div (\sqrt[3]{5a}-\sqrt[3]{4b})$;

(3) $(\sqrt[3]{x^2y}-\sqrt[3]{16xy^2}+\sqrt[3]{4y^3}) \div (\sqrt[3]{x}-\sqrt[3]{2y})$;

(4) $(\sqrt{8x^2y}-2y\sqrt{x}-x\sqrt{x}) \div (\sqrt{2y}-\sqrt{x})$;

(5) $(x^2\sqrt[4]{27xy^3}+2xy\sqrt{2xy}) \div (\sqrt[4]{3x^3y}+\sqrt{2xy})$;

(6) $(y\sqrt{2xy}-xy\sqrt{xy}) \div (\sqrt[6]{2xy^3}-\sqrt{xy})$;

(7) $(x\sqrt[6]{a^5x}+2a\sqrt[6]{ax^5}-3ax) \div (\sqrt[3]{ax^2}-\sqrt{ax})$.

§ 33. Raising a Root to a Power and Taking the Root of a Root

2.167. Raise the following expressions to the indicated power:

(a) (1) $(\sqrt[4]{a^3})^4$; (2) $(\sqrt[4]{4x^2})^2$; (3) $(-a\sqrt[8]{a^2b^3})^7$;

(4) $(-a\sqrt[5]{a^3x})^4$;

(5) $(\sqrt{(x-y)^2})^4$; (6) $(\sqrt[3]{(x+y)^2})^5$;

(b) (1) $(\sqrt{3}-\sqrt{2})^2$; (2) $(\sqrt[6]{2}-\sqrt{3})^2$;

(3) $(\sqrt{3}-2\sqrt[3]{2})^2$; (4) $(\sqrt{2}+3\sqrt[3]{3})^2$;

(5) $(\sqrt{3+\sqrt{5}}+\sqrt{3-\sqrt{5}})^2$;

(c) (1) $(\sqrt{\sqrt{7}-\sqrt{3}}-\sqrt{\sqrt{7}+\sqrt{3}})^2$;

(2) $(\sqrt{11+6\sqrt{2}}-\sqrt{11-6\sqrt{2}})^2$;

(3) $(\sqrt{7+2\sqrt{6}}-\sqrt{7-2\sqrt{6}})^2$;

(d) (1) $\left(\frac{b}{4}\sqrt{ab}-\frac{2}{\sqrt{a}}\right)^2$; (2) $\left(\frac{a}{2}\sqrt{\frac{a}{b}}-\frac{3}{\sqrt{ab}}\right)^2$;

(3) $(a\sqrt{a}+a\sqrt{2a})^3$; (4) $(a\sqrt{b}-2a\sqrt{2b})^3$;

(e) (1) $\left(\frac{a}{a+b}\sqrt[3]{\frac{a+b}{a^2}}\right)^4$; (2) $\left(-\frac{a+b}{a-b}\sqrt[3]{\frac{(a-b)^2}{a+b}}\right)^5$;
(3) $\left(\frac{a\sqrt[3]{(a-b)^2}}{a-b\cdot}\right)^4$; (4) $\left(\frac{a(a-b)}{5a^2(a-b)^4}\right)^6$;
(5) $\frac{(\sqrt[4]{a}+\sqrt[4]{b})^2+(\sqrt[4]{a}-\sqrt[4]{b})^2}{2(a-b)} \div \frac{1}{\sqrt{a^3}-\sqrt{b^3}} - 3\sqrt{ab}$.

2.168. Take the root:

(a) (1) $\sqrt[4]{1296}$; (2) $\sqrt[4]{2401}$; (3) $\sqrt[6]{15625}$; (4) $\sqrt[4]{20736}$;
(5) $\sqrt[9]{262144}$; (6) $\sqrt[10]{59049}$;

(b) (1) $\sqrt{\sqrt{2}}$; (2) $\sqrt[3]{\sqrt{3}}$; (3) $\sqrt{\sqrt{5}}$; (4) $\sqrt[3]{\sqrt[3]{3}}$;
(5) $\sqrt[3]{\sqrt[3]{2}}$; (6) $\sqrt[3]{2\sqrt{5}}$;

(c) (1) $\sqrt{\sqrt[3]{a}}$; (2) $\sqrt[3]{\sqrt[4]{a^3}}$; (3) $\sqrt[3]{\sqrt[5]{a^4}}$; (4) $\sqrt{\sqrt[5]{a^3}}$;
(5) $\sqrt[4]{\sqrt{256a^{10}}}$; (6) $\sqrt[9]{\sqrt[3]{512a^{18}}}$; (7) $\sqrt{x^3\sqrt[3]{x\sqrt[4]{x}}}$;
(8) $\sqrt{2x\sqrt[3]{2x^2y3y\sqrt{3xy^3}}}$;

(d) (1) $\sqrt{(m/n)\sqrt{(n/m)\sqrt{m/n}}}$;
(2) $\sqrt{(a/b)\sqrt{(a/b)\sqrt[3]{(b^3/a)}}}$;
(3) $\sqrt[3]{(a/b)\sqrt{(b^2/a)\sqrt{1/a^2}}}$; (4) $\sqrt[3]{(a/x)\sqrt{(1/(ax))\sqrt{a/x^3}}}$.

2.169. Factor:

(a) (1) $\sqrt{24}+\sqrt{12}$; (2) $\sqrt[4]{8}+2$; (3) $\sqrt[4]{8}-3$;
(4) $25\sqrt[4]{2}+2\sqrt{5}$; (5) $\sqrt{250}+5\sqrt[4]{8}$; (6) $\sqrt[3]{9}-\sqrt[3]{4}$;

(b) (1) $\sqrt{x^2-a^2}-x+a$; (2) $\sqrt{1-x^2}+x-1$;
(3) $1+x+\sqrt{1+x}$; (4) $\sqrt{1-x}+x-1$; (5) $x+4\sqrt{x}+3$;
(6) $\sqrt{x^3}-\sqrt{y^3}+\sqrt{x^2y}-\sqrt{xy^2}$; (7) $a-5\sqrt{a}+4$.

2.170. Reduce the given fractions:

(1) $\frac{\sqrt{15}-\sqrt{6}}{\sqrt{35}-\sqrt{14}}$; (2) $\frac{\sqrt{10}+\sqrt{15}}{\sqrt{8}+\sqrt{12}}$; (3) $\frac{a+\sqrt{ab}}{b+\sqrt{ab}}$;
(4) $\frac{\sqrt[3]{x^2y}-\sqrt[3]{xy^2}}{\sqrt[3]{ax}-\sqrt[3]{ay}}$; (5) $\frac{\sqrt[4]{27}+3}{\sqrt[4]{3}+\sqrt[3]{3}}$; (6) $\frac{\sqrt[4]{125}-5}{\sqrt[4]{5}-\sqrt[3]{5}}$.

2.171. Rationalize the denominator of the given fraction:

(a) (1) $\frac{18}{\sqrt{6}}$; (2) $\frac{46}{\sqrt{8}}$; (3) $\frac{6}{\sqrt[4]{12}}$; (4) $\frac{a^2-b^2}{\sqrt[3]{a-b}}$;

(5) $\frac{a-b}{\sqrt[3]{a^2-b^2}}$;

(b) (1) $\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}}$; (2) $\frac{a}{1-\sqrt{a}}$; (3) $\frac{2a}{3+\sqrt{2a}}$;

(4) $\frac{5-2\sqrt{2}}{3\sqrt{5}-4\sqrt{22}}$; (5) $\frac{3\sqrt{5}-2\sqrt{2}}{2\sqrt{5}-3\sqrt{2}}$; (6) $\frac{x-x-a^2}{x+x-a^2}$;

(7) $\frac{7\sqrt{15}-2\sqrt{3}}{10\sqrt{3}+8\sqrt{5}}$; (8) $\frac{7-4\sqrt{3}}{\sqrt[3]{26}-15\sqrt{3}}$;

(9) $\frac{\sqrt{x^2-a^2}+\sqrt{x^2+a^2}}{\sqrt{x^2-a^2}-\sqrt{x^2+a^2}}$;

(c) (1) $\frac{\sqrt{x^2+1}}{\sqrt{x^2+1}-\sqrt{x^2-1}}$; (2) $\frac{1}{\sqrt{5}+\sqrt{7}+\sqrt{11}}$;

(3) $\frac{b}{\sqrt{3}+\sqrt{5}+\sqrt{7}}$; (4) $\frac{a}{2+\sqrt{2}+\sqrt{3}+\sqrt{6}}$;

(5) $\frac{n}{\sqrt[3]{a}+\sqrt[3]{b}}$; (6) $\frac{m}{\sqrt[3]{a}-\sqrt[3]{b}}$; (7) $\frac{6}{\sqrt[3]{7}-\sqrt[3]{4}}$;

(d) (1) $\frac{n}{\sqrt[3]{a^2}-\sqrt[3]{ab}+\sqrt[3]{b^2}}$; (2) $\frac{1}{\sqrt[3]{9}-\sqrt[3]{6}+\sqrt[3]{4}}$;

(3) $\frac{n}{\sqrt[3]{a^2}+\sqrt[3]{ab}+\sqrt[3]{b^2}}$; (4) $\frac{1}{\sqrt[3]{49}+\sqrt[3]{35}+\sqrt[3]{25}}$;

(5) $\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}+\sqrt[3]{c}}$; (6) $\frac{1}{\sqrt{2}-\sqrt[3]{3}}$; (7) $\frac{a}{\sqrt[6]{3}-\sqrt[6]{2}}$;

(8) $\frac{1}{\sqrt[4]{2}+\sqrt[4]{4}+\sqrt[4]{8}+2}$; (9) $\frac{1}{1+\sqrt[3]{2}+\sqrt[3]{3}}$.

2.172. Rationalize the numerator:

(a) (1) $\frac{\sqrt{3}}{3}$; $\frac{\sqrt[3]{9}}{6}$; $\frac{\sqrt[4]{125}}{2}$; $\frac{\sqrt[3]{16}}{2}$; $\frac{\sqrt{mn}}{m}$; $\frac{\sqrt[5]{n^3}}{n}$;

(2) $\frac{2+\sqrt{3}}{3}$; $\frac{\sqrt{2}+1}{3}$; $\frac{\sqrt{5}+\sqrt{2}}{6}$; $\frac{\sqrt{7}+\sqrt{12}}{15}$;

(b) (1) $\frac{\sqrt{a}+\sqrt{b}}{a-b}$; (2) $\frac{x\sqrt{y}-y\sqrt{x}}{xy}$; (3) $\frac{\sqrt{a}+1}{a}$;

(4) $\frac{1-\sqrt{m}}{m}$; (5) $\frac{\sqrt[3]{(x-y)^2}}{x^2-y^2}$; (6) $\frac{3+\sqrt{m}}{3-\sqrt{m}}$;

(7) $\frac{a\sqrt{x}-b\sqrt{y}}{a\sqrt{x}+b\sqrt{y}}$; (8) $\frac{a+b\sqrt{x}}{a-b\sqrt{x}}$;

(c) (1) $\frac{\sqrt{x-2\sqrt{x-1}}}{\sqrt{x-1}-1}$; (2) $\frac{\sqrt{a}-\sqrt{b}}{a+b}$; (3) $\frac{a\sqrt{b}+b\sqrt{a}}{ab}$;

(4) $\frac{\sqrt{4\sqrt{5}+2\sqrt{11}}}{4\sqrt{5}+2\sqrt{11}}$; (5) $\frac{\sqrt{x-3}-\sqrt{x+3}}{\sqrt{x-3}+\sqrt{x+3}}$;

(6) $\frac{3+\sqrt{2}+\sqrt{3}}{3-\sqrt{2}-\sqrt{3}}$; (7) $\frac{2-\sqrt{2}-\sqrt{3}}{2+\sqrt{2}-\sqrt{3}}$; (8) $\frac{\sqrt{21+8\sqrt{5}}}{4+\sqrt{5}}$.

2.173. Prove the following identities:

(1) $\sqrt{a+\sqrt{b}}=\sqrt{\frac{a+\sqrt{a^2-b}}{2}}+\sqrt{\frac{a-\sqrt{a^2-b}}{2}}$;

$a>0;\ b>0;\ a^2-b>0$;

(2) $\sqrt{a-\sqrt{b}}=\sqrt{\frac{a+\sqrt{a^2-b}}{2}}-\sqrt{\frac{a-\sqrt{a^2-b}}{2}}$;

$a>0;\ b>0;\ a^2-\sqrt{b}>0$;

(3) $\sqrt{6m+2\sqrt{9m^2-n^2}}-\sqrt{6m-2\sqrt{9m^2-n^2}}=2\sqrt{3m-n}$;

(4) $\left(\frac{a\sqrt{a}+b\sqrt{b}}{(\sqrt{a}+\sqrt{b})}-\sqrt{ab}\right)\left(\frac{\sqrt{a}+\sqrt{b}}{a-b}\right)^2=1$;

(5) $(4+\sqrt{15})\cdot(\sqrt{10}-\sqrt{6})\sqrt{4-\sqrt{15}}=2$;

(6) $\sqrt{3-\sqrt{5}}(3+\sqrt{5})\cdot(\sqrt{10}-\sqrt{2})=8$;

(7) $\frac{\sqrt[3]{\sqrt{3}+\sqrt{6}}\cdot\sqrt[6]{9-6\sqrt{2}}-\sqrt[6]{18}}{\sqrt[6]{2}-1}=-\sqrt[3]{3}$;

(8) $5\sqrt[3]{6\sqrt{32}}-3\sqrt[3]{9\sqrt{162}}-11\sqrt[6]{18}+2\sqrt[3]{75\sqrt{50}}=0$;

(9) $\frac{25\sqrt[4]{2}+2\sqrt{5}}{\sqrt{250}+5\sqrt[4]{8}}-\sqrt{\sqrt{2}/5+5/\sqrt{2}+2}=-1$;

(10) $\frac{\sqrt{\sqrt[4]{27}+\sqrt{\sqrt{3}-1}}-\sqrt{\sqrt[4]{27}-\sqrt{\sqrt{3}-1}}}{\sqrt{\sqrt[4]{27}-\sqrt{2\sqrt{3}+1}}}=\sqrt{2}$;

(11) $\left(\frac{4}{3-\sqrt{5}}\right)^2-\left(\frac{6-5\sqrt{6}}{5-\sqrt{6}}\right)^2=2\sqrt{61+24\sqrt{5}}$;

(12) $\left(\frac{\sqrt[4]{8}+2}{\sqrt[4]{2}+\sqrt[3]{2}}-\sqrt[3]{4}\right)\div\left(\frac{\sqrt[4]{8}-2}{\sqrt[4]{2}-\sqrt[3]{2}}-3\sqrt[12]{128}\right)^{1/2}=$

$-\sqrt[4]{2}$.

§ 34. Exercises on All Operations with Roots. Powers with Fractional Exponents

2.174. Perform the indicated operations:

(a) (1) $\left(\frac{3}{\sqrt[3]{64}-\sqrt[3]{25}}+\frac{\sqrt[3]{40}}{\sqrt[3]{8}+\sqrt[3]{5}}-\frac{10}{\sqrt[3]{25}}\right)\div$
$(\sqrt[6]{8}+\sqrt[6]{5})+\sqrt[6]{5}$;

(2) $\frac{9}{5-\sqrt{7}}+\frac{22}{6+\sqrt{5}}-\frac{1}{\sqrt{7}+\sqrt{5}}$;

(3) $\frac{5}{4-\sqrt{11}}+\frac{1}{3+\sqrt{7}}-\frac{6}{\sqrt{7}-2}-\frac{1}{2}(\sqrt{7}-5)$;

(4) $\frac{\sqrt{5-2\sqrt{6}}}{(\sqrt[4]{3}+\sqrt[4]{2})(\sqrt[4]{3}-\sqrt[4]{2})}$;

(5) $\sqrt{\left(\frac{9-2\sqrt{3}}{\sqrt{3}-\sqrt[3]{2}}+3\sqrt[3]{2}\right)\cdot\sqrt{3}\div(3+\sqrt[6]{108})}$;

(6) $\left(\frac{4-2x+x^2}{4-2x}+\frac{6x^2+8+12x}{4-x^2}-\frac{x^2+2x+4}{2x+4}\right)^{-1/3}\cdot(x+2)$;

(b) (1) $\sqrt{\frac{x}{x-a^2}}\div\left(\frac{\sqrt{x}-\sqrt{x-a^2}}{\sqrt{x}+\sqrt{x-a^2}}-\frac{\sqrt{x}+\sqrt{x-a^2}}{\sqrt{x}-\sqrt{x-a^2}}\right)$;

(2) $2x+\sqrt{x^2-1}\cdot\left(1+\frac{x^2}{x^2-1}\right)-\frac{1+x\div\sqrt{x^2-1}}{x+\sqrt{x^2-1}}$;

(3) $\left(\sqrt{a(1-a)}+\frac{\sqrt{a^3}}{\sqrt{1-a}}\right)\div\left(\frac{1}{1+\sqrt{a}}+\frac{\sqrt{a}}{1-a}\right)$,

where $a<1$;

(4) $\left((a-b)\cdot\sqrt{\frac{a+b}{a-b}}-a-b\right)\cdot\left((a-b)\sqrt{\frac{a+b}{a-b}}-a+b\right)$;

(c) (1) $\left(\frac{y+\sqrt{y^2-a^2}}{y-\sqrt{y^2-a^2}}+\frac{y-\sqrt{y^2-a^2}}{y+\sqrt{y^2-a^2}}\right)\div\frac{y\sqrt{2}-a}{a^3}$;

(2) $\frac{(\sqrt{x}-\sqrt{y})^3+\frac{2x^2}{\sqrt{x}}+y\sqrt{y}}{x\sqrt{x}+y\sqrt{y}}+\frac{3\sqrt{xy}-3y}{x-y}$;

(3) $\frac{a-x}{\sqrt{a}-\sqrt{x}}-\left(\frac{a+\sqrt[4]{ax^3}}{\sqrt{a}+\sqrt[4]{ax}}-\sqrt[4]{ax}\right)$;

(4) $\left(\sqrt{x}-\frac{\sqrt{xy}+y}{\sqrt{x}+\sqrt{y}}\right)\cdot\left(\frac{\sqrt{x}}{\sqrt{x}+\sqrt{y}}+\frac{\sqrt{y}}{\sqrt{x}-\sqrt{y}}+\frac{2\sqrt{xy}}{x-y}\right)$;

(d) (1) $\dfrac{(\sqrt{a}+\sqrt{b})^2-4b}{(a-b)\div\left(\sqrt{\frac{1}{b}}+3\sqrt{\frac{1}{a}}\right)}\div\dfrac{a+9b+6\sqrt{ab}}{\frac{1}{\sqrt{b}}+\frac{1}{\sqrt{a}}}$;

(2) $\left(\dfrac{\sqrt{x-a}}{\sqrt{x+a}+\sqrt{x-a}}+\dfrac{x-a}{\sqrt{x^2-a^2}-x+a}\right)\div\sqrt{\dfrac{x^2}{a^2}-1}$;

(3) $\dfrac{\sqrt[4]{x^5}+\sqrt[4]{xy^4}-\sqrt[4]{x^4y}-\sqrt[4]{y^5}}{\sqrt{x}-\sqrt{y}}\cdot(\sqrt[4]{x}+\sqrt[4]{y})$;

(4) $\left(\dfrac{\sqrt{2+a}}{\sqrt{2+a}-\sqrt{2-a}}+\dfrac{2-a}{\sqrt{4-a^2}-2+a}\right)\div$

$\dfrac{\sqrt{\frac{4}{a^2}-1}+\frac{2-a}{a}}{\sqrt{2-a}}$;

(5) $\dfrac{8-m}{\sqrt[3]{m}+2}\div\left(2+\dfrac{\sqrt[3]{m^2}}{\sqrt[3]{m}+2}\right)+\left(\sqrt[3]{m}+\dfrac{2\sqrt[3]{m}}{\sqrt[3]{m}-2}\right)\times$

$\dfrac{\sqrt[3]{m^2}-4}{\sqrt[3]{m^2}+2\sqrt[3]{m}}$.

2.175. Simplify:

(a) (1) $\sqrt{x^2-12x+36}-\sqrt{x^2}$;

(2) $\sqrt{y^2-6y+9}-|y-9|+2$;

(3) $\sqrt{1-2\sqrt{m-m^2}}$; (4) $\sqrt{a-2+\dfrac{1}{a}}$, $a>0$;

(5) $\sqrt{x+2\sqrt{x-1}}-\sqrt{x-2\sqrt{x-1}}$;

(6) $\sqrt{17-4\sqrt{9+4\sqrt{5}}}$; (7) $\sqrt{x^2+4x+4}+|x-6|$;

(8) $2\sqrt{3+\sqrt{5-\sqrt{13+\sqrt{48}}}}$;

(b) (1) $\dfrac{\sqrt{4x+4+x^{-1}}}{\sqrt{x}\cdot|2x^2-x-1|}$; (2) $\dfrac{x^2+4}{x\sqrt{4+\left(\frac{x^2-4}{2x}\right)^2}}$;

(3) $\dfrac{(x-1)\sqrt{(x-1)^2+4x}}{x^2+1+2|x|}$; (4) $\dfrac{\sqrt{(1+2x)^2-8x}}{\sqrt{x}-2/\sqrt{x}}$;

(5) $\dfrac{\sqrt{1+\left(\frac{x^2-1}{2x}\right)^2}}{(x+1)\cdot(1/x)}$; (6) $\dfrac{a\sqrt{(a^2-4)^2/(4a^2)+4}}{a^2+4}+2$.

2.176. Prove the following identities:

(1) $\dfrac{\sqrt{x-a}}{\sqrt{x+a}+\sqrt{x-a}}+\dfrac{x-a}{\sqrt{x^2-a^2}-x+a}\div\sqrt{\dfrac{x^2}{a^2}-1}=1$;

(2) $\dfrac{4x(x+\sqrt{x^2-1})^2}{(x+\sqrt{x^2-1})^4-1}=\dfrac{1}{\sqrt{x^2-1}}$, $|x|>1$;

(3) $\dfrac{\sqrt[3]{m+4\sqrt{m-4}}\cdot\sqrt[3]{\sqrt{m-4}+2}}{\sqrt[3]{m-4\sqrt{m-4}}\cdot\sqrt[3]{\sqrt{m-4}-2}}\cdot\dfrac{m-4\sqrt{m-4}}{2}=\dfrac{m-8}{2}$;

(4) $\dfrac{\sqrt{(2p+1)^3}+\sqrt{(2p-1)^3}}{\sqrt{2p+1}+\sqrt{2p-1}}=4p-\sqrt{4p^2-1}$, $p\geqslant\dfrac{1}{2}$;

(5) $a^3+b^3+c^3=3abc$ if $a+b+c=0$;

(6) $(a+b+c)^3=27abc$ if $\sqrt[3]{a}+\sqrt[3]{b}+\sqrt[3]{c}=0$.

2.177. Rationalize the following expressions:

(a) (1) $\sqrt[3]{a^2}$; (2) $\sqrt[5]{a^3}$; (3) $\sqrt[4]{a^{-3}}$;
(4) $\sqrt[3]{a^{-2}}$; (5) $\sqrt[5]{a^{-3}b^4}$;

(b) (1) $\sqrt[4]{a^3b^{-2}}$; (2) $\sqrt[-3]{a^{-5}}$; (3) $\sqrt{a^2+b^2}$;
(4) $\sqrt[3]{a^3-b^3}$; (5) $\sqrt[-3]{\dfrac{a^3-b^3}{a^{-1}b^2}}$; (6) $\sqrt[-2]{\dfrac{a^2b^{-3}}{a^2-b^2}}$;

(c) (1) $\sqrt[m]{x^{n+1}}$; (2) $\sqrt[mp+nq]{a^{p+q}}$;
(3) $\sqrt[m]{\dfrac{x^2}{y^n}}$; (4) $\sqrt[n]{a^2+2ab+b^2}$.

2.178. Replace the following expressions by radicals:

(a) (1) $a^{5/6}$; (2) $a^{2/3}$; (3) $a^{3/7}$; (4) $a^{-3/4}$;
(5) $3a^{-1/2}\cdot(a-b)^{3/8}$;

(b) (1) $4a^{-2/3}\cdot(a+b)^{-1/2}$; (2) $(a+b)^{0.25}$; (3) $c^{-1/x}$;
(4) $c^{x^2/2}$; (5) $a^{(n-1)/n}$; (6) $b^{(n^2-1)/(3m)}$; (7) $c^{(5n-3)/(2m)}$.

2.179. Compute:

(1) $25^{1/2}-27^{2/3}+81^{3/4}$; (2) $16^{0.5}+(-1/16)^{0.75}-(-1/2)^{-6}$;

(3) $(1/2)^{12}\cdot4^{\sqrt{3}/2}\cdot\left(\dfrac{1}{8}\right)^{\sqrt{27}}\cdot16^3$; (4) $9^{-0.5}-8^{1/3}+(0.25)^{-3/2}$;

(5) $((3/4)^0)^{-0.5}-7.5\left(\sqrt[-4/3]{4}\right)^2-(-2)^4+81^{0.25}$;

(6) $\dfrac{12^{\sqrt{48}}}{4^{\sqrt{108}}} \cdot \dfrac{2^{27\sqrt{3}}}{6^{\sqrt{27}}}$;

(7) $4^{-1/4} + \left(\dfrac{1}{2^{-3/2}}\right)^{-4/3} \cdot (4^{-0.25} - (2\sqrt{2})^{-4/3})$;

(8) $\left((a^{1/3} - x^{1/3})^{-1} \cdot (a - x) - \dfrac{a+x}{\sqrt[3]{a} + \sqrt[3]{x}}\right) \cdot 2\,(ax)^{-1/3}$;

(9) $0.027^{-1/3} - (-1/6)^{-2} + 256^{0.75} - 3^{-1} + 5.5^{0}$.

2.180. Perform the indicated operations:

(a) (1) $a^{2/3} \cdot b^{3/5} \cdot a^{3/4} \cdot b^{2/3}$; (2) $a^{5/2} \cdot a^{2/3} \cdot a^{2/3} \cdot a^{3/4}$;

(3) $(a^{7/12} b^{5/6}) \div (a^{2/3} b^{3/4})$; (4) $(a^{11/15} b^{2/3}) \div (a^{3/5} b^{5/6})$;

(5) $(x^{-4} + x^{2} + x^{-1}) \div x^{-1}$; (6) $(ax^2 + bx) \cdot x^{-2}$;

(7) $(x^2 - y^2) \div (x^{-1} + y^{-1})$; (8) $(x^3 + y^3) \div (x^{-1} + y^{-1})$;

(b) (1) $(a^{3/2} - b^{3/2}) \div (a^{1/2} - b^{1/2})$;

(2) $(a^{3/2} + b^{3/2}) \div (a^{1/2} + b^{1/2})$;

(3) $(a^{1/2} + a^{1/4} b^{1/4} + b^{1/2}) \cdot (a^{1/2} - a^{1/4} b^{1/4} + b^{1/2})$;

(4) $(a^{3n/2} + b^{3n/2}) \div (a^{n/2} + b^{n/2})$;

(5) $(a^{1/2} - b^{1/2} - c^{1/2} + 2b^{1/4} c^{1/4}) \div (a^{1/4} + b^{1/4} - c^{1/4})$;

(6) $\dfrac{m-n}{\sqrt{m} - \sqrt{n}} - \dfrac{m^{3/2} - n^{3/2}}{m-n}$;

(7) $\dfrac{a-b}{\sqrt[3]{a} - \sqrt[3]{b}} - \dfrac{a+b}{\sqrt[3]{a} + \sqrt[3]{b}}$;

(c) (1) $\dfrac{\dfrac{a^{1.5} + b^{1.5}}{a^{0.5} + b^{0.5}} - a^{0.5} b^{0.5}}{a-b} + \dfrac{2b^{0.5}}{a^{0.5} + b^{0.5}}$;

(2) $\left(\dfrac{49}{c+27} - \dfrac{c^{1/3} + 3}{c^{2/3} - 3c^{1/3} + 9}\right) \cdot \dfrac{c^{4/3} + 27c^{1/3}}{16 - c^{2/3}} + \dfrac{40 - c^{2/3}}{4 + c^{1/3}}$;

(3) $\dfrac{3^{1.5}}{3^{1.5} a^{1/6} - a^{2/3}} + \dfrac{a^{5/6} + 3^{1.5} a^{1/3}}{(a^{1/3} + 3)^2} \cdot \left(\dfrac{3a^{1/3}}{a - 27} + \dfrac{1}{a^{1/3} - 3}\right)$;

(4) $\dfrac{a^{1/6}}{a^{1/6} - b^{1/3}} + \dfrac{a^{1/3} + a^{1/6} b^{1/3} + b^{2/3}}{a^{-1/4} b^{1/6}} \cdot \left(\dfrac{1}{a^{1/2} - b} - \dfrac{1}{a^{1/2} - a^{1/4} b^{1/2}}\right)$;

(5) $\left(\dfrac{x^{1/2} - y^{1/2}}{xy^{1/2} + x^{1/2} y} + \dfrac{x^{1/2} + y^{1/2}}{xy^{1/2} - x^{1/2} y}\right) \cdot \dfrac{x^{3/2} y^{1/2}}{x+y} - \dfrac{2y}{x-y}$;

(6) $\dfrac{bc^{1/2}}{b^{1/2} - c^{1/2}} - \dfrac{b}{b^{1/2} c^{1/2} + c} \cdot \left(\dfrac{b^{1/2} + c^{1/2}}{bc^{1/2} - c^{-1/2}} - \dfrac{4}{b-c}\right)$;

(7) $\left(\dfrac{a^{1/3}}{a^{1/2} - b^{1/2}} - \dfrac{a^{1/6} - b^{1/6}}{a^{1/3} + a^{1/6} b^{1/6} + b^{1/3}}\right) \cdot \dfrac{ab^{-1/6} - b^{5/6}}{2a^{1/6} - b^{1/6}}$.

2.181. Evaluate:

(1) $(1+x^{-1})^{-2}+(1-x^{-1})^{-2}$ for $x=2\frac{1}{2}$;

(2) $\dfrac{x^3-a^{-2/3}b-1}{x^2\sqrt{b^3}}\,(a^2+b^2)\,x+b^{1/2}$ for $x=a^{2/3}\cdot b^{-1/2}$;

(3) $\dfrac{a^{3/2}+b^{3/2}}{(a^2-ab)^{2/3}} \div \dfrac{a^{-2/3}\sqrt[3]{a-b}}{a\sqrt{a}-b\sqrt{b}}$ for $a=1.2,\quad b=\dfrac{3}{5}$;

(4) $((a^{1/2}+b^{1/2})(a^{1/2}+5b^{1/2})-(a^{1/2}+2b^{1/2})(a^{1/2}-2b^{1/2}))\div$
$(2a+3a^{1/2}b^{1/2})$ for $a=54,\ b=6$;

(5) $\left(\dfrac{6+4\cdot x^{1/2}}{x^{1/2}+6^{1/2}+4\cdot x^{1/2}}+\dfrac{6-4\cdot x^{1/2}}{x^{1/2}-6^{1/2}-4\cdot x^{1/2}}\right)^2$ for $x=2$;

(6) $a^2(1-a^2)^{-1/2}-\dfrac{1}{1+[a(1-a^2)^{-1/2}]^2}\times$
$\dfrac{(1-a^2)^{1/2}+a^2(1-a^2)^{-1/2}}{1-a^2}$ for $a=\dfrac{\sqrt{35}}{6}$;

(7) $\dfrac{1}{a^{1/4}+a^{1/8}+1}+\dfrac{1}{a^{1/4}-a^{1/8}+1}-\dfrac{2a^{1/4}-2}{a^{1/2}-a^{1/4}+1}$ for $a=\dfrac{1}{4}$.

2.182. Compute:

(1) $\dfrac{3a^{3n-3}}{0.125x^3x^{-7}}\cdot\sqrt[3]{\dfrac{27^{-1}x^{-15}}{0.008a^{6n-12}x^{-6}}}$;

(2) $\dfrac{1}{2}a^{13/8}\cdot\sqrt[5]{a^{-3}}-\dfrac{a}{\sqrt[8]{a}}$;

(3) $\left(\dfrac{3x^{-1/3}}{x^{2/3}-2x^{-1/3}}-\dfrac{x^{1/3}}{x^{4/3}-x^{1/3}}\right)^{-1}-\left(\dfrac{1-2x}{3x-2}\right)^{-1}$
for $x=\sqrt{2}+0.5$;

(4) $\left(\dfrac{4a-9a^{-1}}{2a^{1/2}-3a^{-1/2}}+\dfrac{a-4+3a^{-1}}{a^{1/2}-a^{-1/2}}\right)^2$
for $a=(2+\sqrt{3})^{1/2}(2-\sqrt{3})^{1/2}$;

(5) $\left(\dfrac{x\sqrt{x}-x}{\left(\dfrac{\sqrt[4]{x^3}-1}{\sqrt[4]{x}-1}-\sqrt{x}\right)\left(\dfrac{\sqrt[4]{x^3}+1}{\sqrt[4]{x^3}+1}-\sqrt{x}\right)}\right)^3$ for $x=\sqrt[3]{7}$;

(6) $\left(\dfrac{(\sqrt{a}+\sqrt{b})^2-(2\sqrt{b})^2}{a-b}-(a^{1/2}-b^{1/2})(a^{1/2}+b^{1/2})^{-1}\right)\div$
$\dfrac{(4b)^{3/2}}{a^{1/2}+b^{1/2}}$.

2.183. Perform the indicated operations:

(a) (1) $\sqrt{\frac{2+\sqrt{3}}{\sqrt{3}-1}+\frac{2+2\sqrt{3}}{\sqrt{3}-3}}\cdot\sqrt{(1-\sqrt{3})^2}\cdot\sqrt{3}$;

(2) $\frac{1}{(a^{2/3}-b^{2/3})^{-1}}\cdot\left(\frac{a^{1/3}-b^{1/3}}{a^{1/3}b^{1/3}+b^{2/3}}-\frac{a^{1/3}+b^{1/3}}{a^{1/3}b^{1/3}-b^{2/3}}+\frac{7a^{1/3}b^{1/3}}{a^{2/3}b^{1/3}-b}\right)$;

(3) $(\sqrt{a}-b^{3/4}a^{-1/4})^{2/3}\div\left(\frac{a^{1/4}-b^{1/4}}{a^{1/4}}+\frac{b^{1/4}}{a^{1/4}-b^{1/4}}\right)^{2/3}\times$
$\left(\frac{a-b}{\sqrt{a}+\sqrt{b}}\right)^{3/5}\cdot(\sqrt{a}-\sqrt{b})^{-4/15}$;

(4) $\left(\frac{a^{1/4}-b^{1/4}}{a^{1/2}+a^{1/4}b^{1/4}}+\frac{a^{1/4}+b^{1/4}}{a^{1/2}-a^{1/4}b^{1/4}}-\frac{2^{1/2}}{a^{3/4}-a^{1/4}b^{1/2}}\right)\cdot(b^{1/2}-a^{1/2})$;

(b) (1) $\left(\frac{(1-a)\sqrt[3]{1+a}}{a}\cdot\sqrt{\frac{3a^2}{4-8a+4a^2}}\right)^{-1}\div\frac{3a\sqrt{a}}{2\sqrt{1-a^2}}$;

(2) $\frac{2x^{-1/3}}{x^{2/3}-3x^{-1/3}}-\frac{x^{2/3}}{x^{5/3}-x^{2/3}}-\frac{x+1}{x^2-4x+3}$;

(3) $\left(\frac{\sqrt[4]{x^3}-\sqrt[4]{x}}{1-\sqrt{x}}+\frac{1+\sqrt{x}}{\sqrt[4]{x}}\right)^2\cdot\left(1+\frac{2}{\sqrt{x}}+\frac{1}{x}\right)^{-1/2}$;

(4) $\left(\left(\frac{2^{3/2}+27y^{3/5}}{\sqrt[4]{2}+3\sqrt[5]{y}}+3\sqrt[10]{32y^2}-2\right)3^{-2}\right)^5$;

(c) (1) $\left(\frac{1+\sqrt{x}}{\sqrt{1+x}}-\frac{\sqrt{1+x}}{1+\sqrt{x}}\right)^2-\left(\frac{1-\sqrt{x}}{\sqrt{1+x}}-\frac{\sqrt{1+x}}{1-\sqrt{x}}\right)^2$;

(2) $\frac{a^{4/3}-8a^{1/3}b}{a^{2/3}+2\sqrt[3]{ab}+4b^{2/3}}\cdot\left(1-2\sqrt[3]{\frac{b}{a}}\right)^{-1}$;

(3) $\left[\frac{x^{3/4}-y^{3/4}}{(x^{3/4}-y^{3/4})^{-1}}\cdot(x^{1/2}-y^{1/2})^{-1}-\frac{1}{(x-y)^{-1/2}}\right]\cdot\frac{2\sqrt{x^2y}-2\sqrt{y^2x}}{x^2-y^2}$;

(4) $\left[\frac{(a^{3/4}-b^{3/4})(a^{3/4}+b^{3/4})}{a^{1/2}-b^{1/2}}-\sqrt{ab}\right]\frac{2\sqrt{2.5}\,(a+b)^{-1}}{\sqrt[-2]{10}}$.

2.184. Prove the following equalities:

(1) $\frac{a^{7/3}-2a^{5/3}b^{2/3}+ab^{4/3}}{a^{5/3}-a^{4/3}b^{1/3}-ab^{2/3}+a^{2/3}b}\div a^{1/3}=a^{1/3}+b^{1/3},\ a\neq b\neq 0$;

(2) $\frac{x-1}{x^{3/4}+x^{1/2}}\cdot\frac{x^{1/2}+x^{1/4}}{x^{1/2}+1}\cdot x^{1/4}+1=\sqrt{x}$;

(3) $\frac{1-x^{-2}}{x^{1/2}-x^{-1/2}}-\frac{2}{x^{3/2}}+\frac{x^{-2}-x}{x^{1/2}-x^{-1/2}}=-\sqrt{x}\left(1+\frac{2}{x^2}\right)$;

(4) $\left((1-x^2)^{-1/2}+1+\frac{1}{(1-x^2)^{-1/2}-1}\right)^{-2} \div$
$(2-x^2-2\sqrt{1-x^2})=1-x^2,\ x\neq 0,\ |x|<1;$

(5) $(a^2-x^2)^{1/2}-x^2(a^2-x^2)^{-1/2}+$
$a^2\frac{(a^2-x^2)^{1/2}+x^2(a^2-x^2)^{-1/2}}{(a^2-x^2)\left(1+\frac{\sqrt{a^2-x^2}}{x}\right)^{-2}}=2\sqrt{a^2-x^2};$

(6) $\left(\frac{x+4}{2x^2-2x-4}+\frac{x+2}{2(x^2+3x+2)}\right)\sqrt{2x}-$
$\left(\sqrt{2}+\sqrt{x}-\frac{x+6}{\sqrt{x}+\sqrt{2}}\right)\div(x^{1/2}-2^{1/2})^2=\sqrt{2};$

(7) $\frac{(1-x^2)^{-1/2}+1}{(1+x)^{-1/2}+(1-x)^{1/2}}\div\frac{\sqrt{1-x}}{x-2}+$
$(x+1)\left(\frac{1}{x+1}+\frac{4}{x^2-4x}-\frac{5}{x^2-3x-4}\right)=\frac{1}{x(x-1)};$

(8) $\left(\frac{1}{x^{1/2}-4x^{-1/2}}+\frac{2\sqrt[3]{x}}{x\sqrt[3]{x}-4\sqrt[3]{x}}\right)^{-2}-$
$\sqrt{x^2+8x+16}=-4\sqrt{x}$ if $x>-4.$

§ 35. Quadratic Equations. Pure Quadratic Equations

2.185. Solve the following equations:

(a) (1) $x^2+1=82$; (2) $x^2+5=30$; (3) $5x^2=20$; (4) $5x^2-80=0$; (5) $\frac{5}{9}x^2-3380=0$; (6) $67-6x^2=13$; (7) $79-7x^2=16$; (8) $2\frac{1}{4}-\frac{2}{9}x^2=2\frac{1}{8}$; (9) $8x^2-0.75=0.53$; (10) $4.3-6x^2=2.8$; (11) $7.25-0.6x^2=3.5$; (12) $4x^2-75=0$; (13) $5x^2-4=0$; (14) $2x^2-7=0$; (15) $0.2x^2-6=0$;

(b) (1) $x^2-4x=0$; (2) $5x^2-20x=0$; (3) $x^2-0.5x=0$; (4) $4x^2+6x=9x^2-15x$; (5) $13x+7x^2=5x^2+8x$; (6) $12x^2-5x=9x^2+7x$; (7) $8.5x-3x^2=3.5x+2x^2$;

(c) (1) $\frac{4}{x^2-10x+25}+\frac{1}{25-x^2}-\frac{1}{x+5}=0;$

(2) $\frac{6}{7x-21}-\frac{1}{x^2-6x-9}+\frac{1}{x^2-9}=0$;

(3) $\frac{x+5}{x^2-25}-\frac{3}{2x+10}+\frac{35x+25}{2x^2-50x}$;

(4) $\frac{1}{4y-1}+\frac{2}{1-16y^2}+\frac{y-3}{12y+3}=0$;

(5) $\frac{x+56}{9x^2-16}+\frac{1}{8-6x}=\frac{18}{3x^2+4x}$;

(6) $\frac{x+5}{25x^2-10x}+\frac{x+4}{4x(5x+2)}-\frac{9}{25x^2-4}=0$;

(7) $\frac{x+1}{2x^2-3x}-\frac{4x+1}{4x^2+6x}-\frac{10}{4x^2-9}=0$;

(8) $\frac{y+2}{6y^2+20y}+\frac{2}{100-9y^2}+\frac{y+1}{3y^2-10y}=0$;

(d) (1) $ax^2+bx=0$; (2) $x^2/a-x/b=0$; (3) $(x+a)\times(x-b)+(x+b)(x-a)=2a^2-2ab$; (4) $(a+x)\times(a+2x)-(a-x)(a-2x)=(a+3x)^2-a^2-9b^2$; (5) $(ax+b)^2-(a-bx)^2-4abx+a^2(x^2-1)=0$;

(6) $\frac{x+a}{x-a}+\frac{x-a}{x+a}=\frac{a(3x+2a)}{x^2-a^2}$;

(7) $\frac{x-a}{a}-\frac{a}{x-a}=0$; (8) $\frac{a^2-1}{ax-1}+\frac{a-x}{a}=1$;

(9) $\frac{ax+b}{a}=\frac{ab}{a^2-x}$; (10) $\frac{ax^2}{x-1}=(a+1)^2$;

(11) $\frac{ax+b}{x-a}=\frac{x-b}{x+a}$; (12) $\frac{x}{a+b}+\frac{2a-x}{a-b}-\frac{a+b}{x}=1$.

§ 36. Affected Quadratics and Equations Reducible to Them

2.186. Solve the following equations by separating a perfect square:

(a) (1) $x^2+8x-33=0$; (2) $x^2+12x-64=0$; (3) $x^2-8x=20$; (4) $x^2-4x=45$; (5) $3x+40=x^2$; (6) $x^2+5x=14$;

(b) (1) $x^2+12x=-35$; (2) $x^2+14x+24=0$; (3) $x^2-11x+30=0$; (4) $x^2-11x=60$;

(c) (1) $x^2+x-30=0$; (2) $x^2-x-12=0$; (3) $x^2-x-20=0$; (4) $x^2-7x+12=0$;

(d) (1) $4x^2 + 4x - 8 = 0$; (2) $9x^2 - 12x - 12 = 0$; (3) $25x^2 + 30x - 40 = 0$; (4) $36x^2 + 12x - 80 = 0$.

2.187. Solve the following equations by separating a perfect square:

(a) (1) $x^2 - 2ax - 3a^2 = 0$; (2) $x^2 - 6ax + 5a^2 = 0$; (3) $x^2 - 11ax - 60a^2 = 0$; (4) $x^2 - 3ax + 2a^2 - ab - b^2 = 0$;

(b) (1) $x^2 - 4ax + 4a^2 = b^2$; (2) $x^2 - (2a - b)\,x + a^2 - ab - 2b^2 = 0$;

(3) $x^2 - 5\,(a - b)\,x + 6a^2 - 13ab + 6b^2 = 0$;

(4) $x^2 - (2a + 1)\,x + 5a - 3 - \frac{3}{4} = 0$;

(c) (1) $x^2 - 5ax + 6a^2 = 0$; (2) $x^2 - 2mx + m^2 - n^2 = 0$; (3) $x^2 - 2\,(p + q)\,x + 4pq = 0$; (4) $x^2 + 8ax - 20a^2 = 0$.

2.188. Solve the following equations:

(a) (1) $x^2 - 6x + 8 = 0$; (2) $x^2 - 2x - 35 = 0$; (3) $x^2 - x - 12 = 0$; (4) $x^2 - 18x + 77 = 0$;

(b) (1) $x^2 - 26x + 120 = 0$; (2) $x^2 - 36x + 320 = 0$; (3) $x^2 - 40x + 300 = 0$; (4) $x^2 - 52x + 640 = 0$;

(c) (1) $x^2 - 134x + 1240 = 0$; (2) $x^2 - 372x + 3620 = 0$; (3) $x^2 + 412x + 4800 = 0$; (4) $x^2 - 502x + 60\,600 = 0$;

(d) (1) $x^2 - 4x + 2 = 0$; (2) $x^2 - 6x - 66 = 0$; (3) $x^2 - 24x - 56 = 0$; (4) $x^2 - 28x + 100 = 0$.

2.189. Solve the following equations:

(a) (1) $x^2 - 5.6x + 6.4 = 0$; (2) $x^2 + 2.4x - 13 = 0$; (3) $x^2 - \frac{5}{3}x + 26 = 0$; (4) $x^2 - 4\frac{1}{2}x + 4\frac{1}{2} = 0$;

(5) $x^2 - 3\frac{5}{12}x + 2 = 0$; (6) $x^2 - 1.6x - 2 = 0$;

(b) (1) $3x^2 - 5x - 2 = 0$; (2) $2x^2 - 7x + 6 = 0$; (3) $4x^2 + x - 3 = 0$; (4) $5x^2 - 8x + 3 = 0$; (5) $10x^2 - 3x - 1 = 0$; (6) $4x^2 - 17x - 15 = 0$;

(c) (1) $3x^2 + 2x - 8 = 0$; (2) $(3x - 1)(x + 2) = 20$; (3) $(x - 4)(4x - 3) + 3 = 0$; (4) $(x - 3)^2 + (x + 4)^2 - (x - 5)^2 = 17x + 24$; (5) $(x + 5)^2 + (x - 2)^2 + (x - 7) \times (x + 7) = 11x + 30$; (6) $(x + 3)(x - 2) + (x + 2)^2 - 3x - 10 = 0$; (7) $(x - 5)^2 + (3 - x^2) - 4(x + 5) \times (3 - x) - 48 = (x + 1)^2$; (8) $(x - 1)(x - 2)(x - 3) - (x^2 + 3)(x - 5) + 2x - 33 = 0$;

(d) (1) $\frac{x^2}{6} - \frac{2x}{3} = \frac{3x-10}{4}$; (2) $\frac{x(x+4)}{2} - 3 = \frac{7x}{4} - \frac{5x-4}{6}$; (3) $\frac{(x-11)^2}{10} - \frac{(6x-1)^2}{5} = 7 - \frac{7x-3}{2}$;

(4) $\frac{x-3}{4} + \frac{2x+3}{6} = \frac{x^2-11}{12}$;

(5) $\frac{(5x-x)^2}{3} - \frac{(5x-11)^2}{4} = 6 - \frac{(7-x)^2}{2}$;

(6) $6x + \frac{(3+5x)^2}{2} = \frac{8-2x}{5} - \frac{(x+3)(x+7)}{2}$;

(7) $x - 7 + \frac{(x-6)^2}{3} = \frac{(x+4)^2}{2} - \frac{(x+2)(x+6)}{4}$.

2.190. Solve the following equations by factoring their left-hand sides:

(a) (1) $x^3 - 3x + 2 = 0$; (2) $x^3 - 4x + 3 = 0$; (3) $x^3 - x^2 - x + 1 = 0$; (4) $x^3 - 25x^2 + 114x = 0$; (5) $x^3 - 3x^2 - 6x + 8 = 0$; (6) $x^3 - 7x^2 - 21x + 27 = 0$;

(b) (1) $8x^3 - 4x^2 - 2x + 1 = 0$; (2) $8x^3 - 14x^2 - 7x + 1 = 0$; (3) $3x^4 + x^3 - 12x^2 - 4x = 0$; (4) $x^4 - 10x^3 + 90x - 81 = 0$; (5) $9x^4 - 30x^3 + 25x^2 - 4 = 0$ (6) $x^4 - 25x^2 + 60x - 36 = 0$; (7) $x^4 + 10x^3 + 25x^2 - 36 = 0$; (8) $x^4 - 5x^3 - 3x^2 + 45x - 54 = 0$.

2.191. Solve the following equations:

(a) (1) $\frac{5(x-1)}{4} = \frac{x}{6} + \frac{6}{x}$; (2) $\frac{5x-1}{9} + \frac{3x-1}{5} = \frac{2}{x} + x - 1$, (3) $\frac{12}{x} - \frac{7x-6}{6} + 5x - 26 = 0$; (4) $\frac{5(x-1)}{8} = \frac{x}{10} + \frac{10}{x}$;

(5) $\frac{7}{x} - \frac{21+65x}{7} + 8x + 11 = 0$; (6) $x + \frac{1}{x} - \frac{1}{3} = 3$;

(b) (1) $\frac{3x-7}{x+5}=\frac{x-3}{x+2}$; (2) $\frac{2x-5}{x-1}=\frac{5x-3}{3x+5}$;

(3) $\frac{5-x}{2x-1}=\frac{15-4x}{3x+1}$; (4) $\frac{x^2+1}{x-4}-\frac{x^2-1}{x+3}=23$;

(5) $\frac{3x-2}{x-3}-\frac{x-4}{x+3}=\frac{15x-3}{x^2-9}$;

(6) $\frac{3x-1}{x+2}-\frac{18}{2-x}=\frac{7x^2-28}{x^2-4}+\frac{7}{2+x}$;

(c) (1) $\frac{1}{x-2}+\frac{1}{x+7}=\frac{1}{x-1}+\frac{1}{x+1}$;

(2) $\frac{1}{x-3}+\frac{1}{x-2}=\frac{1}{x-4}+\frac{1}{x+6}$;

(3) $\frac{1}{x+2}+\frac{1}{x+20}=\frac{1}{x+4}+\frac{1}{x+8}$;

(4) $\frac{1}{x-6}+\frac{1}{x-4}=\frac{1}{x+2}+\frac{1}{x-7}$;

(5) $\frac{1}{x-9}+\frac{1}{x-7}=\frac{1}{x+18}+\frac{1}{x-10}$;

(6) $\frac{1}{x-8}-\frac{1}{x-2}=\frac{1}{x-11}-\frac{1}{x-10}$.

2.192. Solve the following equations:

(1) $\frac{2}{8x^3+4x^2-2x-1}+\frac{1}{1-4x^2}+\frac{x}{4x^2+4x+1}=0$;

(2) $\frac{5}{x^3+2x^2-9x-18}+\frac{2}{4-x^2}=\frac{1}{x^2-9}$;

(3) $\frac{1}{(3x-2)^2}+\frac{x+1}{9x^3-4x}+\frac{3x}{27x^3-18x^2-12x+8}=0$;

(4) $\frac{6}{4x^2-1}+\frac{1}{25-x^2}+\frac{27}{4x^3-20x^2-x+5}=0$;

(5) $\frac{1}{x^3+8x^2+32x+64}-\frac{9x}{x^3-64}+\frac{6}{x^2-16}=0$;

(6) $\frac{2}{x^2-9}+\frac{11x+4}{x^3+27}-\frac{x}{x^3-6x^2+18x-27}=0$;

(7) $\frac{4}{x^2-16}-\frac{1}{x^2+8x+16}-\frac{10}{x^3-16x-4x^2+64}=0$.

2.193. Solve the following equations by introducing a new variable:

(1) $(x^2-6x)^2-2(x-3)^2=81$; (2) $(x^2+2x)^2-(x+1)^2=55$;

(3) $\frac{x^2+1}{x}+\frac{x}{x^2+1}=2.9$; (4) $\frac{21}{x^2-4x+10}-x^2+4x=6$;

(5) $\frac{1}{x^2+2}-\frac{1}{x^2+3}=\frac{1}{12}$;

(6) $\frac{4}{x^2+4}+\frac{5}{x^2+5}=2$;

(7) $\frac{x^2+x-5}{x}+\frac{3x}{x^2+x-5}+4=0$;

(8) $\frac{1}{x(x+2)}-\frac{1}{(x+1)^2}=\frac{1}{12}$.

2.194. Solve the following equations with literal coefficients:

(a) (1) $x^2+2ax-3a^2=0$; (2) $x^2-11ax-60a^2=0$; (3) $x^2-5ax+6a^2-ab-b^2=0$; (4) $x^2-4ax+4a^2=b^2$;

(b) (1) $x^2-(2a-b)x+a^2-ab-2b^2=0$;

(2) $x^2-5(a-b)x+6a^2-13ab+6b^2=0$;

(3) $(a^2-b^2)x^2-4abx=a^2-b^2$;

(4) $(ax-b)^2+(a-bx)^2+4abx=2(a^2+b^2)x$.

2.195.

(a) (1) $x+\frac{1}{x}=a+\frac{1}{a}$; (2) $\frac{x-b}{a-b}=\frac{x^2}{a^2}$;

(3) $\frac{3x-a}{a}=\frac{4x-a}{2x}$; (4) $\frac{x^2+4ab}{b^2-x^2}=\frac{x-a}{x+b}-\frac{x+a}{x-b}$;

(5) $\frac{2x}{x-b}+\frac{12x^2}{b^2-x^2}=\frac{b-x}{b+x}$;

(6) $\frac{2ax}{2ax-b}=\frac{3b}{2ax+b}-\frac{a^2x^2+2b^2}{b^2-4a^2x^2}$;

(b) (1) $\frac{x}{x-a}-\frac{2a}{x+a}=\frac{8a^2}{x^2-a^2}$;

(2) $\frac{m^2+2n}{nx+2n+mx+2m}-\frac{m}{x+2}-\frac{x}{m+n}+\frac{x}{x+2}=0$;

(3) $\frac{ax^2}{x-1}=(a+1)^2$; (4) $\left(\frac{x^2+6}{Q^2-4}\right)^2=\left(\frac{5x}{4-Q^2}\right)^2$;

(5) $\frac{x}{ax-3a}-\frac{a+3}{ax-3a-6x+3b}=\frac{1}{3-x}-\frac{x}{a^2-ab}$;

(6) $\frac{x}{a^2+ab}-\frac{x}{ax+3a}+\frac{1}{x+3}=\frac{a+3}{ax+3a+bx+3b}$.

§ 37. Properties of the Roots of a Quadratic Equation

2.196. Given the roots, set up quadratic equations:

(a) (1) 7, -5; (2) 3, -3; (3) 1/2, $-1/4$; (4) -7 and 0; (5) 3/4 and 4/3; (6) 0.6 and 0.5; (7) 0.4 and 0.01; (8) 1.6 and 1.4; (9) 2.4 and 3.2;

(b) (1) $2-\sqrt{3}$ and $2+\sqrt{3}$; (2) $4-\sqrt{5}$ and $4+\sqrt{5}$; (3) $7-4\sqrt{3}$ and $7+\sqrt{12}$; (4) $\frac{1}{10-2\sqrt{10}}$ and $\frac{1}{10+\sqrt{40}}$;

(5) $\frac{1}{10-\sqrt{72}}$ and $\frac{1}{10+6\sqrt{2}}$;

(6) $\frac{6}{\sqrt{15}-3}$ and $\frac{6}{\sqrt{15}+3}$;

(7) $\frac{12}{4-\sqrt{7}}$ and $\frac{16-4\sqrt{7}}{9}$; (8) $\frac{22}{5-\sqrt{14}}$ and $\frac{22}{5+\sqrt{14}}$.

2.197.

(a) (1) $3a-2b$ and $2b$; (2) $a/3$ and $3/a$;

(3) a/b and $-b/a$; (4) $2a-b$ and $a-2b$;

(5) $\frac{m+n}{2}$ and $\frac{m-n}{2}$; (6) $\frac{a+b}{a-b}$ and 1;

(7) $\frac{a}{1-a}$ and $\frac{b}{1-a}$; (8) $\frac{a+b}{a-b}$ and $\frac{a-b}{a+b}$;

(b) (1) $a\sqrt{5}$ and $b\sqrt{3}$; (2) $2\sqrt{a}-3\sqrt{b}$ and $-3\sqrt{b}$; (3) $3m+2n\sqrt{3}$ and $3m-2n\sqrt{3}$; (4) $a+b\sqrt{m}$ and $a-b\sqrt{m}$;

(5) $\frac{a}{\sqrt{a}-\sqrt{a-b}}$ and $\frac{a}{\sqrt{a}+\sqrt{a-b}}$;

(6) $\frac{2}{a+a\sqrt{b}}$ and $\frac{2}{a-a\sqrt{b}}$.

2.198. Set up a quadratic equation whose roots are twice the roots of the given equation:

(1) $x^2-5x+6=0$; (2) $3x^2+10x+3=0$;
(3) $2x^2-3x+1=0$; (4) $6x^2-7x+1=0$;
(5) $5x^2-12x+4=0$; (6) $8x^2+14x+3=0$.

2.199. Set up a quadratic equation whose roots are by unity greater than those of the given equation:

(1) $x^2 - 10x + 9 = 0$; (2) $x^2 - 2x - 35 = 0$;
(3) $8x^2 - 11x + 2 = 0$; (4) $4x^2 + 5x + 1 = 0$;
(5) $2x^2 + 7x + 3 = 0$; (6) $6x^2 + 17x + 5 = 0$.

2.200. Set up a quadratic equation whose roots are by 0.2 greater than those of the given equation:

(1) $x^2 - 12x + 20 = 0$; (2) $x^2 - 18x + 32 = 0$;
(3) $5x^2 - x - 2 = 0$; (4) $11x^2 + 8x - 3 = 0$;
(5) $5x^2 + 22x + 24 = 0$; (6) $3x^2 + 10x + 8 = 0$.

2.201. Set up a quadratic equation whose roots are inverse to those of the given equation:

(1) $x^2 - 8x + 12 = 0$; (2) $2x^2 - x - 1 = 0$;
(3) $3x^2 + 5x + 8 = 0$; (4) $4x^2 - 3x - 1 = 0$;
(5) $x^2 + px + q = 0$; (6) $ax^2 + bx + c = 0$.

2.202. Set up a quadratic equation whose roots $y_1 = x_1 + x_2$ and $y_2 = x_1 \cdot x_2$ if x_1 and x_2 are the roots of the given equation:

(1) $x^2 - 6x - 40 = 0$; (2) $4x^2 - 5x + 1 = 0$;
(3) $x^2 + px + q = 0$; (4) $ax^2 + bx + c = 0$.

2.203. Find:

(1) the sum of squares of the roots of the equation;
(2) the difference of squares of the roots of the equation;
(3) the sum of their cubes;
(4) the difference of their cubes if the following equations are given:

(a) $x^2 + px + q = 0$; (b) $ax^2 + bx + c = 0$;
(c) $x^2 - 3x + 2 = 0$; (d) $2x^2 - 5x + 2 = 0$.

2.204. Find k if $6x_1 + x_2 = 0$, where x_1 and x_2 are the roots of the given equation:

(1) $x^2 - 15x + k = 0$; (2) $x^2 - 5x + k = 0$;
(3) $x^2 - kx - 6 = 0$; (4) $3x^2 - 5x + k = 0$;
(5) $kx^2 - 5x + 2 = 0$; (6) $kx^2 - 20x + 4 = 0$.

2.205. It is known that $x_1 - x_2 = 1$;
(a) find k for each equation:

(1) $5x^2 + kx + 1 = 0$; (2) $5x^2 - kx - 1 = 0$;
(3) $kx^2 - 6x + 4 = 0$; (4) $4x^2 - kx + 3 = 0$;

(b) it is known that $x_1 - x_2 = 2$. Find k for each equation:

(1) $25x^2 - 30x + k = 0$; (2) $kx^2 - 4x + 4\sqrt{3} = 0$;
(3) $2x^2 - kx + 1\frac{1}{4} = 0$; (4) $kx^2 - 12x + 5 = 0$.

2.206. (a) The ratio of the roots of the equation is equal to 6. Find k if:

(1) $32x^2 + kx + 75 = 0$; (2) $kx^2 - 7x + 24 = 0$;
(3) $3x^2 - 63x + k = 0$; (4) $2x^2 - 7x + k = 0$.

(b) The ratio of the roots of the equation is equal to 12. Find k if:

(1) $26x^2 - kx + 78 = 0$; (2) $16x^2 - 78x + k = 0$;
(3) $kx^2 - 52x + 8 = 0$; (4) $4x^2 + kx + 3 = 0$.

2.207. Find m and n if:

(1) in the equation $3x^2 + mx + n = 0$ $x_1 = -1\frac{1}{3}$; $x_2 = -m$;

(2) in the equation $3x^2 + mx + n = 0$ $x_1 = 1/3$; $x_2 = -n - 7$;

(3) in the equation $mx^2 - 5x + n = 0$ $x_1 = 1/2$; $x_2 = m$.

2.208. Find c knowing the sum or the difference of the squares of the given equation:

(1) $x_1^2 - x_2^2 = 1.75$; $2x^2 + 7x + c = 0$;

(2) $x_1^2 - x_2^2 = 1\frac{6}{7}$; $7x^2 - 10x + c = 0$;

(3) $x_1^2 - x_2^2 = 3\frac{1}{3}$; $3x^2 - 14x + c = 0$;

(4) $x_1^2 + x_2^2 = 13/36$; $6x^2 - 5x + c = 0$;

(5) $x_1^2 + x_2^2 = 1\frac{1}{64}$; $8x^2 - 9x + c = 0$;

(6) $x_1^2 + x_2^2 = 5/4$; $16x^2 - 24x + c = 0$;
(7) $x_1^2 + x_2^2 = 10/3$; $6x^2 - 2x + c = 0$.

2.209. (a) x_1 and x_2 are the roots of the equation $x^2 - 6x + q = 0$; find q if $3x_1 + 2x_2 = 20$;

(b) x_1 and x_2 are the roots of the equation $x^2 + 3x + q = 0$; find q if: (1) $x_1 - x_2 = 6$; (2) $3x_1 - x_2 = 4$; (3) $x_1^2 + x_2^2 = 34$;

(c) solve the equation $x^2 + px + 35 = 0$ if the sum of the squares of its roots is equal to 74;

(d) x_1 and x_2 are the roots of the equation $x^2 - x + q = 0$; find q if $x_1^3 + x_2^3 = 19$;

(e) find p and q if in the equation $x^2 + px + q = 0$ the difference of the roots is equal to 5, and the difference of the cubes of the roots is equal to 35;

(f) find m if in the equation $9x^2 - 18(m-1)x - 8m + 24 = 0$ one root is twice the other.

2.210. (a) Compute $\frac{6x_1^2 \cdot x_2 - 4x_1^3 + 6x_1x_2^2 - 4x_2^3}{3x_1^2 + 5x_1x_2 + 3x_1^2}$, where x_1 and x_2 are roots of the following equations:

(1) $x^2 - 3x + 17 = 0$; (2) $x^2 + px + q = 0$;
(3) $ax^2 + bx + c = 0$; (4) $2x^2 - 5x + 2 = 0$;

(b) find $x_1^7 \cdot x_2^2 + x_1^2 \cdot x_2^7$, where x_1 and x_2 are the roots of the following equations:

(1) $3x^2 + 5x + 6 = 0$; (2) $x^2 + px + q = 0$;
(3) $ax^2 + bx + c = 0$; (4) $3x^2 - 10x + 3 = 0$.

§ 38. Factorization of a Quadratic Trinomial into Linear Factors

2.211. Factor:

(a) (1) $x^2 - 4x + 3$; (2) $x^2 - 10x + 9$; (3) $x^2 - 2x - 35$; (4) $x^2 - 4x - 60$; (5) $x^2 + 7x + 10$; (6) $x^2 + 25x + 114$;

(b) (1) $a^2 - 17a + 72$; (2) $a^2 - 29a + 198$; (3) $2a^2 - 5a + 2$; (4) $3a^2 - 2a - 1$; (5) $5m^2 + m - 4$; (6) $2m^2 - m - 3$; (7) $x^2 - 6ax + 4a^2 - 9b^2$;

(c) (1) $4x^2 + 9bx + 5b^2$; (2) $5x^2 + ax + 6a^2$; (3) $5y^2 - 2by - 35b^2$; (4) $2a^2 - ab - 3b^2$.

2.212. Reduce the following fractions:

(1) $\frac{a^2+6a-91}{a^2+8a-105}$; (2) $\frac{2a^2+8a-90}{3a^2-36a+105}$;

(3) $\frac{a^2-9ab+14b^2}{a^2-ab-2b^2}$; (4) $\frac{2a^2-ab-3b^2}{2a^2-5ab+3b^2}$;

(5) $\frac{5a^2-13a-6}{(a-2)^3-1}$; (6) $\frac{27-(b+1)^3}{7b^2-9b-10}$;

(7) $\frac{6a^2-5a-4}{27a^3-(a-1)^3}$; (8) $\frac{3c^2-11c+10}{(5-c)^3-8c^3}$;

(9) $\frac{x^2-4x+3}{x^2-12x+27}$; (10) $\frac{5a^2-7a-24}{24-9-7a^2}$;

(11) $\frac{27x^3+21x^2-7x-1}{27x^2-6x-1}$.

2.213. Are there such real values of x for which the given trinomials attain equal values:

(a) (1) $x^2 + 11x - 60$ and $2x^2 + 7x + 5$;
(2) $6x^2 + 8x + 3$ and $3x^2 + 5x - 2$; (3) $8x^2 + 2x - 1$ and $2x^2 - 11x + 7$; (4) $5x^2 + 8x + 3$ and $3x^2 + 3x + 1$;
(5) $9x^2 + 9x + 2$ and $2x^2 + 12x + 6$; (6) $2x^2 - 7x - 54$ and $x^2 - 8x - 24$;

(b) (1) $3x^2 + ax + 26a^2$ and $2x^2 + 15ax - 25a^2$;
(2) $5x^2 + 8x + 12a^2$ and $3x^2 + 3ax + 10a^2$;
(3) $2x + 12ax - 9a^2$ and $x^2 + 10ax - 6a^2$?

2.214. Are there such real values of x for which the given trinomials attain opposite values:

(a) (1) $7x^2 + 5x - 8$ and $2x^2 + 7x + 3$;
(2) $x^2 + 10x - 15$ and $5x^2 + x + 5$;
(3) $5x^2 + 7x - 11$ and $2x^2 + 5x - 8$;
(4) $11x^2 + 9x - 3$ and $4x^2 + 6x + 7$;
(5) $4x^2 + 3x - 2$ and $5x^2 + x + 7$;

(b) (1) $8x^2 - ax - 11a^2$ and $2x^2 - 13ax + 3a^2$;
(2) $6x^2 + 4ax + 7a^2$ and $5x^2 - 2ax - 10a^2$;
(3) $17x^2 + 15ax + 11a^2$ and $15x^2 + 10ax + 9a^2$?

§ 39. Investigating the Roots of a Quadratic Equation by Its Discriminant and Coefficients

2.215. Without solving the equations, determine which of them have different real roots, equal real roots, or no real roots:

(a) (1) $x^2 + 6x + 5 = 0$; (2) $x^2 - 6x + 8 = 0$;
(3) $x^2 - 10x + 25 = 0$; (4) $4x^2 + 6x + 9 = 0$;
(5) $4x^2 - 10x + 5 = 0$; (6) $5x^2 + 11x - 60 = 0$;

(b) (1) $2x^2 + 3x + 2 = 0$; (2) $25x^2 + 30x + 9 = 0$;
(3) $3x^2 + 12x + 3 = 0$; (4) $4x^2 - 4x + 13 = 0$;
(5) $2x^2 - 18x + 65 = 0$; (6) $7x^2 - 12x + 19 = 0$;

(c) (1) $x^2 - 6ax + 5a^2 = 0$; (2) $ax^2 - 2x - a = 0$;
(3) $mx^2 - (m + n)\,x + n = 0$; (4) $x^2 - 4ax + 4a^2 = 2$;
(5) $a^2x^2 - 2ax + 8 = 0$; (6) $a^2x^2 - 2ax + 1 = 0$;
(7) $5a^2x^2 - 4ax + 1 = 0$.

2.216. Determine the sign of the roots of the following equations:

(a) (1) $x^2 - 6x + 5 = 0$; (2) $x^2 + 4x - 5 = 0$;
(3) $x^2 + 20x + 19 = 0$; (4) $x^2 - x - 20 = 0$;

(b) (1) $4x^2 - 10x + 5 = 0$; $8x^2 - 2x - 1 = 0$;
(3) $4x^2 + 9x + 2 = 0$; (4) $2x^2 + 5x + 2 = 0$;

(c) (1) $x^2 - 2ax - 8a^2 = 0$; (2) $ax^2 - 2x + 7a = 0$;
(3) $5x^2 - 6ax + 10a^2 + 25 = 0$; (4) $a^4x^2 - 10x + 3a^2 + a^4 = 0$.

2.217. For what values of a does each of the following equations have equal real roots:

(1) $x^2 + ax + 9 = 0$; (2) $x^2 + 12x + a = 0$;
(3) $ax^2 + 4x + 1 = 0$; (4) $9x^2 + 6x + a = 0$;
(5) $4x^2 + ax + 9 = 0$; (6) $3a^2x^2 - 10x + 3 = 0$;
(7) $(a - 1)\,x^2 + 2\,(a + 1)\,x + a - 2 = 0$; (8) $2x^2 + (a - 9)\,x + a^2 + 3a + 4 = 0$; (9) $x^2 + (a - 1)\,ax + 36 = 0$;
(10) $(2a - 1)\,x^2 - 4\,(a - 1)\,x + 3 = 0$; (11) $x^2 - 2ax\sqrt{a^2 - 3} + 4 = 0$; (12) $x^2 - 2\,(a - 4)\,x + a^2 +$

$6a + 2 = 0$; (13) $2(a + 2)x^2 - 24x + a + 3 = 0$;
(14) $(2a + 1)x^2 - 3(a - 1)x - a + 1 = 0$.

2.218. (a) Prove that the left-hand side of the given equation is a perfect square if its discriminant equals zero: (1) $x^2 + px + q = 0$; (2) $ax^2 + bx + c = 0$.

(b) Prove that the equation $\frac{a^2}{x} + \frac{b^2}{x-1} = 1$ has real roots if a and b are real numbers not equal to zero simultaneously.

(c) Prove that the equation $(a^2 + b^2 + c^2)x^2 + 2(a + b + c)x + 3 = 0$ has no real roots if $a \neq b \neq c$.

(d) Prove that the equation $\frac{1}{x-a} + \frac{1}{x-b} = \frac{1}{c^2}$ has real roots for any real values of a, b, and c.

§ 40. Biquadratic Equations

2.219. Solve the following equations:

(a) (1) $x^4 - 10x^2 + 9 = 0$; (2) $x^4 - 13x^2 + 36 = 0$;
(3) $x^4 - 29x^2 + 100 = 0$; (4) $x^4 - 5x^2 + 4 = 0$;
(5) $x^4 - 17x^2 + 16 = 0$; (6) $x^4 - 37x^2 + 36 = 0$;
(7) $4x^4 - 5x^2 + 1 = 0$; (8) $3x^4 - 28x^2 + 9 = 0$;
(9) $2x^4 - 19x^2 + 9 = 0$; (10) $3x^4 - 7x^2 + 2 = 0$;

b) (1) $a^2b^2x^4 = b^4x^2 - a^2b^2 + a^4x^2$;
(2) $x^4 - 25x^2 = m^2x^2 - 25m^2$; (3) $x^4 + 9n^2 = n^2x^2 + 9x^2$;
(4) $4x^4 + a^2 = x^2 + 4a^2x^2$; (5) $x^4 - \frac{50}{2x^4 - 7} = 14$;
(6) $x^2(x - 4)(x + 4) = 24 - 2x^2(x^2 + 5)$;
(7) $\frac{3(x^2+1)}{2-x} + 2 + x = \frac{9}{6x^2(2-x)}$;
(8) $\frac{5x^3-5x^2+1}{x^2} + \frac{4(2-x^2)}{x+1} + \frac{10}{x(x+1)} = \frac{1+10x}{x^2}$.

2.220. Factor the following polynomials:

(1) $x^4 - 5x^2 + 4$; (2) $x^4 - 13x^2 + 36$; (3) $4x^4 - 5x^2 + 1$;
(4) $x^4 - 8x^2 + 7$; (5) $x^4 - 12x^2 + 35$; (6) $4x^4 - 17x^2 + 4$;
(7) $9x^4 - 22x^2 + 8$; (8) $x^4 - 11x^2 + 30$;
(9) $x^4 - (a + b)x^2 + ab$; (10) $x^4 - (1 + ab)x^2 + ab$.

2.221. Reduce the following fractions:

(1) $\frac{x^4-10x^2+9}{x^4-13x^2+36}$; (2) $\frac{x^4-17x^2+16}{x^4-50x^2+49}$;

(3) $\frac{4x^4-13x^2+3}{x^4-7x^2+12}$; (4) $\frac{x^4-9x^2+20}{x^4-10x^2+24}$;

(5) $\frac{2x^4-7x^2+6}{3x^4+3x^2-6}$; (6) $\frac{5a^4+5a^2-3a^2b-3b}{a^4+3a^2+2}$;

(7) $\frac{x^4-a^2x^2-b^2x^2+a^2b^2}{x^4-x^2-b^2x^2+b^2}$; (8) $\frac{x^4-n^2x^2-m^2x^2+m^2n^2}{x^4-c^2x^2-n^2x^2+c^2n^2}$;

(9) $\frac{9x^4-x^2-9a^2x^2+a^2}{4x^4-x^2-4a^2x^2+a^2}$; (10) $\frac{25x^4-a^2x^2-25b^2x^2+a^2b^2}{4x^4-a^2x^2-4b^2x^2+a^2b^2}$.

2.222. (1) Prove that the sum of all the roots of the biquadratic equation $x^4 + px^2 + q = 0$ is equal to zero, while the product of the roots is equal to q.
(2) Prove that the sum of all the roots of the biquadratic equation $ax^4 + bx^2 + c = 0$ is equal to zero, and the product of the roots equals c/a.
(3) It is known that one of the roots of the equation $x^4 - 10x^2 + q = 0$ is equal to unity. Find q and solve this equation.
(4) It is known that one of the roots of the equation $2x^4 - bx^2 + 9 = 0$ is equal to 3. Find b and solve this equation.
(5) One of the roots of the equation $ax^4 - 7x^2 + 2 = 0$ is equal to $\sqrt{2}$. Find the roots of this equation.
(6) In the equation $x^4 + px^2 + q = 0$ $x_1 = 2$ and $x_2 = 3$. Find p and q and solve this equation.

2.223. Set up a biquadratic equation given its roots:

(a) (1) ± 2 and ± 3; (2) ± 1 and ± 6; (3) $\pm 2/3$ and ± 4;
(4) $\pm\sqrt{3}$ and $\pm\sqrt{2}$; (5) $\pm 2\sqrt{3}$ and $\pm 2\sqrt{5}$;
(6) $\pm\sqrt{a}$ and ± 1; (7) $\pm a\sqrt{2}$ and $\pm b\sqrt{2}$;
(8) $\pm\left(\frac{2}{3}\right)\sqrt{a}$ and $\pm\left(\frac{1}{3}\right)\sqrt{b}$; (9) $\pm ab$ and $\pm bc$;

(b) (1) 3 and 4; (2) 5 and 7; (3) 1 and $\sqrt{2}$;
(4) $\sqrt{a}$ and $\sqrt{b}$; (5) $3\sqrt{2}$ and $2\sqrt{3}$; (6) $\sqrt{5}$ and 2;
(7) $a\sqrt{2}$ and $2\sqrt{a}$; (8) $a\sqrt{b}$ and 2; (9) a/b and b/a.

§ 41. Irrational Equations

2.224. Find the domain of permissible values of the unknown for each of the following equations:

(a) (1) $\sqrt{x-4}=3$; (2) $\sqrt{x-1}+\sqrt{x+4}=5$;

(3) $\sqrt{x+1}+\sqrt{4-x}=3$; (4) $\sqrt{3x-6}+\sqrt{1+x}=2$;

(5) $\sqrt{x}\cdot\sqrt{x-2}=3$; (6) $\sqrt[4]{x^2}\cdot\sqrt[4]{x}=2\sqrt{2}$;

(b) (1) $\sqrt{2x+1}+\sqrt[5]{x-3}=4$; (2) $\sqrt[4]{x+4}+\sqrt[6]{x+9}=5$;

(3) $x^2+\sqrt{x-1}=10x+\sqrt[3]{x+2}$; (4) $\sqrt[3]{x}+\sqrt[3]{x+6}=\sqrt[3]{x+8}$;

(5) $\sqrt{x^2-2x+2}=5$; (6) $\sqrt{(5-x)/x^2}+\sqrt{x-7}=11$.

2.225. Without solving the given equations, explain why they have no solution:

(1) $\sqrt{x+1}+\sqrt{x+2}=0$; (2) $\sqrt{2x-1}=-5$;

(3) $\sqrt{x^2+1}+\sqrt{x-1}=-3$; (4) $\sqrt{1-x}+\sqrt{2-x}=-1$;

(5) $\sqrt{x+1}+\sqrt{x+2}=-6$; (6) $\sqrt{x}+\sqrt{x+9}=2$.

2.226. Solve the following equations:

(1) $3+\sqrt{x-2}=4$; (2) $\sqrt{5+\sqrt{x-4}}=3$;

(2) $\sqrt{2x^2-x-2}=x$; (4) $\sqrt{24-10x}=3-4x$;

(5) $\sqrt{4+x}\cdot\sqrt{5-x}=2\sqrt{2}$; (6) $\sqrt{2-\sqrt{2+x}}=x$;

(7) $\sqrt{4+2x-x^2}=x-2$; (8) $x-1=\sqrt{2x^2-3x-5}$;

(9) $\sqrt{2x+5}+\sqrt{x-1}=8$; (10) $\sqrt{x+1}+\sqrt{2x+3}=1$;

(11) $\sqrt{22-x}-\sqrt{10-x}=2$; (12) $\sqrt{2x+1}+\sqrt{x-3}=4$;

(13) $\sqrt{x^2+6x+9}+\sqrt{x^2-4x+4}=5$;

(14) $\sqrt{x+8+2\sqrt{x+7}}+\sqrt{x+1-\sqrt{x+7}}=4$;

(15) $\dfrac{2x+3}{\sqrt{2x-1}}=\sqrt{6x+1}$; (16) $\dfrac{x}{x+1}-2\sqrt{\dfrac{x+1}{x}}=3$;

(17) $\dfrac{1}{x-\sqrt{x^2-x}}-\dfrac{1}{x+\sqrt{x^2-x}}=\sqrt{3}$;

(18) $\frac{4}{x+\sqrt{x^2+x}}-\frac{1}{x-\sqrt{x^2+1}}=\frac{3}{x}$;

(19) $\frac{1}{2x+\sqrt{x}}+\frac{1}{2x-\sqrt{x}}=\frac{x}{4x-1}$;

(20) $\sqrt{x+1}-\sqrt{9-x}=\sqrt{2x-12}$;

(21) $\sqrt{5+\sqrt[3]{x}}+\sqrt{5-\sqrt[3]{x}}=\sqrt[3]{x}$;

(22) $\sqrt{x+2}+\sqrt{x-2}=\sqrt{3x-2}$;

(23) $\sqrt{3x^2+1}+\sqrt{x^2+3}=\sqrt{6x^2+10}$;

(24) $\sqrt{x+1}+\sqrt{4x+13}=\sqrt{3x+12}$;

(25) $\sqrt{2x+5}+\sqrt{5x+6}=\sqrt{12x+25}$;

(26) $\sqrt{7x+1}-\sqrt{3x-18}=\sqrt{2x+7}$;

(27) $\frac{x^2}{\sqrt{5x+4}}+\sqrt{5x+4}=\frac{4}{3}x+2$;

(28) $\frac{\sqrt[7]{x-\sqrt{2}}}{2}-\frac{\sqrt[7]{x-\sqrt{2}}}{x^2}=\frac{x}{2}\sqrt[7]{\frac{x^3}{x+\sqrt{2}}}$;

(29) $\frac{\sqrt{x+2}}{x}+\frac{x}{\sqrt{x+2}}=\frac{4}{3}\sqrt{3}$;

(30) $\sqrt[6]{1.5}\cdot\sqrt[6]{x-1/x}-\sqrt{x-1/x}=0$.

2.227. Solve the following equations:

(1) $\sqrt[3]{x^4}-4\sqrt[3]{x^2}+4=0$; (2) $\sqrt{x\sqrt[5]{x}}-\sqrt[5]{x\sqrt{x}}-56=0$;

(3) $8.4\sqrt[12]{x^{-7}}-0.2\sqrt[4]{x^{-1}\sqrt[3]{x^2}}=\sqrt[12]{x^{11}}$;

(4) $\sqrt[3]{x}+2\sqrt[3]{x^2}-3=0$; (5) $3\sqrt[3]{x}-5\sqrt[3]{x^{-1}}=2x^{-1}$;

(6) $\frac{x\sqrt[5]{x}-1}{\sqrt[5]{x^3}-1}+\frac{\sqrt[5]{x^3}-1}{\sqrt[5]{x}-1}=16$;

(7) $5\sqrt[15]{x^{22}}+\sqrt[15]{x^{14}\sqrt{x}}-22\sqrt[15]{x^7}=0$;

(8) $2\sqrt[3]{x}+5\sqrt[6]{x}-18=0$;

(9) $\frac{\sqrt{x}+\sqrt[3]{x}}{\sqrt{x}-\sqrt[3]{x}}=3$; (10) $\frac{x-1}{\sqrt{x}+1}=4-\frac{1-\sqrt{x}}{2}$:

(11) $\frac{x-4}{\sqrt{x}+2}=x-8$; (12) $\frac{\sqrt[3]{x^4}-1}{\sqrt[3]{x^2}-1}-\frac{\sqrt[3]{x^2}-1}{\sqrt[3]{x}+1}=4$;

(13) $\sqrt[5]{x^4}-7\sqrt[5]{x^{-2}}+6x^{-1}=0$;

(14) $\sqrt{x^3+8}+\sqrt[4]{x^3+8}=6$;

(15) $\sqrt[5]{\frac{16x}{x-1}}+\sqrt[5]{\frac{x-1}{16x}}=2.5$;

(16) $\sqrt{\frac{3-x}{2+x}}+3\sqrt{\frac{2+x}{3-x}}=4$; (17) $\sqrt[3]{\frac{x+3}{5x+2}}+\sqrt[3]{\frac{5x+2}{x+3}}=\frac{13}{6}$;

(18) $\sqrt[7]{\frac{5-x}{x+3}}+\sqrt[7]{\frac{x+3}{5-x}}=2$;

(19) $\sqrt[3]{x+b}+\sqrt[3]{x+b+1}+\sqrt[3]{x+b+2}=0$;

(20) $2x^2+\sqrt{2x^2-4x+12}=4x+8$;

(21) $2x^2+3x-5\sqrt{2x^2+3x+9}+3=0$;

(22) $\sqrt{3x^2-2x+15}+\sqrt{3x^2-2x+8}=7$;

(23) $x^2+3x+4\sqrt{x^2+3x-6}=18$;

(24) $3x^2+15x+2\sqrt{x^2+5x+1}=2$;

(25) $x^2+2x+\sqrt{x^2+2x+8}-12=0$;

(26) $\sqrt{x^3+x^2-1}+\sqrt{x^3+x^2+2}=3$;

(27) $\sqrt{x^2+x+4}+\sqrt{x^2+x+1}=\sqrt{2x^2+2x+9}$;

(28) $\sqrt[4]{x+8}-\sqrt[4]{8-x}=2$:

(29) $\sqrt[4]{18+5x}+\sqrt[4]{64-5x}=4$.

2.228. Solve the following equations with the help of formulas $(a\pm b)^3=a^3\pm b^3\pm 3ab(a\pm b)$:

(1) $\sqrt[3]{24+\sqrt{x}}-\sqrt[3]{5+\sqrt{x}}=1$;

(2) $\sqrt[3]{x+45}-\sqrt[3]{x-16}=1$; (3) $\sqrt[3]{1+x}+\sqrt[3]{34-x}=5$;

(4) $\sqrt[3]{5x+7}-\sqrt[3]{5x-12}=1$; (5) $\sqrt[3]{x+34}-\sqrt[3]{x-3}=1$;

(6) $\sqrt[3]{x}+\sqrt[3]{x-16}=\sqrt[3]{x-8}$;

(7) $\sqrt[3]{x}+\sqrt[3]{2x-3}=\sqrt[3]{12(x-1)}$;

(8) $\sqrt[3]{x}+\sqrt[3]{3x-4}=\sqrt[3]{4(x+2)}$;

(9) $\sqrt[3]{x+1}+\sqrt[3]{2x-3}=\sqrt[3]{3x-2}$;

(10) $\sqrt[3]{x+1}+\sqrt[3]{x+2}+\sqrt[3]{x+3}=0$;

(11) $\sqrt[3]{x-1}+\sqrt[3]{x-2}=\sqrt[3]{2x-3}$;

(12) $\sqrt[3]{9-\sqrt{x+1}}+\sqrt[3]{7+\sqrt{x+1}}=4$;

(13) $\sqrt[3]{x+5}+\sqrt[3]{x+6}=\sqrt[3]{2x+11}$;

(14) $\sqrt[3]{(x+a)^2}+\sqrt[3]{(x-a)^2}+\sqrt[3]{x^2-a^2}=\sqrt[3]{a^2}$;

(15) $\sqrt[3]{(a+x)^2}-3\sqrt[3]{(a-x)^2}=3\sqrt[3]{(a^2-x^2)}$.

2.229. Solve the following equations with parameters:

(1) $\sqrt{a+x}+\sqrt{a-x}=\sqrt{2a}$; (2) $\sqrt{a-x}+\sqrt{x-b}=\sqrt{a-b}$;

(3) $\sqrt{x^2+3a^2}-\sqrt{x^2-3a^2}=x\sqrt{2}$;

(4) $\sqrt{x}-\sqrt{a}+\sqrt{x+a-b}=\sqrt{b}$;

(5) $\sqrt{x+4a-4n}+\sqrt{x}=2\sqrt{x-2n+a}$;

(6) $\frac{a+b\sqrt{x}}{a\sqrt{x}+b}=\frac{2ab}{a^2+b^2}$; (7) $\frac{a\sqrt{x}+b}{a-b\sqrt{x}}=\frac{a+b}{a-b}$, $a\neq 0$, $a\neq b$;

(8) $\frac{a\sqrt{x}+b}{a\sqrt{x}+b-2a}=\frac{b\sqrt{x}+a}{b\sqrt{x}-a}$, $a\neq 0$, $a\neq b$;

(9) $\sqrt[4]{a+x}+\sqrt[4]{a-x}=4$, $a>0$;

(10) $\sqrt{ax-2ab}+\sqrt{ax+b^2}=2a+b$;

(11) $\sqrt{4a+b-5x}+\sqrt{4b+a-5x}=3\sqrt{a+b-2x}$;

(12) $\sqrt[3]{\sqrt{a}+\sqrt{x}}=\sqrt{\sqrt[3]{a}+\sqrt[3]{x}}$;

(13) $\sqrt[7]{(ax-b)^3}+\frac{1}{\sqrt[7]{(bx-b)^3}}=\frac{65}{8}$;

(14) $\sqrt[4]{x^2+2b^2x+2b^4}-\sqrt{x+b^2}=b$;

(15) $\sqrt[3]{a+\sqrt{x}}+\sqrt[3]{a-\sqrt{x}}=\sqrt[3]{b}$;

(16) $\sqrt[n]{\frac{a-x}{b+x}}+\sqrt[n]{\frac{b+x}{a-x}}=2$; (17) $\frac{1-ax}{1+ax}\sqrt{\frac{1+bx}{1-bx}}=1$;

(18) $\sqrt[3]{(a+x)^2}+4\sqrt[3]{(a-x)^2}=5\sqrt[3]{a^2-x^2}$.

2.230. Solve graphically the following equations:

(1) $\sqrt{x-1}=x-1$; (2) $\sqrt{-x}=-x-2$;

(2) $\sqrt{-x}=2x^2-1$; (4) $\sqrt{2-x}=-4/x$;

(5) $\sqrt{x-2}=x-4$; (6) $\sqrt{-x+9}-\sqrt{-x+4}=1$;

(7) $\sqrt{2x-9}=\sqrt{6-x}$; (8) $\sqrt{3x}=6-x$; (9) $\sqrt{2x+1}=8-x$.

§ 42. Systems of Second-degree Equations and Systems Reducible to Them

2.231. Solve the given systems of equations:

(a) (1) $\begin{cases} x^2-y^2=32, \\ x-2y=2; \end{cases}$ (2) $\begin{cases} x^2+y^2=41, \\ y-x=1; \end{cases}$

(3) $\begin{cases} x^2+6xy+8y^2=91, \\ x+3y-10=0; \end{cases}$ (4) $\begin{cases} 2x^2+10xy+17y^2=218, \\ 2x+5y-20=0; \end{cases}$

(5) $\begin{cases} x^2+3xy-y^2=92, \\ x+3y=18; \end{cases}$ (6) $\begin{cases} 2x^2-2xy+x=-9, \\ 2y-3x=1; \end{cases}$

(b) (1) $\begin{cases} x^2+2xy-4y^2-5x+4=0, \\ 1+x=y; \end{cases}$

(2) $\begin{cases} xy=0.5, \\ x/y=2; \end{cases}$ (3) $\begin{cases} 1/(xy)=5, \\ x/y=20; \end{cases}$ (4) $\begin{cases} xy=99, \\ (x+y)/(x-y)=10; \end{cases}$

(5) $\begin{cases} (x+y)/y=2, \\ 1+xy/3=4; \end{cases}$ (6) $\begin{cases} xy-x/y=16/3, \\ xy-y/x=9/2; \end{cases}$

(c) (1) $\begin{cases} \dfrac{x+y}{y}=a, \\ 1+\dfrac{xy}{a+1}=a^2; \end{cases}$ (2) $\begin{cases} \dfrac{x}{a-b}-\dfrac{a-b}{y}=0, \\ x-y=0; \end{cases}$

(3) $\begin{cases} x-y=\dfrac{2a}{a^2-1}, \\ -\dfrac{1}{x}+\dfrac{1}{y}=\dfrac{2}{a}; \end{cases}$ (4) $\begin{cases} \dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1, \\ \dfrac{x}{a}+\dfrac{y}{b}=1; \end{cases}$

(5) $\begin{cases} x+y=a, \\ x^2+y^2=bxy; \end{cases}$ (6) $\begin{cases} x^2+xy=a, \\ y^2+xy=b; \end{cases}$ (7) $\begin{cases} x^2+xy+y^2=2a, \\ x^2-xy+y^2=2b. \end{cases}$

2.232. Solve the following systems of equations applying Viète's theorem:

(a) (1) $\begin{cases} x+y=5, \\ xy=4; \end{cases}$ (2) $\begin{cases} x+y=8, \\ xy=7; \end{cases}$ (3) $\begin{cases} x+y=12, \\ xy=-15; \end{cases}$

(4) $\begin{cases} x+y=-5, \\ xy=-36; \end{cases}$ (5) $\begin{cases} x-y=7, \\ xy=18; \end{cases}$ (6) $\begin{cases} x-y=2, \\ xy=15; \end{cases}$

(b) (1) $\begin{cases} x-y=16, \\ xy=-48; \end{cases}$ (2) $\begin{cases} x+2y=7, \\ xy=6; \end{cases}$ (3) $\begin{cases} 2x+y=7, \\ xy=5; \end{cases}$

(4) $\begin{cases} x-2y=1, \\ xy=6; \end{cases}$ (5) $\begin{cases} x+y=a, \\ xy=-2a^2; \end{cases}$ (6) $\begin{cases} \sqrt{x}+\sqrt{y}=5, \\ xy=36; \end{cases}$

(c) (1) $\begin{cases} x^2+y^2=13, \\ xy=6; \end{cases}$ (2) $\begin{cases} x^2+y^2=10, \\ xy=3; \end{cases}$ (3) $\begin{cases} x^2+y^2=41, \\ xy=20. \end{cases}$

2.233. Solve the systems of equations whose left-hand sides are homogeneous with respect to x and y:

(a) (1) $\begin{cases} x^2-5y^2=-1, \\ 3xy+7y^2=1; \end{cases}$ (2) $\begin{cases} 3y^2-2xy=160, \\ y^2-3xy-2x^2=8; \end{cases}$

(3) $\begin{cases} x^2-3xy+y^2=-1, \\ 3x^2-xy+3y^2=13; \end{cases}$ (4) $\begin{cases} 3x^2-4xy+2y^2=17, \\ x^2-y^2=-16; \end{cases}$

(b) (1) $\begin{cases} x^2-2xy-y^2=2, \\ xy+y^2=4; \end{cases}$ (2) $\begin{cases} x^2+xy+4y^2=6, \\ 3x^2+8y^2=14; \end{cases}$

(3) $\begin{cases} 5x^2-6xy+5y^2=29, \\ 7x^2-8xy+7y^2=43; \end{cases}$ (4) $\begin{cases} 3x^2+5xy-4y^2=38, \\ 5x^2-9xy-3y^2=15. \end{cases}$

2.234. Solve the following systems of equations by introducing an auxiliary unknown:

(a) (1) $\begin{cases} \dfrac{x}{y}+\dfrac{y}{x}=\dfrac{34}{15}, \\ x^2+y^2=34; \end{cases}$ (2) $\begin{cases} \dfrac{x}{y}+\dfrac{y}{x}=\dfrac{26}{5}, \\ x^2-y^2=24; \end{cases}$

(3) $\begin{cases} \dfrac{x}{y}-\dfrac{y}{x}=\dfrac{9}{20}, \\ x^2-y^2=9; \end{cases}$ (4) $\begin{cases} \dfrac{x}{y}-\dfrac{y}{x}=\dfrac{5}{6}, \\ x^2+y^2=13; \end{cases}$

(b) (1) $\begin{cases} \sqrt{\dfrac{x}{y}}+\sqrt{\dfrac{y}{x}}=\dfrac{5}{2}, \\ x+y=10; \end{cases}$

(2) $\begin{cases} \sqrt{\frac{x}{y}}+\sqrt{\frac{y}{x}}=\frac{41}{20}, \\ x+y=41; \end{cases}$

(3) $\begin{cases} \sqrt{\frac{3x-2y}{2x}}+\sqrt{\frac{2x}{3x-2y}}=2, \\ x^2-18=2y(4y-9); \end{cases}$

(c) (1) $\begin{cases} \sqrt{\frac{5x}{x-y}}-\sqrt{\frac{x-y}{5x}}=\frac{21}{10}, \\ xy+x+y=11; \end{cases}$

(2) $\begin{cases} \sqrt{\frac{y}{x}}-2\sqrt{\frac{x}{y}}=1, \\ \sqrt{5x+y}+\sqrt{5x-y}=4; \end{cases}$

(3) $\begin{cases} \sqrt{x+y}+\sqrt{2x+y+2}=7, \\ 3x+2y=23; \end{cases}$

(4) $\begin{cases} \sqrt{\frac{x+a}{y}}+\sqrt{\frac{y}{x+a}}=2, \\ x+y=xy+a. \end{cases}$

2.235. Solve the following systems:

(a) (1) $\begin{cases} x+y+x^2+y^2=18, \\ xy+x^2+y^2=19; \end{cases}$ (2) $\begin{cases} x+y+x^2+y^2=18, \\ xy+x^2+y^2=12; \end{cases}$

(3) $\begin{cases} x^3+y^3=72, \\ x^2-xy+y^2=12; \end{cases}$ (4) $\begin{cases} x^3-y^3=218, \\ x^2+xy+y^2=109; \end{cases}$

(b) (1) $\begin{cases} \sqrt{x^2+y^2}-\sqrt{x^2-y^2}=7, \\ x^4-y^4=144; \end{cases}$

(2) $\begin{cases} x^2+y^2-xy=61, \\ x+y-\sqrt{xy}=7; \end{cases}$

(3) $\begin{cases} x^2+y^2+xy=19, \\ \frac{1}{y}-\frac{1}{x}=\frac{1}{6}; \end{cases}$ (4) $\begin{cases} xy=2, \\ yz=6, \\ xz=3; \end{cases}$ (5) $\begin{cases} xy=6, \\ yz=12, \\ xz=8; \end{cases}$

(c) (1) $\begin{cases} \sqrt{\frac{x+y}{2}}+\sqrt{\frac{x-y}{3}}=14, \\ \sqrt{\frac{x+y}{8}}-\sqrt{\frac{x-y}{12}}=3; \end{cases}$

(2) $\begin{cases} x^2+xy+y^2=91, \\ x+\sqrt{xy}+y=13; \end{cases}$

(3) $\begin{cases} x^2+y^2+x-3y=2, \\ x^2+y^2-5y-y=2; \end{cases}$ (4) $\begin{cases} x+y=1, \\ y+z=2, \\ xz=6; \end{cases}$

(d) (1) $\begin{cases} x-y=2, \\ y+z=7, \\ x^2+z^2=41; \end{cases}$ (2) $\begin{cases} x^2-yz=3, \\ y^2-xz=5, \\ z^2-xy=-1; \end{cases}$

(3) $\begin{cases} y-x=3, \\ y-z=4, \\ x^2+y^2+z^2=30; \end{cases}$ (4) $\begin{cases} x^2+xy+y^2=7, \\ y^2+yz+z^2=3, \\ z^2+zx+x^2=1; \end{cases}$

(e) (1) $\begin{cases} x+y=-3, \\ x+z=-2, \\ xy+xz+yz=2; \end{cases}$ (2) $\begin{cases} xy+xz=-4, \\ yz+yx=-1, \\ zx+zy=-9; \end{cases}$

(3) $\begin{cases} xy+yz=5, \\ yz+xz=9, \\ xy+xz=8; \end{cases}$ (4) $\begin{cases} x+yz=2, \\ y+zx=2, \\ z+xy=2. \end{cases}$

§ 43. Functions and Graphs. Ways of Representing a Function. Domain and Range. Zeros of a Function

2.236.

(1) $f(x)=2x^2-3x+1$. Find: $f(0)$; $f(3)$; $f(-3)$;

(2) $f(x)=\sqrt{x^2-5x+4}$. Find: $f(0)$; $f(1)$; $f(m+1)$;

(3) $f(x)=\dfrac{|2-x|}{1+x}$. Find: $f(0)$; $f(3)$; $f(-3)$; $f(2)$; $f(-2)$;

(4) $f(x)=x^2+x+1/x+1/x^2$. Prove that $f(a)=f(1/a)$;

(5) $f(x)=\begin{cases} x+1 \text{ if } -\infty<x\leqslant 0, \\ 1 \text{ if } 0<x<+\infty. \end{cases}$ Find: $f(-1/2)$; $f(-1)$; $f(1)$; $f(0.36)$; $f(-3)$;

(6) $f(x)=\begin{cases} x^2-1 \text{ if } x\leqslant 2, \\ x-1 \text{ if } x>2. \end{cases}$ Find: $f(-1.5)$; $f(3.1)$; $f(4.5)$.

2.237. (1) Find $f(-1)$; $f(1)$; $f(0)$; $f\left(\frac{1}{3}\right)$; $f(3)$; $f(5)$ if the function is given by the table:

x	-4	-2	-1	0	1	2	3	4
y	17	5	2	1	2	5	10	17

(2) Plot the graph of this function.

(3) Represent this function analytically.

2.238. Given the functions:

(1) $f(x)=\begin{cases} x+2 & \text{if } -\infty<x\leqslant -2, \\ 4-x^2 & \text{if } -2<x<2, \\ x-2 & \text{if } 2\leqslant x<\infty; \end{cases}$

(2) $f(x)=\begin{cases} x+6 & \text{if } x<-2, \\ x^2-x-2 & \text{if } |x|\leqslant 2, \\ 0 & \text{if } x>2; \end{cases}$

(3) $f(x)=\begin{cases} 1.5x+3 & \text{if } x<0, \\ 3-2x & \text{if } x\geqslant 0; \end{cases}$ (4) $f(x)=\begin{cases} 2x & \text{if } x>0, \\ 1 & \text{if } x\leqslant 0. \end{cases}$

Find: $f(-3)$; $f(-2)$; $f(0)$; $f(2)$; $f(5)$; $f(8)$; $f(2.75)$.

2.239. Represent the given function by one formula using the modulus sign:

(1) $f(x)=\begin{cases} x & \text{if } x\geqslant 0, \\ -x & \text{if } x<0; \end{cases}$ (2) $f(x)=\begin{cases} 2x-3 & \text{if } x\geqslant 1.5, \\ 3-2x & \text{if } x<1.5; \end{cases}$

(3) $f(x)=\begin{cases} 1 & \text{if } x>0, \\ -1 & \text{if } x<0; \end{cases}$ (4) $f(x)=\begin{cases} x+1 & \text{if } x>0, \\ 1-x & \text{if } x<0. \end{cases}$

2.240. Given two sets: $x = \{4; 6; 9; 11\}$ and $y = \{1/4; 1/6; 1/9; 1/11\}$.

Show the correspondence between the sets x and y which is a function: (1) by arrows; (2) by pairs; (3) by a table.

2.241. The function f is given with the help of pairs:

(1) (1, 1); (2, 1/2), (5, 1/5); (2/7, 3, 5); (−0.01, −100);

(2) (1, 1); (2, 4); (3, 9); (4, 16);

(3) (−1/2, 0); (−1, −1); (0, 1); (1, 3); $\left(2\frac{1}{2}, 6\right)$;

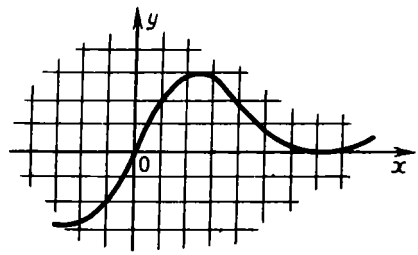

FIG. 1

(4) (−3, −4); (−2, −6); (−1, −12); (−1/2, −24); (−1/4, 48); (1, 12); (3, 4); (4, 3).

Find: $f(-1)$; $f(-1/2)$; $f(0)$; $f(2)$.

2.242. A function is given by the formula $y=1/x$, where x {1/9; 1/7; 1/5; 1/3; 1}. For each value of x find the corresponding value of y.

2.243. Suppose we are given: $X = \{-5; -1; 5; 6\}$, $Y = \{5; 29; 35\}$, $x \in X$, $y \in Y$.

Can a function with domain x and range y be represented by the formula: (1) $y = x^2 + 4$; (2) $y = 6 \mid x \mid - 1$?

2.244. A function is given by the graph (Fig. 1). Using this graph, find the roots of the function.

2.245. Find the roots of the given function:

(a) (1) $y = x - 2$; (2) $y = -2x^2 + 5$; (3) $y = x^2 - 2x + 1$; (4) $y = x^2 - 2 \mid x \mid - 3$;

(b) (1) $y = x^2 - 6x + 8$; (2) $y = x^2 - 4$; (3) $y = (x + 1)/(x - 1)$; (4) $y = \mid x - 2 \mid /(x + 3)$;

(c) (1) $y = \sqrt{4x+3}$; (2) $y = \sqrt{x^2+9}$; (3) $y = \sqrt{(x+8)/x}$; (4) $y = \sqrt{(x^2-2x+1)/(8x)}$.

§ 44. Coordinates of a Point

2.246. (a) Construct the line segment $[AB]$ if:
(1) A (5, 4), B (−3, −2); (2) A (−4, 2), B (5, −3);
(3) A (8, 2), B (−6, 3); (4) A (10, 13), B (−14, −3);
(5) A (−5, 7), B (3, −9); (6) A (−1.5, 2), B (4,−1/5);

(b) find the coordinates of the midpoint of the line segment $[AB]$ if:

(1) A (3, 0), B (7, 0); (2) A (0, 2), B (0, 2);
(3) A (−3, 2), B (5, −2); (4) A (0, −3), B (0, 5);
(5) A (−4.5, 5), B (3.5, 3.5); (6) A (−6.5, 4.5), B (−6.5, 4.5);
(7) $A\left(-3\frac{3}{4}, 5\frac{1}{2}\right)$, $B(-0.8, -1.4)$; (8) $A\left(8\frac{1}{2}, -5\frac{1}{2}\right)$, $B(0, 1)$.

2.247. (a) Given: points A (4, 6); B (−3, 2); C (8.5, 4.5); and D (−5, −4). Construct the points symmetric to the given ones with respect to: (1) the axis of abscissas; (2) the axis of ordinates; (3) the origin. Show the difference between the abscissas and ordinates of these points.

(b) The following points are given on the coordinate plane: A (1, 3); B (2, 5); C (3.5, −5); D (1, −3); E (−2, −5); F (3.5, 5); K (−1, 3); L (−3, 5.4); M (−1, 3); N (−1, −3); O (0, 0); $P\left(2.5, 4\frac{3}{4}\right)$; R (8, 5); $S\left(-2.5, 4\frac{3}{4}\right)$.

Determine which pairs of these points are symmetric with respect to: (1) the axis of abscissas; (2) the axis of ordinates; (3) the origin.

(c) On what line do the points whose abscissa and ordinate are equal both in length and sign lie? How is this line situated? What angle does it form with the coordinate axes?

(d) On what line do the points lie whose abscissa and ordinate are equal in absolute value but have opposite sign. What angles does it form with the coordinate axes?

2.248. Construct the triangles given the coordinates of their vertices:

(1) A (6, 12); B (9, 1); C (−6, 5). Find the coordinates of the midpoints of its sides.

(2) A (3, −4); B (−5, 12); C (0, 8). Find the coordinates of the vertices of the triangle which is symmetric to the given one with respect to the axis of abscissas.

(3) A (6, 0); B (2, −7); C (−4, 5). Find the coordinates of the vertices of the triangle which is symmetric to the given one with respect to the axis of ordinates.

(4) A (−7, 3); B (−3, −5); C (5, −1). Find the coordinates of the vertices of the triangle which is symmetric to the

given one with respect to the midpoint of the greater side.
(5) A (8, −2); B (3, 5); C (−4, −6). Find the coordinates of the point which is symmetric to the midpoint of the smaller side.
(6) A (−8, 2); B (−2, 2); C (−4, 6). Find the coordinates of the vertices of the triangle which is symmetric to the triangle ABC with respect to the origin.

2.249. (a) Construct the quadrilaterals given the coordinates of their vertices and find the coordinates of the midpoints of all the sides:
(1) A (0, 0); B (1, 3); C (8, 5); D (9, 1);
(2) A (−3, 8); B (10, 6); C (5, −5); D (−7, −4).

(b) Construct a square given the coordinates of its vertices: A (4, 1); B (3, 5); C (−1, 4); D (0, 0). Find the coordinates of the midpoints of its sides and compute the area of the square.

(c) Find the coordinates of the vertices C and D of the square $ABCD$ if A (3, 1); B (3, −4).

2.250. (a) The coordinates of the two opposite vertices of the square $ABCD$ are A (1, −6), and C (1, 2). Find the coordinates of two other vertices.

(b) The length of the side of the square $ABCD$ is equal to 6, and the coordinates of one of its vertices are (−2, 3). Find the coordinates of other vertices if the side AD is parallel to the axis of ordinates, and the origin lies inside the square.

§ 45. Direct Proportional Relationship

2.251. Construct the graphs of the following functions: (1) $y = x$; (2) $y = -4x$; (3) $y = (1/2)\,x$; (4) $y = x/4$; (5) $y = -0.3x$; (6) $y = 6x$; (7) $y = 3x$; (8) $y = -2.5x$.

2.252. Construct the graphs of the functions: $y = 2x$ and $y = -2x$. How are these graphs arranged with respect to the coordinate axes?

2.253. A straight line passes through the origin and has a slope equal to 4. (a) Write the equation of this line and draw it; (b) does the point M (2, 5) lie on the given straight line?

2.254. Construct the graphs of the following equations:
(a) (1) $3x - 2y = 0$; (2) $4x - 7y = 0$; (3) $y = |x|$;

(4) $|y| = x$; (5) $y = -\frac{x}{|x|}$; (6) $y = |x+1| - x$;

(b) (1) $y = \frac{\sqrt{x^2}}{x}$; (2) $y = |x+1| - x + 1$;

(3) $y = \sqrt{(x-1)^2} - x - 1$; (4) $y = x - 2 + \sqrt{(x+2)^2}$;

(5) $y = \sqrt{(3x-5)^2} + 3x - 5$.

2.255. Construct the graph of the function $y = 3x$.

(a) Find the value of y if x is equal to: (1) 1; (2) 1.5; (3) 2; (4) 2.5; (5) 3; (6) 10.5;

(b) if $x \in [1, 3]$, then to what set do the corresponding values of y belong?

(c) for what value of x is the value of y equal to: (1) 3; (2) -3; (3) 0; (4) 12?

(d) for what values of x does the variable y take on: (1) positive values? (2) negative values? (3) zero value?

2.256. Construct the graph of the function $y = -0.5x$.

(a) Find $f(0)$; $f(1)$; $f(-0.5)$; $f(-2)$; $f(-4)$; $f(8)$;

(b) for what value of x does the value of y equal: (1) -150; (2) 20.02; (3) 0; (4) -4; (5) -450;

(c) how do the values of y change with an increase in x?

2.257. Solve graphically the following equations:

(1) $5x + 1 = 6$; (2) $5x + 3 = 8$; (3) $3x - 2 = 4x + 2$; (4) $3x - 2 = x + 4$; (5) $5x - 1 = x + 11$; (6) $|x| = 1$; (7) $8 = 2|x|$; (8) $2|x| - 3 = |x| + 6$; (9) $5 - 2|x| = 7 + |x|$; (10) $1 - 3|x| = -7 + |x|$; (11) $|x - 3| = 5$; (12) $|2x - 3| = 3 - x$; (13) $|5 - x| = |x + 4|$; (14) $|x/2 - 5/2| = x - 1$; (15) $|x - 3| = |4x - 1|$.

§ 46. Linear Functions

2.258. (a) Graph the following functions:

(1) $y = x - 1$; (2) $y = 2x + 3$; (3) $y = 0.5x - 2$; (4) $y = -\frac{1}{2}x + 7$; (5) $y = -\frac{2}{3}x + 5$.

(b) Graph the given equations and find k:

(1) $0x + 3y = 6$; (2) $0.8x - 2y + 3 = 0$; (3) $4y - 2.8x + 12 = 0$; (4) $(3x - 2y)/5 = 0.4$.

2.259. What is the mutual position of the graphs:

(1) $y = 2x - 7$ and $y = 2x + 3$;
(2) $y = 0.5x + 1$ and $y = -0.5x - 2$;
(3) $y = 4x + 2$ and $y = -0.25x + 1$;
(4) $y = 0.5x + 3$ and $y = -2x + 1$;
(5) $y = 2x + 3$ and $y = -2x - 3$;
(6) $y = 2.5x - 8$ and $y = 2.5x + 4$;
(7) $y = -0.75x + 4$ and $y = -3/4 + 5$;
(8) $y = \frac{2}{3}x + 2$ and $y = -\frac{2}{3}x + 2$.

2.260. Determine whether the given points belong to the graph of the equation $y = \frac{8+3x}{2}$: (2, 7); (2, 8); (3, 9); (2/3, 5); (1/3, 4.5); (−2, 1); (−1/2, 5); (−2, −7); (−5, −3.5).

2.261. Graph the following functions:

(1) $y = x - 4$; (2) $y = 0.5x + 2$; (3) $y = -2x + 3$;
(4) $y = -3x + 2$; (5) $y = -0.7x$; (6) $y = 0.9x + 1.1$.

For what values $xy = 0$; $y > 0$; $y < 0$?

How does the variable y change (increase or decrease) with an increase in x?

On what set does the closed interval $[-2, 4]$ map?

2.262. Construct on one drawing the graphs of the following functions: $y = 3x$; $y = 3x + 6$; and $y = 3x - 6$.

(1) How are these straight lines situated?
(2) What is the length of the y-intercept cut off by each line as measured from the origin?
(3) For what values of x do all the functions take on: (a) negative values; (b) positive values?
(4) For what values of x do the functions take on values of opposite sign?

2.263. Find the value of k if:

(1) the graph of the function $y = kx + 2$ passes through the point P (−7, −12);
(2) the graph of the function $y = kx + 6$ passes through the point M (3, 0.5);
(3) the graph of the function $y = kx - 2$ passes through the point A (1, 2);

(4) the graph of the function $y = kx + 3$ passes through the point B $(-2, 6)$.

2.264. Find the value of b if:
(1) the graph of the function $y = 3x + b$ passes through the point A $(1, 4)$;
(2) the graph of the function $y = -1.5x + b$ passes through the point B $(-2, 9)$;
(3) the graph of the function $y = -2.6x + b$ passes through the point C $(1, 1/2)$;
(4) the graph of the function $y = 6x + b$ passes through the point D $(0, -5/4)$.

2.265. Graph the function $y = -0.8x + 2$. Determine how y changes with an increase in x. Find the length of the x- and y-intercepts cut by the graph as measured from the origin. For what values of x $y < 0$; $y = 0$; $y > 0$?

2.266. Construct the graph of the equation $y - x = 5$. Where on the coordinate plane do we find the set of the points for which the difference between the ordinate and abscissa is less than 5; equal to 5; greater than 5?

2.267. Find the slope of the straight line if:

(1) $2x - 3y = 8$: (2) $\frac{1}{2}x - 2y = -3$; (3) $5x - 4y = 11$;

(4) $3\frac{1}{4}x - \frac{1}{4}y = 10$; (5) $5\frac{1}{3}x + \frac{2}{3}y = \frac{10}{27}$;

(6) $2.5x - 0.5y = 4.5$; (7) $5x + 6y = 30$.

2.268. Find the area of the triangle bounded by the straight line: (1) $y = 2x - 3$; (2) $y = 0.5x + 4$; (3) $y = -2x + 0.8$; (4) $y = -2.5x - 5$; (5) $y = -1.2x + 6$ and x- and y-intercepts.

2.269. Construct the graph of the equation $x + y = 10$. Where on the coordinate plane can one find the points the sum of whose abscissa and ordinate is less than 10?

2.270. Graph the following functions:
(1) $y = |x| - 2$; (2) $y = |x - 2|$; (3) $y = |x - 2| + 4$; (4) $y = |3x - 1| - 2$; (5) $y = |5 - x| + 1$; (6) $y = |x - 1| + |x + 2|$; (7) $y = |3x + 1| - 5$; (8) $y = ||x| - 6|$; (9) $y = |x| + |x - 1|$; (10) $y = 2|x + 2| - |x + 1|$; (11) $|x| + |y| = 1$; (12) $|x| - |y| = 1$; (13) $x + |x| = y + |y|$; (14) $|2x - 3y| = 5$; (15) $|3x + 5y| = 1$.

2.271. Solve graphically the following equations:

(a) (1) $5x + 1 = 6$; (2) $5x + 3 = 18$; (3) $|x/2 - 5/2| = x - 1$; (4) $|2x - 3| + |x - 3| - |4x - 1| = 0$;

(b) (1) $|x - 3| = 5$; (2) $|5 - x| = |x + 4|$; (3) $|2x - 3| = 3 - x$; (4) $|3x - 2| = 2 - 3x$.

2.272. Solve graphically the following systems of equations:

(a) (1) $\begin{cases} x+y=7, \\ x-y=3; \end{cases}$ (2) $\begin{cases} y=-3x-9, \\ y=\frac{1}{2}x-2; \end{cases}$ (3) $\begin{cases} y=2x-14, \\ y=\frac{2}{3}x-2; \end{cases}$

(4) $\begin{cases} y+5=4x, \\ 2x+y=7; \end{cases}$ (5) $\begin{cases} 2x+y=6, \\ 3x+5y=2; \end{cases}$ (6) $\begin{cases} y=2x-2, \\ 2x+y=10; \end{cases}$

(7) $\begin{cases} 3x-5y=15, \\ 2x+3y=10; \end{cases}$ (8) $\begin{cases} (x-y)/4=1, \\ 3(x-y)/4=3; \end{cases}$ (9) $\begin{cases} y=2x-14, \\ y=1.5-12; \end{cases}$

(b) (1) $\begin{cases} |x+y|=5, \\ |x-y|=1; \end{cases}$ (2) $\begin{cases} |2x+3y|=5, \\ |2x-3y|=1; \end{cases}$

(3) $\begin{cases} |x-1|+|y-5|=1, \\ |x-1|-y=-5; \end{cases}$ (4) $\begin{cases} |x+1|+|y-1|=5, \\ |x+1|=4y-4; \end{cases}$

(5) $\begin{cases} 2x-3|y|=1, \\ |x|+2y=4; \end{cases}$ (6) $\begin{cases} 3|x|+|y+2|=6, \\ 2x-|y|=2. \end{cases}$

2.273. Investigate the given systems of equations and show the result on the graph:

(a) (1) $\begin{cases} x+y=5, \\ x-y=1; \end{cases}$ (2) $\begin{cases} x-y=5, \\ x-2y=2; \end{cases}$ (3) $\begin{cases} x+2y=3, \\ 2x+4y=6; \end{cases}$

(4) $\begin{cases} x+y=3, \\ 2x+2y=7; \end{cases}$ (5) $\begin{cases} 3x+y=9, \\ x+2y=-2; \end{cases}$ (6) $\begin{cases} x-y=3, \\ 2x-2y=6; \end{cases}$

(b) (1) $\begin{cases} x=4-y, \\ 2y=8-2x; \end{cases}$ (2) $\begin{cases} x+y=5, \\ 2y-5=-2x; \end{cases}$

(3) $\begin{cases} x-y=4, \\ 2x-2y=5; \end{cases}$ (4) $\begin{cases} x=2-y, \\ y=2-x; \end{cases}$

(5) $\begin{cases} x=7-y, \\ 2y+x=11; \end{cases}$ (6) $\begin{cases} 5x-4y=20, \\ 4x+5y=10; \end{cases}$

(c) (1) $\begin{cases} 2x+y=3, \\ 4x+2y=5; \end{cases}$ (2) $\begin{cases} 2x-y=11, \\ 3x+2y=20; \end{cases}$

(3) $\begin{cases} 3x-5y=15, \\ 2x+3y=10; \end{cases}$ (4) $\begin{cases} -3x+7y=21, \\ x+2y=-6; \end{cases}$

(5) $\begin{cases} 2y-x=4, \\ 5y+3x=15.5; \end{cases}$ (6) $\begin{cases} 5x-8y=19, \\ 3x+5y=23; \end{cases}$

(d) (1) $\begin{cases} x-4y=-3, \\ 2x+y=3; \end{cases}$ (2) $\begin{cases} 0x+2y=5, \\ 2x+y=4; \end{cases}$

(3) $\begin{cases} x-2y=1, \\ -3x+6y=2; \end{cases}$ (4) $\begin{cases} 4x-2y=1, \\ 2x-y=0.5. \end{cases}$

2.274. (a) Find the length of the perimeter of the triangle bounded by the straight line $8x - 15y = 120$ and the x- and y-intercepts.

(b) Find the length of the perimeter of the triangle bounded by the straight line $5x + 12y = 60$ and the x- and y-intercepts.

2.275. (a) The straight lines intersect. Find k.

(1) $y = (1/2)\,x - 3$; $y = -3x + 4$ and $y = kx$;
(2) $y = (1/3)\,x + 4$; $y = 1.5x + 11$ and $y = kx$.

(b) The lines are parallel. Find k.

(1) $3x - 2y = 8$; $6x - 4y = 10$; $y = kx$;
(2) $x - 5y = -1$; $3x - 15y = -5$; $y = kx$;
(3) $x + 2y = 7$; $3x + 6y = 11$; $y = kx$;
(4) $3x - y = 5$; $1.5x - 0.5y = 3$; $y = kx$.

§ 47. Inverse Proportional Relation

2.276. Graph the following functions and equations: (1) $y = 12/x$; (2) $y = -12/x$; (3) $y = 8/x$; (4) $y = -4/x$; (5) $y = 1/x$; (6) $xy = -5$; (7) $xy = -9$; (8) $3xy = -12$.

2.277. Given the graph of the equation $xy = 10$, find the abscissa of the point whose ordinate is equal to: 2; 5; 10; 20; -2; -5; -10.

2.278. Construct the graph of the equation $xy = -18$. Using the graph, find the abscissa of the point whose ordinate is equal to: -6; -2; -1; 0; 3; 4; 12; 24.

2.279. Construct the graph of the function $y = 24/x$ and find the set of values of x for which: (1) $y < 0$; (2) $y > 0$; (3) $y = 4$.

2.280. Construct the graph of the function $y = \frac{6}{x-2}$. Indicate:
(1) the domain of definition of the function;
(2) the range of change of the function;
(3) how are the straight lines $x = 2$ and $y = 0$ called for the constructed graph?
(4) for what values of x does the function: increase, decrease, take on negative values, take on positive values?

2.281. Sketch the graphs of the following functions:

(a) (1) $y = \frac{6}{x-1}$; (2) $y = \frac{8}{x+3}$; (3) $y = \frac{3}{x-5}$;
(4) $y = 1/x + 6$; (5) $y = -1/x + 1$; (6) $y = -2/x - 3$;

(b) (1) $y = \frac{x+5}{x+1}$; (2) $y = \frac{2x+3}{x+1}$; (3) $y = \frac{2x-1}{x+1}$;
(4) $y = \frac{2x-5}{x-2}$; (5) $y = \frac{x+3}{x-1}$; (6) $y = \frac{3x-12}{x-3}$;

(c) (1) $y = \frac{2}{|x|}$; (2) $y = \frac{4}{|x-1|}$; (3) $y = \frac{8}{|x-2|}$;
(4) $y = \frac{6}{2-|x|}$; (5) $y = \frac{8}{|x|-1}$; (6) $y = |1/x + 2|$;

(d) (1) $y = \left|\frac{2x+3}{x+1}\right|$; (2) $y = \left|\frac{x+1}{x-1}\right|$; (3) $y = \left|\frac{x+4}{x-2}\right|$;
(4) $y = \left|\frac{6}{x-5} + 4\right|$; (5) $y = \left|\frac{2}{x-1} - \frac{1}{2}\right|$.

2.282. Sketch the graphs of the following functions: (1) $y = 1 + 6/x$; (2) $y = 2 - 8/x$.
Indicate for what values of x the values of y are negative; positive; equal to zero.

2.283. Graph the following functions:

(1) $y = \frac{x-2}{|x^2-5x+6|}$; (2) $y = \frac{|x^2+5x+6|}{x+3}$;
(3) $y = \frac{x-2}{|x^2-6x+8|}$; (4) $y = \frac{|2x^2-5x+2|}{2x-1}$.

2.284. Solve graphically the following equations:

(1) $2x - 5 = -2/x$; (2) $3x - 10 = -3/x$;
(3) $4x - 5 = -1/x$; (4) $2x - 1 = 6/x$;
(5) $x - 1 = 12/x$; (6) $2x + 1 = 3/x$.

2.285. Solve graphically the following systems of equations:

(1) $\begin{cases} xy = -12, \\ x + 2y = 2; \end{cases}$ (2) $\begin{cases} x - y = 11, \\ xy = -12; \end{cases}$ (3) $\begin{cases} xy = -24, \\ x + y = 10; \end{cases}$
(4) $\begin{cases} xy = 10, \\ 2x + y = 6; \end{cases}$ (5) $\begin{cases} x - y = 0, \\ xy = 16; \end{cases}$ (6) $\begin{cases} x - y = 6, \\ xy = 12; \end{cases}$
(7) $\begin{cases} y = x - 3, \\ y = 1/x; \end{cases}$ (8) $\begin{cases} xy = -6, \\ x - y = 0; \end{cases}$ (9) $\begin{cases} xy = 1, \\ x + y = 0. \end{cases}$

§ 48. The Graphs of Quadratic Functions

2.286. Given functions: $y = x^2$; $y = 2x^2$; $y = 0.5x^2$; $y = -x^2$; $y = -2x^2$; $y = -0.5x^2$.
(1) Construct the graphs of these functions on the same coordinate axes.
(2) How does each of these functions vary as the argument x changes from $-\infty$ to 0 and from 0 to $+\infty$?
(3) For what values of x does each function have the least value? Write this value.

2.287. Construct the graphs of the following functions:

(1) $y = 3x^2$; (2) $y = -2.5x^2$; (3) $y = x^2/5$; (4) $y = x\,|\,x\,|$;
(5) $y = 0.1x^2$; (6) $y = 0.2|x| \cdot x$; (7) $y = x\sqrt{4x^2}$;
(8) $y = 3x\sqrt{16x^2}$.

2.288. Given functions: $y = x^2$; $y = x^2 + 2$; $y = x^2 - 2$; $y = -x^2$; $y = -x^2 + 2$; $y = +x^2 - 2$.
(1) Construct the graphs of these functions on the same coordinate axes; (2) what is the difference between the positions relative to the coordinate axes of the graphs of the functions: $y = \pm x^2 + 2$ and $y = \pm x^2 - 2$ and the graph of the function $y = x^2$? (3) for what values of x does each of the functions have the least value? (4) find the domain of definition of each of the functions; (5) how does each of the functions

vary if: (a) the argument x increases from $-\infty$ to 0; (b) the argument x increases from 0 to $+\infty$?

2.289. Investigate the functions $y = -x^2$; $y = -x^2 + 5$; $y = -x^2 - 5$ using the questions of the preceding problem.

2.290. How should the parabola $y = x^2$ be translated so that the new equations of the parabola are: (a) $y = x^2 + 1$; (b) $y = x^2 - 1$; (c) $y = -x^2 + 5$; (d) $y = -x^2 - 1$.

2.291. Given functions: $y = x^2$; $y = (x - 2)^2$; $y = (x + 2)^2$. (1) Find the domain of definition of each function; (2) draw the graphs of these functions using the same coordinate axes; (3) for what x does each of the functions: (a) decrease; (b) vanish; (c) increase?

2.292. Graph the functions: (a) $y = -(x + 5)^2$; (b) $y = 2(x - 1)^2$; (c) $y = -3(x - 6)^2$; (d) $y = -\frac{1}{2}(x + 2)^2$.

2.293. Given a function: $y = x^2 - 4x + 4$. (1) Find the domain and the range of the function; (2) write this function in the form of the square of a binomial; (3) for what values of x does the function: (a) decrease; (b) increase; (c) have the least value; (d) vanish?

2.294. Using the questions of the preceding problem, investigate the following functions: (a) $y = x^2 + 2x + 1$; (b) $y = x^2 - x + 1/4$; (c) $y = -x^2 + 6x - 9$; (d) $y = -x^2 - 8x - 16$.

2.295. What is the difference between the graphs of the functions: $y = 2x^2 - 4x + 2$ and $y = 2x^2$?

2.296. Graph the functions: (1) $y = -\frac{1}{3}x^2 - 2x - 3$; (2) $y = 4 - 4x + x^2$; (3) $y = -3x^2 - 6x - 3$; (4) $y = -\frac{1}{2}x^2 - 2x - 2$.

2.297. Given functions: $y = (x - 2)^2$ and $y = (x - 2)^2 + 3$. (1) Using the same coordinate axes, construct the graphs of these functions; (2) what is the difference between the graphs of the functions? (3) for what x does the function $y = (x - 2)^2 + 3$: (a) vanish; (b) decrease; (c) increase; (d) have the least value; (e) positive; (f) negative?

2.298. Given a quadratic function: $y = x^2 - 6x + 4$. (1) Separate a perfect square; (2) compute the values of x for which the function vanishes; (3) find the coordinates of the vertex of the parabola; (4) determine the values of x for

which the function: (a) decreases; (b) increases; (c) has the least value.

2.299. Using the questions of the preceding problem, investigate the functions: (a) $y = x^2 + 4x + 5$; (b) $y = x^2 - 2x + 4$; (c) $y = 9x^2 - 6x + 1$.

2.300. Construct the graphs of the functions: $y = 4 - x^2$ and $y = |4 - x^2|$, indicate the values of x for which $y > 0$, $y < 0$, $y = 0$.

2.301. Construct the graphs of the functions: $y = x^2 - 9$ and $y = |x^2 - 9|$. Indicate the values of x for which $y > 0$, $y < 0$, $y = 0$.

2.302. Graph the following functions: (1) $y = \frac{|x^3|}{x} + 1$; (2) $y = \frac{-2|x^3|}{x} + 3$; (3) $y = |x^2 - 3| - 1$; (4) $y = |4 - x^2| + 3$.

2.303. The quadratic function is a perfect square. Find c and construct the following graphs: (1) $y = x^2 - 6x + c$; (2) $y = 4x^2 + 12x + c$; (3) $y = 9x^2 - 6x + c$; (4) $y = \frac{1}{4}x^2 - x + c$.

2.304. How many points does the graph of the given function have in common with the axis of abscissas: (1) $y = x^2 + 16x + 64$; (2) $y = 4x^2 - 4x + 1$; (3) $y = x^2 - x + 1$; (4) $y = x^2 - 6x - 7$; (5) $y = 2x^2 - 5x + 2$; (6) $y = 5x^2 - 2x + 5$.

2.305. Without sketching the graph of the given function, find the coordinates of the points of intersection of this graph and the coordinate axes: (1) $y = x^2 - 5x + 4$; (2) $y = 8x - x^2$; (3) $y = -x^2 + 2x + 3$; (4) $y = (x - 4)(x + 3)$; (5) $y = 2x^2 + 7x$; (6) $y = (2x + 3)(x - 2)$.

2.306. Given a function: $y = -2x^2 + 4x - 6$. (1) Prove that the function has no real roots. (2) Find the set of values of x on which the function either increases or decreases. (3) Prove that for any real values of x $y < 0$. (4) For what values of x does the function have the greatest value? Write this value. (5) Construct the graph of the function. (6) Check that the coordinates of the vertex of the parabola $y = -2x^2 + 4x - 6$ are determined by the formulas: $x = -\frac{b}{2a}$; $y = \frac{4ac - b^2}{4a}$. (7) Find the set of values of the function $E(f)$.

2.307. Using the questions of the preceding problem, investigate the following functions and construct their graphs: (1) $y = x^2 + 2x - 35$; (2) $y = x^2 + 4x$; (3) $y = x^2 + 2x - 8$; (4) $y = 2x^2 - 4x + 6$; (5) $y = x^2 + 16x - 13$; (6) $y = \frac{1}{2}x^2 - 4x + 6$.

2.308. Find the coefficients of the trinomial $ax^2 + bx + c$ if it vanishes for $x = 6$ and its least value is equal to -8 for $x = 4$.

2.309. Find the coefficients of the trinomial $ax^2 + bx + c$ if its greatest value is equal to 25 for $x = \frac{1}{2}$, and for $x = 0$ it takes on the value equal to 24.

2.310. The vertex of the parabola $y = ax^2 + bx + c$ has the coordinates $x = 6$, $y = -12$. The parabola opens upward and intersects the x-axis at the point (8, 0). Find a, b, and c.

2.311. The parabola $y = ax^2 + bx + c$ opens upward and intersects the y-axis at the point (0, 15). The coordinates of the vertex of the parabola is $(-2, 7)$. Find a, b and c.

2.312. Graph the following functions:

(1) $y = x^2 - |x| + 1$; (2) $y = x^2 - 2|x| - 3$;
(3) $y = |9 - x^2|$; (4) $y = |-x^2 + 36|$; (5) $y = -|x^2 - 4|$; (6) $y = |x^2 + 5x + 6|$; (7) $y = x^2 + 5|x| + 6$;
(8) $y = |-x^2 + 4|x| - 5|$; (9) $y = \frac{|x-1|}{x-1} \cdot (x^2 - 4)$;
(10) $y = \frac{\sqrt{(x-1)^2}}{x-1} - x^2$; (11) $y = x|x| - x^2$;
(12) $y = |x - 2|x - 3$; (13) $y = |-4x^2 - 6x + 1|$;
(14) $y = |x^2 - |x| + 4|$;
(15) $y = |x - 1| \cdot |x - 2| - 4$.

§ 49. Graphical Solution of Quadratic Equations and Systems of Second-degree Equations

2.313. Solve graphically the following equations:

(1) $x^2 - 2x - 3 = 0$; (2) $x^2 - 5x + 6 = 0$;
(3) $x^2 - 7x + 12 = 0$; (4) $x^2 + x - 6 - 0$;
(5) $x^2 + 3x + 2 = 0$; (6) $x^2 - 3x - 4 = 0$;
(7) $3x^2 - 10x + 3 = 0$; (8) $2x^2 - 3x - 4 = 0$;
(9) $2x^2 - 5x + 2 = 0$.

2.314. Solve graphically the following systems:

(1) $\begin{cases} x^2 - y = 4, \\ y + 6 = 1; \end{cases}$ (2) $\begin{cases} x^2 - y = 8, \\ x - y = 2; \end{cases}$ (3) $\begin{cases} x^2 + y^2 = 25, \\ xy = 12; \end{cases}$

(4) $\begin{cases} x^2 + y = 4, \\ x + y^2 = 10; \end{cases}$ (5) $\begin{cases} y^2 = 2x, \\ x^2 + y^2 = \sqrt{3}; \end{cases}$ (6) $\begin{cases} 2x + y = 11, \\ x^2 + y^2 = 25; \end{cases}$

(7) $\begin{cases} x^2 + y^2 = 25, \\ x - 7y = 25; \end{cases}$ (8) $\begin{cases} x^2 - y = 13, \\ x^2 + y^2 = 25; \end{cases}$

(9) $\begin{cases} x^2 + y = 2x - 1, \\ x + y = -1; \end{cases}$ (10) $\begin{cases} x^2 + y^2 = 4, \\ xy = 1; \end{cases}$

(11) $\begin{cases} x^2 + y^2 = 36, \\ y - 0.5x = 0; \end{cases}$ (12) $\begin{cases} y = x^2 - 3x - 10, \\ x = 0.25y^2 - 5; \end{cases}$

(13) $\begin{cases} x + y = 8, \\ x^2 + y^2 = 81; \end{cases}$ (14) $\begin{cases} x + 3y = -1, \\ x^2 + 2xy + y = 3; \end{cases}$

(15) $\begin{cases} y - 9 + x^2 = 0, \\ x^2 = 25 - y^2; \end{cases}$ (16) $\begin{cases} xy = 8, \\ y = x^2. \end{cases}$

2.315. Prove that each of the following equations is an equation of a circle:

(1) $3x^2 + 3y^2 = 27$; (2) $x^2/2 + y^2/2 = 8$;

(3) $x^2/4 + y^2/4 = 9$; (4) $0.1x^2 = 10 - 0.1y^2$;

(5) $x^2 - 6x + 4y + y^2 = 12$; (6) $x^2 - 2x + y^2 - 4y = 4$.

2.316. On the coordinate plane find the point of intersection of:

(1) the ellipse $x^2/8 + y^2/2 = 1$ and the circle $x^2 + y^2 = 5$;

(2) the ellipse $x^2/3 + y^2/2 = 1$ and the circle $2x^2 + 2y^2 = 8$;

(3) the ellipse $x^2/4 + y^2/9 = 1$ and the circle $x^2 + y^2 = 100$;

(4) the ellipse $x^2/81 + y^2/9 = 1$ and the circle $x^2 + y^2 = 36$.

2.317. Compute the area of the square inscribed into the ellipse:

(1) $x^2 + 9y^2 = 81$;

(2) $x^2/36 + y^2/9 = 1$;

(3) $x^2/16 + y^2/25 = 1$;

(4) $x^2/49 + y^2/16 = 1$.

§ 50. General Properties of Inequalities. Solving First-degree Inequalities and Systems of First-degree Inequalities

2.318. Subtract the second inequality from the first one:

(1) $6 > 3$; $2 < 5$; (2) $-8 < -5$; $-2 > -7$;
(3) $2x > b^2$; $a^2 < 9 - x$; (4) $2a^2 > b$; $x^2 < 9$.

2.319. Multiply both sides of the inequality by the indicated multiplier:

(1) $5 > -2$ by 5; (2) $-7 < -5$ by -2; (3) $a^2 > b$ by $-b$;
(4) $a - 1 < b$ by $-m$; (5) $x - 1 > 7 - x$ by $a^2 + 4$;
(6) $x + 2 < c$ by $-x^2 - 4$.

2.320. Divide both sides of the inequality by the indicated divisor:

(1) $-6 < 9$ by 3; (2) $-15 > -35$ by -5; (3) $a^3 < a^2$ by $(-a)$; (4) $(a - b)^3 < (a - b)^2$ by $(a - b)$; (5) $(a - b)^4 > 9$ by $(-b^2 - 1)$.

2.321. Multiply the given inequalities termwise:
(1) $5 > 3$; $7 > 2$; (2) $2 > -5$; $-3 > -7$; (3) $-3 < 5$; $-5 < 2$; (4) $-13 < -7$; $-9 > -13$; (5) $-5 < 3$; $-6 < 2$.

2.322. Determine whether the given inequalities are equivalent:

(1) $3x \leqslant 0$ and $5x + \frac{11}{x} \leqslant \frac{11}{x}$;

(2) $x - 4 > 0$ and $x - 4 + \frac{1}{64 - x^2} > \frac{1}{64 - x^2}$;

(3) $2x + 3 > 0$ and $2x + 3 + (x - 8) > x - 8$;
(4) $7x > 3$ and $7x^2 > 3x$; (5) $2x > 3$ and $6 \cdot 2x > 6 \cdot 3$;
(6) $2x > 3$ and $-6 \cdot 2x < -6 \cdot 3$;

(7) $2x > 3$ and $\frac{2x}{9 - x^2} > \frac{3}{9 - x^2}$;

(8) $2x > 3$ and $2x^3 > 3x^2$; (9) $2x > 3$ and $\frac{2x}{\sqrt{x}} > \frac{3}{\sqrt{x}}$;

(10) $x^2 > x - 1$ and $x^2 + 1 > x$.

2.323. Solve the following inequalities:

(1) $x + 4 > 2 - 3x$; (2) $3 + 5x < 7x + 4$;

(3) $4(x - 1) > 2 + 7x$; (4) $3(x - 2) < 4x - 9$;

(5) $\frac{3x}{2} - \frac{3}{5} < 4x - 3$; (6) $\frac{x}{5} - 3\frac{1}{3} > 1\frac{3}{4} - \frac{5}{2}x$;

(7) $\frac{1}{2}(3x - 1) + \frac{x}{5} < 7x + 10.1$;

(8) $4 - \frac{3}{2}x > \frac{13}{8} - \frac{1}{6}(4x - 3)$;

(9) $\frac{37 - 2x}{3} + 9 < \frac{3x - 8}{4} - x$; (10) $3 - \frac{3x}{2} > \frac{5}{8} - \frac{4x - 3}{6}$;

(11) $\frac{7 - 6x}{2} + 12 < \frac{8x + 1}{3} - 10x$;

(12) $8 + \frac{3x - 4}{5} > \frac{x + 1}{6} - \frac{5x + 3}{8}$;

(13) $\frac{x - \frac{1}{3}(4x - 9)}{6} - \frac{3x}{2} + \frac{5(2 - 3x)}{4} - 1 < x + 27\frac{2}{9}$;

(14) $\frac{1}{2} + |(x^2 - 1)x + 2| \cdot \frac{1}{4} > 3\frac{1}{2}x + \frac{x^3 - 1}{4}$.

2.324. (a) For what values of x are the following expressions positive:

(1) $2x - 16$; (2) $\frac{x - 2}{3} + \frac{x}{2}$; (3) $\frac{5x - 8}{8} + \frac{3 - 2x}{4}$;

(4) $\frac{x + 1}{2} - 2x + 2\frac{1}{2}$; (5) $10 - \frac{3x - 1}{2} - \frac{6x + 3}{11}$;

(6) $\frac{x - 4}{5} + \frac{2x - 41}{9} - 9$?

(b) For what values of x are the following expressions negative:

(1) $3x + 15$; (2) $\frac{x - 5}{2} - \frac{2x - 1}{3} + 2$;

(3) $\frac{x - 5}{2} + \frac{x - 1}{8} - \frac{x - 3}{4} - \frac{x - 4}{3}$;

(4) $\frac{4x - 3}{2} - \frac{5 - 2x}{3} - \frac{3x - 4}{3} - 5$;

(5) $2x + \frac{3x - 1}{2} - \frac{5x - 2}{3} - 2$;

(6) $\frac{4x}{3} \cdot 17 + \frac{3x-17}{4} - \frac{x+3}{2}$;

(7) $9 - \left(\frac{x-2}{4} + \frac{2}{3}\right)$?

2.325. Solve graphically the following inequalities:

(1) $2x - 1 > 3$; (2) $3 - 2x > 7$; (3) $x + 1 > 2x - 1$;
(4) $3 > 1 - x$.

2.326. Solve the following systems:

(a) (1) $\begin{cases} 5(x+1)+6(x+2) > 9(x+3), \\ 7x-3(2x+3) > 2(x-18); \end{cases}$

(2) $\begin{cases} (x-3)(x-4) < (x+1)(x+2), \\ x(x+1)+x(x+2) < (2x-1)(x+3); \end{cases}$

(3) $\begin{cases} 6x(x-1)-(2x-1)3x < x, \\ 0.5x-5.7 < 0.2x-0.7; \end{cases}$

(4) $\begin{cases} x < 39+3.6(5x-1)-2(2x-1.8), \\ x(2.5x-1)-3x > x(15x+2)-11; \end{cases}$

(5) $\begin{cases} 3x+7 > 7x-9, \\ x-3 > -3x+1; \end{cases}$ (6) $\begin{cases} 6x-7 > 5x-1, \\ 3x+6 > 8x-4; \end{cases}$

(7) $\begin{cases} 2x-1 < 0, \\ \frac{2}{x+1} > 9; \end{cases}$ (8) $\begin{cases} 2 - \frac{5+x}{7} < 1 - \frac{9-x}{14}, \\ 12 - \frac{1}{3}\left(47 - \frac{60}{x}\right) > 3; \end{cases}$

(9) $\begin{cases} \frac{7-x}{2} - 3 < \frac{3+x}{5} - 4, \\ \frac{5}{3}x + 5(4-x) < 2(4-x); \end{cases}$

(10) $\begin{cases} x+2 < \frac{2x-8}{6} - \frac{18-4x}{3}, \\ 9 - \left(\frac{x-2}{4} + \frac{2}{3}\right) > x. \end{cases}$

(b) Solve the following inequalities:

(1) $\frac{5-2a}{8+5a} > 0$; (2) $\frac{x-1}{x+3} > 2$; (3) $\frac{5x-1}{x+6} < 1$;

(4) $\frac{3x+7}{2-6x} > 0$; (5) $\frac{3x-8}{2x+4} < 0$; (6) $\frac{3-5x}{x+1} > 4$.

2.327. Find the integral solutions of the following systems:

(1) $\begin{cases} 17x-5(x+0.6)<3x, \\ 2(3.5-x)+5(2x-2.4)>x-26; \end{cases}$

(2) $\begin{cases} 2x-\frac{x-1}{8}>x, \\ x-1<3-\frac{x+1}{2}; \end{cases}$

(3) $\begin{cases} \frac{x-1}{2}-\frac{x-4}{3}>2x-1, \\ 2x-\frac{x-5}{3}>x-3; \end{cases}$

(4) $\begin{cases} 1-\frac{3x-88}{7}>5x, \\ 4x+5-\frac{1}{6}\left(25x+29\frac{1}{2}\right)<0; \end{cases}$

(5) $\begin{cases} \frac{2x-11}{4}+\frac{19-2x}{2}<2x, \\ \frac{2x+15}{9}>\frac{1}{5}(x-1)+\frac{x}{3}; \end{cases}$

(6) $\begin{cases} \frac{x-1}{2}-\frac{2x+3}{3}+\frac{x}{6}<2-\frac{x+5}{2}, \\ 1-\frac{x+5}{8}+\frac{4-x}{3}<3x-\frac{x+1}{4}; \end{cases}$

(7) $\begin{cases} 9+\frac{4x-11}{7}<\frac{x-3}{5}, \\ (2+x)^2+8x^2<(3x-1)^2-12; \end{cases}$

(8) $\begin{cases} 8(3x-2)-7x-5(2-3x)<13x, \\ 20<\frac{2}{3}(7x-10)-\frac{1}{2}; \end{cases}$

(9) $\begin{cases} 2.6x-1\frac{1}{3}>\frac{1}{3}x+2, \\ 0.4x+2\frac{1}{3}<\frac{2}{3}x-1.2, \\ 5x+17>9x-63; \end{cases}$

(10) $\begin{cases} 2x+3<3x-4, \\ x+8>3x+1, \\ 3x-2>x+2, \\ 5x-1<4x+6. \end{cases}$

2.328. Find the middle of the interval satisfying the system of inequalities:

$$\begin{cases} \dfrac{3x-1}{5} - \dfrac{13-x}{2} > \dfrac{7x}{3} - \dfrac{11(x+3)}{6}, \\ \dfrac{2x+7}{3} < \dfrac{3x+5}{7} + 8 + \dfrac{10-3x}{5}. \end{cases}$$

2.329. Find the greatest natural value of x which satisfies the system of inequalities:

$$\begin{cases} 5x - \dfrac{3-2x}{2} > \dfrac{7x-5}{2} + x, \\ \dfrac{7x-2}{3} - 2x < 5 - \dfrac{x-2}{4}. \end{cases}$$

2.330. Find the least integral value of x which satisfies the system of inequalities:

$$\begin{cases} \dfrac{3x-2}{4} + 2\dfrac{1}{2} > \dfrac{2x-1}{3} - \dfrac{3x+2}{6}, \\ \dfrac{2x-5}{3} - \dfrac{3x-1}{2} < \dfrac{3-x}{5} - \dfrac{2x-1}{4}. \end{cases}$$

2.331. Find the greatest negative integral value of x which satisfies the system:

$$\begin{cases} x/8 - x/4 + x/2 > x + 5, \\ (1/8)(x+2) < -(1/7)(x-2). \end{cases}$$

2.332. Solve the following inequalities:

(a) (1) $|x-3| > 5$; (2) $|x+2| < 8$; (3) $|x-2| < 5$;
(4) $|x-1| + |2-x| > 3+x$;
(5) find the integral solutions of the inequality $\left|\dfrac{2}{x-13}\right| > \dfrac{8}{9}$;
(6) $|x+3| > |2x-1|$;
(7) $|x+5| > |3x+4|$; (8) $|x-2| - |x| > 0$;
(9) $|2x+5| - |3x-7| < 0$; (10) $|x+3| + |x-1| + |x-3| < 10$;
(b) (1) $|x-1|-1| < 2$; (2) $|x+1| - |x-1| > 3$;

(3) $|x+2|+|x|+|x-2|<4$;

(4) $|4x-1|-|2x-3|+|x-2|>1$;

(5) $\sqrt{(x-1)^2}+\sqrt{1+2x+x^2}-\frac{|x-3|}{x-3}<5$;

(6) $|x-1|+|2-x|>3-x$; (7) $\left|\frac{3x+1}{x-3}\right|<3$;

(8) $\left|\frac{4-x}{3}-1\right|>7$; (9) $\left|\frac{4-2x}{1+3x}\right|>0$.

§ 51. Proving Inequalities

2.333. Prove that in every triangle the length of the semiperimeter is greater than the length of each of its sides.

2.334. Prove that if a, b, c are the lengths of the three sides of a triangle, then $a^2+b^2+c^2<2(ab+ac+bc)$.

2.335. Prove that the arithmetic mean of two unequal positive numbers is greater than their geometric mean.

2.336. Prove the following inequalities:

(1) $a^2+b^2>2|ab|$;

(2) $3(x^2/y^2+y^2/x^2)-8(x/y+y/x)+10\geqslant 0$;

(3) $a^4+b^4+c^4+d^4\geqslant 4abcd$ if $a>0$, $b>0$, $c>0$, $d>0$;

(4) $x^2+2xy+3y^2+2x+6y+4\geqslant 1$;

(5) $\frac{a}{b}+\frac{b}{c}+\frac{c}{a}>3$ if a, b, and c have the same sign;

(6) $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\geqslant\frac{9}{a+b+c}$ if $a>0$, $b>0$, $c>0$;

(7) $\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}\geqslant\frac{3}{2}$ if $a+b>0$, $a+c>0$, $b+c>0$;

(8) $\frac{a+b}{c}+\frac{b+c}{a}+\frac{a+c}{b}>6$;

(9) $(a+b+c)\cdot\left(\frac{1}{a+b}+\frac{1}{b+c}+\frac{1}{a+c}\right)\geqslant\frac{9}{2}$ if $a+b>0$, $b+c>0$, $a+c>0$;

(10) $\frac{1}{a^2-4a+4}>\frac{2}{a^3-8}$ $(a\neq 2)$;

(11) $\frac{a+b+c+d}{4}\geqslant\sqrt[4]{abcd}$ $(a>0,\ b>0,\ c>0,\ d>0)$.

2.337. Prove the following inequalities:

(a) (1) $x^2 + y^2 \geqslant 1/2$ if $2x + y = 1$;

(2) $a^4 + b^4 \geqslant 1/8$ if $a + b = 1$;

(3) $a^2 + b^2 + c^2 \geqslant 1/3$ if $a + b + c = 1$;

(b) (1) $a^2 + b^2 + c^2 \geqslant (ab + ac + bc) \cdot 2$;

(2) $(a + 1)(b + 1)(a + c)(b + c) > 16abc$ if $a > 1, b > 1, c > 1$;

(3) $(a + b)(b + c)(a + c) \geqslant 8abc$ if $a > 0, b > 0, c > 0$;

(4) $a^4 + b^4 + c^4 \geqslant abc(a + b + c)$ if $a > 0, b > 0$ $c > 0$;

(c) (1) $\sqrt{4a+1} + \sqrt{4b+1} + \sqrt{4c+1} < 5$ if $a + b + c = 1$;

(2) $\sqrt{a^2+b^2+c^2} \leqslant |a| + |b| + |c|$;

(3) $\sqrt{a^2+b^2} > \sqrt[3]{a^3+b^3}$;

(d) (1) $\dfrac{a^2+a+2}{\sqrt{a^2+a+1}} \geqslant 2$;

(2) $x^2 + 2y^2 + 2xy + 6y + 10 > 0$;

(3) $mn(m + n) \leqslant m^3 + n^3$ if $m \geqslant 0, n \geqslant 0$;

(4) $a/b^2 + b/a^2 > 1/a + 1/b$ if $a > 0, b > 0$.

2.338. Prove that of all rectangles having the perimeter P the square has the greatest area.

2.339. Prove the following inequalities:

(a) $k + 1/k \geqslant 2$ $(k > 0)$; (b) $a/b + b/a \geqslant 2$ $(a > 0, b > 0)$;

(c) $(2a)/(1 + a^2) \leqslant 1$; (d) $(a + b)(1/a + 1/b) \geqslant 4$ $(a > 0, b > 0)$.

§ 52. Investigating Linear Equations and Systems of Linear Equations

2.340. Determine for what values of a the following equations have positive solutions:

(1) $5(x-3) = 3(3x-2a)$; (2) $3(x+1) = 4 + ax$;

(3) $3(2-x)=4(a-2x)$; (4) $4-a=\frac{2}{x-1}$; (5) $\frac{a}{4}=\frac{3}{x}$;

(6) $\frac{3}{x+1}=8-a$; (7) $\frac{5}{3+x}=\frac{a}{x}$; (8) $\frac{7}{1+x}=\frac{a}{x}$.

2.341. Determine for what values of a the following equations have negative solutions:

(1) $7-a=\frac{2}{x-1}$; (2) $\frac{3}{4x-a}=\frac{2}{ax-5}$;

(3) $\frac{5}{2x-k}=\frac{3}{4-kx}$; (4) $\frac{1}{x+1}=1-k$;

(5) $k-2=\frac{3x+1}{x+1}$; (6) $\frac{k(x+2)-3(k-1)}{x+1}=1$;

(7) $\frac{4x-1}{x-1}=k+3$; (8) $\frac{5}{3x-k}=\frac{3}{kx-4}$.

2.342. Determine for what values of the parameters the following equations have: (a) a positive solution; (b) a negative solution; (c) a zero solution; (d) infinitely many solutions; (e) have no solution:

(1) $ax-a=3x-5$; (2) $2mx+3=2m-x$;

(3) $a^2x+3=9x+a$; (4) $bx-b=3bx$;

(5) $\frac{a}{3a+x}=\frac{1}{6+x}$; (6) $a^3x+8-2a^2x=a^3$;

(7) $\frac{a}{ax+1}=\frac{6}{x+2}$; (8) $\frac{2a+5}{3}=\frac{5ax+1}{4}$;

(9) $\frac{2a}{3ax-2}=\frac{4}{bx-3}$.

2.343. Determine for what values of the parameter:

(1) the equation $3(x+1)=4+ax$ has a solution exceeding -1;

(2) the equation $\frac{2-t}{x}=\frac{5+t}{3+x}$ has a solution greater than -1;

(3) the equation $2+2.5x=1.5(x-2m)-\frac{m-5}{2}\cdot x$ has a solution lesser than 3;

(4) the equation $\frac{a-4x}{7-ax}=\frac{3}{4}$ has a solution greater than -1.

2.344. (1) Determine for what integral k the equation $\frac{k+1}{2}+k(1+x)+4.5x=5\cdot\frac{k-1+x}{2}$ has a solution lesser than -3;

(2) determine for what integral t the equation $\frac{5+tx}{2x-t}=2$ has a solution exceeding -3;

(3) determine for what natural values of a the equation $\frac{2x(a+1)}{a}=3\cdot(1+x)+\frac{7}{a}$ has a solution lesser than -5.

2.345. Determine for what values of the parameters the given systems have a negative solution if $x<0$, $y<0$:

(1) $\begin{cases}mx+my=8,\\5x+3y=4;\end{cases}$ (2) $\begin{cases}4x-3y=6,\\5x-ay=8;\end{cases}$

(3) $\begin{cases}(t+1)x+ty=t+1,\\4x-y=2;\end{cases}$ (4) $\begin{cases}2x+y=-12,\\4x-5y-a=0;\end{cases}$

(5) $\begin{cases}3x-6y=1,\\5x-ky=2;\end{cases}$ (6) $\begin{cases}2x+3y=8,\\mx+y=n;\end{cases}$

(7) $\begin{cases}mx+(n-1)y=2,\\3x+10y=-1.\end{cases}$

2.346. Determine for what values of the parameters the given systems have a positive solution if $x>0$, $y>0$:

(1) $\begin{cases}2x+5y=7,\\7x-ay=9;\end{cases}$ (2) $\begin{cases}x+y=a,\\3x+2y=10;\end{cases}$ (3) $\begin{cases}3x+7y=m,\\2x+5y=20;\end{cases}$

(4) $\begin{cases}3x-ay-2=0,\\5x-6y-1=0;\end{cases}$ (5) $\begin{cases}2x+5y=7,\\ax-y=a;\end{cases}$

(6) $\begin{cases}6x+(t+1)y=t,\\3x+12y=4;\end{cases}$ (7) $\begin{cases}4x+3y=12,\\2x+ay=a;\end{cases}$

(8) $\begin{cases}3x+ay=10,\\x-ay=7;\end{cases}$ (9) $\begin{cases}ax-5y=9,\\ax+3y=15.\end{cases}$

2.347. (1) Determine for what values of a the system $\begin{cases}(a-1)x+y=-1,\\ 2x-3y=2\end{cases}$ has the solutions $x>0$, $y>1$;

(2) determine for what values of m the system $\begin{cases}3x+y=-2,\\ x-(2-m)y=3\end{cases}$ has the solutions $x>2$, $y>0$;

(3) determine for what values of k the system $\begin{cases}(k+1)x+ky=k+1,\\ 4x-y=2\end{cases}$ has the solutions $x<2/3$, $y<2/3$.

2.348. For what values of a does the system $\begin{cases}ax-4y=a+1,\\ 2x+2ay=-1\end{cases}$ have the solutions $x\in P^+$, $y\in P^-$?

2.349. For what values of m does the system $\begin{cases}(m+1)x-my=4,\\ 3x-5y=m\end{cases}$ have such solutions (x, y) for which $x-y<2$?

2.350. For what values of p does the system $\begin{cases}px+3y=-p,\\ 3x+py=8\end{cases}$ have the solutions $(x, y)\subset P^+\cup 0$?

2.351. For what ratio $a:b$ can one of the variables of the system $\begin{cases}ax-12y=-5,\\ bx-4y=10\end{cases}$ vanish? Which one, namely?

2.352. Determine for what values of the parameters the following systems of equations have either infinitely many solutions or have no solution at all:

(1) $\begin{cases}2x-(m-3n)y=3n,\\ nx+y=6;\end{cases}$ (2) $\begin{cases}x+2y=a,\\ 3x-a+y=6;\end{cases}$

(3) $\begin{cases}x+k^2y=3-2y,\\ 4x+(15k-1)y=5;\end{cases}$ (4) $\begin{cases}16x+my=4,\\ mx+9y=3;\end{cases}$

(5) $\begin{cases} px+my=4, \\ 15x+9y=m; \end{cases}$ (6) $\begin{cases} 6x+y=10, \\ (n+1)x+(1/n)y=n^2+m; \end{cases}$

(7) $\begin{cases} (3+m)x+4y=5-3m, \\ 2x+(5+m)y=8; \end{cases}$ (8) $\begin{cases} (k+1)-3y-4=0, \\ 2x-ky-3=0. \end{cases}$

§ 53. Second-degree Inequalities, Irrational Inequalities and Inequalities Reducible to Them. Systems of Nonlinear Inequalities

2.353. Solve the following inequalities:

(a) (1) $x^2-14x+45>0$; (2) $5x^2-7x+2<0$;
(3) $3x^2-5x-2>0$; (4) $x^2+2x>6x-15$;
(5) $3x^2-7x-6<0$; (6) $3x^2-2x+5>0$;

(b) (1) $\frac{(x-1)(x-2)}{x-3}>0$; (2) $\frac{(x-3)(x-5)}{x-2}>0$;
(3) $\frac{x-1}{x^2+4x+12}<0$; (4) $\frac{x^2-6x+18}{x-4}>0$;
(5) $\frac{1}{2-x}+\frac{5}{2+x}<1$; (6) $\frac{1}{x+2}<\frac{3}{x-3}$;

(c) (1) $\frac{15}{4+3x-x^2}>1$; (2) $\frac{x^2+2x-3}{x^2-2x+8}>0$;
(3) $\frac{x^2+5x+4}{x^2-5x-6}<0$; (4) $\frac{x^2-2x-3}{x^2-4x-12}<0$;
(5) $\frac{x^4-2x^2-8}{x^2+2x+1}<0$; (6) $\frac{x^2-3x+2}{x^2+3x+2}\geqslant 1$;

(d) (1) $\frac{1}{(x-1)(x-2)}<\frac{1}{x(x+1)}$; (2) $2-\frac{x-3}{x-2}>\frac{x-2}{x-1}$;
(3) $3-\frac{2x-17}{x+2}\geqslant\frac{x-5}{x+2}$; (4) $\frac{1}{3x-2-x^2}-\frac{3}{7x-4-3x^2}>0$;
(5) $\frac{x^2-10x+3}{x^2-10x+25}>0$; (6) $\frac{4-x}{x-5}>\frac{1}{1-x}$;
(7) $\frac{x^3+3x^2-x-3}{x^2+3x-1}<0$;

(e) (1) $x^6-9x^3+8>0$;
(2) $x^8-6x^7+9x^6-x^2+6x-9<0$;
(3) $x^4+x^3-x-1>0$; (4) $\frac{x^3-x^2+x-1}{x+8}\leqslant 0$;

(f) (1) $|x^2-5x|<6$; (2) $(x-3)^2-|x-3|-2<0$;
(3) $\left|\frac{x^2-5x+4}{x^2-4}\right|\leqslant 1$; (4) $\frac{|x-3|}{x^2-5x+6}\geqslant 2$;
(5) $x^2-4|x|+3>0$.

2.354. Determine the values of m for which each given inequality is satisfied for any x:

(1) $x^2+2x+m>0$; (2) $x^2-5x+m>0$; (3) $2x^2+7x+3m>0$;
(4) $x^2+2(m+1)x+9m-5>0$; (5) $x^2+6x+(5m-1)(m-1)>0$;
(6) $mx^2+12x-5<0$; (7) $x^2+(m+2)x+8m+1>0$; (8) $(m+3)x^2-5x-4<0$.

2.355. For what values of m will the roots of the following equations be real and distinct:

(a) (1) $x^2+2(m-4)x+m^2+6m=0$;
(2) $mx^2-(1-2m)x+m=0$;
(3) $(m-1)x^2-2(m+1)x+m-2=0$;
(4) $x^2+2(m-1)x+3m^2+5=0$;

(b) (1) $(m-2)x^2-(3m+6)x+6m=0$;
(2) $(m+2)x^2+(2m+3)x+m-1=0$;
(3) $(m+1)x^2-(2m+3)x+m-1=0$;
(4) $2mx^2-2x-3m-2=0$;

(c) (1) $(x-1)(x-3)+m(x-2)(x-4)=0$;
(2) $(m^2-1)x^2+2(m-1)x+3/2=0$;
(3) $(5m+1)x^2+(7m+3)x+3m=0$;
(4) $(3+m)x^2-3(6-m)x+5-18m=0$.

2.356. Solve the following inequalities:

(1) $\sqrt{(1-x)(2+x)}<1$; (2) $\sqrt{x^2+3x+4}>-2$;
(3) $\sqrt{6+x-x^2}<6$; (4) $(x-3)\sqrt{x^2+4}\leqslant x^2-9$;
(5) $\sqrt{9x-20}<x$; (6) $\sqrt{3x-x^2}<4-x$;
(7) $\sqrt{x^2-4x}>x-3$; (8) $\sqrt{x^2-x-12}<x$;
(9) $\sqrt{x+3}<\sqrt{x-1}+\sqrt{x-2}$;

(10) $\sqrt{17-15x-2x^2}/(x+3)>0$;

(11) $4/\sqrt{2-x}-\sqrt{2-x}<2$;

(12) $\sqrt{x+3}+\sqrt{x-2}-\sqrt{2x+4}>0$;

(13) $\sqrt{1/x^2-3/4}<1/x-1/2$;

(14) $\sqrt{x^2-3x-10}<8-x$; (15) $\sqrt{x+1}-\sqrt{3x}>2x-1$;

(16) $\sqrt{\sqrt{x}+5}-\sqrt{x}<2$; (17) $(x-2)\sqrt{x^2+1}>x^2+2$;

(18) $\frac{6x}{x-2}-\sqrt{\frac{12x}{x-2}}-2\sqrt[4]{\frac{12x}{x-2}}>0$;

(19) $\frac{x-7}{\sqrt{4x^2-19x+12}}<0$;

(20) $\frac{2x}{\sqrt{2x+9}}<\sqrt{1+2x}-1$;

(21) $\frac{1-\sqrt{1-4x^2}}{x}<3$; (22) $\frac{1-\sqrt{8x-3}}{4x}\geqslant 1$;

(23) $\frac{x^2+2}{\sqrt{x^2+1}}\geqslant 2$; (24) $\sqrt{\frac{x+1}{x-1}}+3\sqrt{\frac{x-1}{x+1}}<4$;

(25) $\frac{x-2\sqrt{x+3}+1}{x^2-5x+6}>0$; (26) $\frac{\sqrt{1-2x+x^2}+x}{x}>0$;

(27) $\frac{|x+2|-|x|}{\sqrt{4-x^2}}>0$.

2.357. (1) Find the least positive value of x for which the inequality $\frac{x+4}{x^2+2}>\frac{7}{x+5}$ is satisfied.

(2) Find an integral positive value of x for which the inequality $\frac{5x-1}{x-1}>2x+2$ is satisfied.

(3) Find the least integral value of x for which the inequality $\frac{2x-9}{(x-2)(x+3)}+\frac{1}{x-2}>0$ is satisfied.

(4) How many integral solutions has the inequality $(\sqrt{1-x})^2\geqslant 3x^2-7x-8$?

2.358. Find the left- and right-hand bounds of the interval in which the given inequality is satisfied:

(1) $\frac{2-x}{x+3}\leqslant\frac{\sqrt{x}}{2}$; (2) $4x^2-4x-2(\sqrt{x-1})^2\leqslant 23$;

(3) $\sqrt{\frac{1-x}{x+4}}\leqslant\frac{\sqrt{x+1}}{2}$.

2.359. Find the middle of the interval in which the given inequality is satisfied:

(1) $x^2+2(\sqrt{x})^2-3\leqslant 0$; (2) $\frac{x+0.5}{x^2+1}<\frac{1}{x}$;

(3) $\frac{2x-1}{x^2-x+1}<\frac{2}{x+1}$.

2.360. Find the greatest value of x for which the given inequalities are satisfied:

(1) $\frac{x^2-2x+14}{x+8}\geqslant\frac{(\sqrt{5x-3})^2}{5}$;

(2) $\frac{x^2+x-45}{x-6}\geqslant\frac{3x+1}{2}$;

(3) $\frac{x^2-(\sqrt{15-x})^2-30}{x-6}\leqslant\frac{3x+1}{2}$;

(4) $\frac{(\sqrt{-x})^2}{4}+\frac{1}{x}\geqslant 0$; (5) $x^2-4(\sqrt{x})^2\leqslant 5$.

2.361. Find the least value of x for which the following inequalities are satisfied:

(1) $\frac{2x^2-(\sqrt{2x-13})^2-91}{x-7}\leqslant 3x+2$; (2) $x\geqslant\frac{4x-15}{x-4}$;

(3) $x^2+2x+6+(\sqrt{x+1})^2\geqslant 54$;

(4) $x^2-2x+3\ (\sqrt{x-1})^2\geqslant 27$.

2.362. Solve the following systems of inequalities:

(a) (1) $\begin{cases}\sqrt{4x-7}<x,\\ \sqrt{x+5}+\sqrt{5-x}>4;\end{cases}$

(2) $\begin{cases}\sqrt{x^2-9x+20}\leqslant\sqrt{x-1},\\ \sqrt{x-1}\leqslant\sqrt{x^2-13};\end{cases}$

(3) $\begin{cases}\frac{1}{x}<1,\\ \frac{3x-1}{x-2}>2,\\ |3-2x|\leqslant 3;\end{cases}$ (4) $\begin{cases}|\sqrt{3x+2}-3|>-1,\\ \sqrt{4x^2-4x+1}\geqslant 1;\end{cases}$

(b) (1) $\begin{cases} 2a-1 \geqslant 0, \\ 1-\sqrt{2a-1} \geqslant 0; \end{cases}$ (2) $\begin{cases} \sqrt{3-x} > \sqrt{x-2}, \\ \sqrt{3x-5} > \sqrt{x-4}; \end{cases}$

(3) $\begin{cases} \sqrt{3x-1} - \sqrt{5} > 0, \\ \sqrt{4-x} < \frac{1}{2}; \end{cases}$ (4) $\begin{cases} x+1 \geqslant \sqrt[3]{x^2+91}, \\ \sqrt{x-3} \leqslant \sqrt[4]{x-1}. \end{cases}$

2.363. Find the integral solutions of the given systems of inequalities:

(1) $\begin{cases} \sqrt{4-\sqrt{1-x}-\sqrt{2-x}} > 0, \\ x^7+x^6-1 > 0; \end{cases}$ (2) $\begin{cases} 9-2x-x^2 < 1, \\ 2x-3 < 1; \end{cases}$

(3) $\begin{cases} \sqrt{5-x} > 0, \\ x^2-2x-3 > 0; \end{cases}$ (4) $\begin{cases} \frac{5x-7}{x-5} < 4, \\ \frac{3x}{x^2-25} - \frac{x}{5-x} < 0. \end{cases}$

§ 54. Inequalities and Systems of Inequalities in Two Variables

2.364. Solve the following inequalities:

(a) (1) $x + 3y + 1 < 0$; (2) $y > 3x - 0.5$; (3) $y + 8 \leqslant 2x$; (4) $5x + 2y - 4 < 0$; (5) $2x - 2y \geqslant -1$;

(b) (1) $|x| + |y| \leqslant 1$; (2) $|x| + |y| \geqslant 4$; (3) $|y| \geqslant x$; (4) $y \leqslant |x|$; (5) $y \leqslant x^2 + 1$;

(c) (1) $y \geqslant 0.5x$; (2) $y \geqslant 6/x$; (3) $y \leqslant -0.5x$; (4) $x^2 + y^2 > 0$; (5) $x^2 + y^2 \leqslant 0$.

2.365. Solve the following systems of inequalities:

(a) (1) $\begin{cases} y \geqslant 2x+6, \\ y \leqslant -2x+6; \end{cases}$ (2) $\begin{cases} y-3x-6 \leqslant 0, \\ 2y-6x+3 \geqslant 0; \end{cases}$

(3) $\begin{cases} x+y \leqslant 2, \\ y-x \leqslant 2, \\ y \geqslant 0; \end{cases}$

(4) $\begin{cases} x^2+y^2 \leqslant 0.64, \\ y \geqslant 0; \end{cases}$ (5) $\begin{cases} x^2+y^2 \leqslant 16, \\ y \geqslant 4-x; \end{cases}$

(b) (1) $\begin{cases} y \leqslant -2x^2, \\ y+5 \geqslant 0; \end{cases}$ (2) $\begin{cases} x^2+y^2 \geqslant 4, \\ x^2+y^2 \leqslant 16; \end{cases}$

(3) $\begin{cases} y \geqslant x^2, \\ y-4 \leqslant 0; \end{cases}$ (4) $\begin{cases} x^2+y^2 \leqslant 9, \\ y > x^2; \end{cases}$ (5) $\begin{cases} y > x, \\ y < 2x, \\ y < 4; \end{cases}$

(c) (1) $\begin{cases} x+y < 6, \\ 2y \geqslant 7-2x, \\ 0 \leqslant x; \end{cases}$ (2) $\begin{cases} y-x < 2, \\ y+x > 4, \\ x-4 < 0; \end{cases}$

(3) $\begin{cases} y \geqslant 0, \\ x-y \geqslant 0; \end{cases}$ (4) $\begin{cases} x-y \geqslant 3, \\ x+y \leqslant 5, \\ y \geqslant 1; \end{cases}$ (5) $\begin{cases} x+y+2 \geqslant 0, \\ x-y \leqslant 0. \end{cases}$

2.366. Show on the coordinate plane the sets of points whose coordinates satisfy the following inequalities and systems:

(a) (1) $xy \geqslant 0$; (2) $(x-2)(y-3) > 3$; (3) $(x+y) \times (1/x + 1/y) > 0$; (4) $(x-y)(1/x - 1/y) > 0$;

(b) (1) $|x| + |y| \geqslant 1$; (2) $|x| - |y| \geqslant 1$;

(3) $x + |y| \leqslant y + |y|$;

(4) $\begin{cases} 3x-4y+12 > 0, \\ x+y-2 < 0; \end{cases}$

(c) (1) $\begin{cases} x+3 \geqslant x^2+2x, \\ x+y \leqslant 3; \end{cases}$ (2) $x^2-3xy+2y^2 < 0$;

(3) $1 \leqslant x^2+y^2 \leqslant 4$; (4) $\begin{cases} (x-y^2)(x-4y^2) > 0, \\ y \geqslant x^2; \end{cases}$

(5) $(y-x^2)(x-y^2) > 0$.

§ 55. The Idea of a Set

2.367. (Orally). Which of the given sets are finite and which are infinite:

(1) the set of natural numbers;

(2) the set of prime numbers;

(3) the set of even numbers;

(4) the set of molecules of water in an ocean;
(5) the set of days in a week;
(6) the set of rational numbers;
(7) the set of students of the preparatory faculty;
(8) the set of cities of the Soviet Union;
(9) the set of even divisors of the number 21?

2.368. If N is the set of natural numbers, Z is the set of whole numbers; Q is the set of rational numbers, which set-builder notations are correct: $3 \in N$; $5 \notin Q$; $1/2 \notin N$; $0.(3) \in Q$; $0.(31) \in Q$; $5.5 \in N$; $12 \notin N$; $0 \in Q$; $3.27 \in Z$; $0.2.(16) \in Z$; $-12 \in Q$; $133 \notin Q$; $8 \in Z$; $26 \in Q$?

2.369. Given the following sets: N (the set of natural numbers), N_1 (the set of natural numbers whose last digit is 7) and P (the set of even natural numbers).

Using the membership symbol $\in$, show to which of the given sets the numbers: 18; 317; -16; $-28\frac{1}{2}$; 228; -457; 245; 697; 324 belong. Using the symbol $\notin$, show to which sets the indicated numbers do not belong.

2.370. C is the set of all polygons. Mention several elements of this set. Using the equality sign and braces, write the following sets:

(1) B (the set of letters in the word "mathematics");
(2) D (the set of letters in the word "revolution");
(3) P (the set of distinct words in the sentence: "And he saw a blue, blue sea").

2.371. Read the following notations and list the elements of each set:

(1) $E = \{x \in N,\ x < 7\}$; (2) $F = \{x \in N,\ x \leqslant 3\}$;
(3) $K = \{x \in N,\ x < 2.5\}$; (4) $L = \{x \in Z,\ -4 < x < 7\}$;
(5) $P = \{x \in Z,\ -3 \leqslant x \leqslant 6\}$; (6) $y = \{x \in N,\ |x| < 5\}$.

2.372. Does the equation $a + x = b$ have a solution, where a and b are natural numbers:

(1) in the set of all natural numbers;
(2) in the set of all integers;
(3) in the set of all positive rational numbers?

2.373. Does the equation $ax = b$ (a and b integers) have a solution:

(1) in the set of all integers;
(2) in the set of all rational numbers?

2.374. Which values of the variable x from the set $\{-2; -1; 0; 2; 3; 4; 5; 7; 8; 9\}$ are roots of the equation $3(x + 8)(x - 5) = 0$?

2.375. Name the given number sets, show them on the number line, and write them in the form of an inequality:

(1) $(3, \infty)$; (2) $(-\infty, 2)$; (3) $[-2, 4]$; (4) $(-3, 3)$;
(5) $[0, 5]$; (6) $(-4, 0)$; (7) $[-2, 4)$; (8) $(-\infty, -3]$;
(9) $[0, 7)$; (10) $[-2, -1)$.

2.376. Name the greatest integer which belongs to the set:

(1) $[-12, -9]$; (2) $[-1, 17)$; (3) $(-8, -5]$; (4) $(-\infty, 31]$;
(5) $(-\infty, +\infty)$; (6) $(-\infty, 11)$.

2.377. Which of the integers belong to the given set:

(1) $[0, 8]$; (2) $(-3, 3)$; (3) $(-5, 3)$; (4) $[-4, 8]$; (5) $[-5, 5]$;
(6) $(-4, 1)$; (7) $[-1, 0)$; (8) $[-1, 6]$; (9) $[-2, -1]$?

2.378. Show on the number line the set of points x if:

(1) $A = \{x \mid x \geqslant 2\}$; (2) $B = \{x \mid x > 8\}$;
(3) $C = \{x \mid x \leqslant 3\}$; (4) $D = \{x \mid x < -5\}$;
(5) $E = \{x \mid x < 8\}$; (6) $F = \{x \mid -2 < x < 1\}$;
(7) $K = \{x \mid -5 < x < 3\}$; (8) $L = \{x \mid -1 < x < 4\}$;
(9) $M = \{x \mid x < 3.2\}$; (10) $P = \{x \mid x > 3.2\}$;
(11) $A_1 = \{x \mid x \leqslant 4.6\}$; (12) $A_2 = \{x \mid -3.5 < x < 0\}$;
(13) $B_1 = \{x \mid -4.5 < x < 4\}$; (14) $C_1 = \{x \mid -12.9 < x < 12\}$; (15) $L_1 = \{x \mid -16 < x < 1\}$.

2.379. Find the set of integral solutions of the inequality:

(1) $-8 < x < 1$; (2) $-3 < x < 3$;
(3) $-3 \leqslant x < 5$; (4) $-1 \leqslant x \leqslant 1$.

2.380. Write an inequality which has the given set of solutions:

(1) $(10, +\infty)$; (2) $[-3, +\infty)$; (3) $(-6, -5)$;
(4) $(-\infty, 4]$; (5) $(-\infty, -1)$; (6) $(-13, 4]$.

2.381. Read the following set-builder notations and indicate the elements of each set:

(1) $M = \{x \mid 3x = x + 8\}$; (2) $K = \{x \mid 3(5x + 10) = 30 + 15x\}$; (3) $L = \{x \mid 3x + 5 = 2(x + 6)\}$;
(4) $P = \{x \mid x(x + 16) = 0\}$.

2.382. Indicate the common properties of the elements of each set:

(1) $A = \{23; 22; 21; 20; 19; 18; 17; 16; 15\}$;
(2) $B = \{11; 22; 33; 44; 55; 66; 77\}$;
(3) P is the set of squares;
(4) K is the set of isosceles triangles.

2.383. Take a sheet of paper, mark on it a point "O", and show the set of points "M" for which:
(1) $|OM| = 4$ cm; (2) $|OM| < 4$ cm; (3) $|OM| > 4$ cm.

2.384. Determine of which numbers the following infinite sets consist and write the first five elements for each case:

(1) $A = \{x \mid x = 2p,\ p \in N\}$;
(2) $B = \{x \mid x = 2p + 1,\ p \in N\}$;
(3) $C = \{x \mid x = 5p,\ p \in N_0\}$.

2.385. Given a set of numbers x: $A = \left\{x \mid x = \frac{n^2+1}{n^2+4},\ n \in N\right\}$.

Determine whether the following numbers belong to the set A: 2/5; 17/20; $-1/7$; 5/6; $-2/3$; 50/53; $-16/25$; 37/40; $-15/16$; 10/13; 122/125; 145/148; $-26/35$.

2.386. Given a set $A = \left\{x \mid x = \frac{n^3+7}{n^3+15},\ n \in N\right\}$. Write three numbers which belong to the set A.

§ 56. An Empty Set. Equal Sets. A Subset

2.387. Find an empty set among the following sets:
(1) the set of rectangles with equal sides;
(2) the set of rectangles with unequal diagonals;
(3) the set of triangles whose medians do not intersect at one point;
(4) the set of integral roots of the equation $4x^2 - 1 = 0$;
(5) the set of integral roots of the equation $2x^2 - 3x - 90 = 0$;
(6) the set of even numbers between 0 and 13;
(7) the set of triangles in which the sum of interior angles is equal to 181°;
(8) the set of solutions of the inequality $|x - 2| < 0$;
(9) the set of people on the Mars;
(10) the set of right triangles in which the square of the length of the hypotenuse is not equal to the sum of the squares of the lengths of the legs.

2.388. Prove that the following sets are empty:
(1) $A = \{x \mid x \in N,\ x < 0\}$; (2) $B = \{x \mid x \in N,\ 8 < x < 9\}$; (3) $C = \{x \mid x^2 = -1\}$;

(4) $D = x,\ y \left\{ \left| \begin{matrix} 3x+4y=7 \\ 6x+8y=12 \end{matrix} \right. \right\}$; (5) $E = \left\{ x,\ y \left| \begin{matrix} 2x+y=3 \\ 4x+2y=2 \end{matrix} \right. \right\}$;

(6) $F = \left\{ x,\ y \left| \begin{matrix} x-y=1 \\ 3x-3y=4 \end{matrix} \right. \right\}$.

2.389. Formulate the theorem on the intersection of the altitudes of a triangle as an empty set.

2.390. Among the given sets find equal ones:
(1) $A = \{1; 3; 6\}$; (2) $B = \{3; 6; 9\}$; (3) $C = \{3; 2; 6\}$;
(4) $D = \{2; 3; 6\}$; (5) $E = \{6; 2; 3\}$; (6) $F = \{6; 9; 3\}$.

2.391. For each of the words "report", "porter", and "sailor" form the set of its letters. Find equal sets among them.

2.392. Among the given sets find equal ones:
(1) $A = \{x \mid x \in N,\ 2 < x < 5\}$; (2) $B = \{x \mid x \in N,\ 1 < x < 4\}$; (3) $C = \{x \mid x \in N,\ 2 < x < 4\}$; (4) $D = \{x \mid x \in N,\ 1 < x < 8\}$; (5) $E = \{x \mid x \in N,\ 1 \leqslant x \leqslant 5\}$.

2.393. Which of the given sets are equal:
A is the set of squares;
B is the set of rectangles;
C is the set of quadrilaterals with right angles;
D is the set of rectangles with equal sides;
E is the set of rhombi with equal angles?

2.394. Determine whether the sets $B = \{x \mid -3x + 2 = 0\}$; $C = \{x \mid x(-3x + 2) = 0\}$; $A = \{x \mid 3x + 5 = 7\}$ are equal.

2.395. Name three subsets of:
(1) the set of one-valued natural numbers;
(2) the set of two-valued natural numbers;
(3) the set of integers;
(4) the set of all quadrangles;
(5) the set of letters in the word "drum".

2.396. Given sets: $A = \{a; b; c; d; e; f; g; h; i\}$; $B = \{a; d; i\}$; $C = \{b; c; d; e; g; h\}$; $D = \{a; b; c\}$; $E = \{b; c; d; e\}$; and $F = \{a; i\}$.
(1) Which of them is a subset of the set A?
(2) Is the set E a subset of the set B?
(3) A subset of which set is the set D?

2.397. From the set $A = \{1; 2; 3; \ldots; 10\}$ single out the subset of:
(1) odd numbers;
(2) numbers multiple of 3;
(3) prime numbers;
(4) numbers x such that $1 < x \leqslant 4$;
(5) numbers by unity differing from 3;
(6) numbers x such that, when added to the number 3, yield 5;
(7) numbers x such that, when added to the number 3, yield 2.

2.398. Given a set: $K = \{21; 54; 153; 171; 234\}$. Using the elements of the set K, form subsets of:
(1) numbers divisible by 7;
(2) numbers divisible by 9;
(3) numbers indivisible by 4;
(4) numbers indivisible by 5.

2.399. Given sets: $C = \{1; 2; 3; 4; 5; 6\}$; $M = \{4; 1; 3; 2\}$; $K = \{4; 6; 5; 3; 2\}$. Prove that: (1) $M \in C$; (2) $K \in C$.

2.400. Write three subsets of the following sets:
(a) the set of letters in the word "proportion";

(b) the set of two-digit numbers;
(c) the set of all quadrilaterals.

2.401. Write three sets A, B, and C such that:
(1) $C \subset B$ and $A \subset B$; (2) $B \subset C$ and $C \subset A$;
(3) $B \supset A$ and $A \supset C$; (4) $A \subset B$, $B \subset C$, and $B \subset A$.

2.402. Form all the subsets of B if
(1) $B = \{a\}$; (2) $B = \{a; b\}$; (3) $B = \{a; b; c\}$;
(4) $B = \{a; b; c; d\}$.

2.403. Check whether the following notations are correct:
(1) $\{1; 2\} \subset \{1; 2; 3; 4\}$; (2) $\{1\} \in \{1; 2\}$;
(3) $\{7; 5; 3; 1\} \subset \{1; 7; 3; 5\}$; (4) $1 \in \{1; 2\}$;
(5) $1 \notin \{1; 2\}$; (6) $\{4; 3; 2; 1\} \subset \{1; 2\}$;
(7) $\{1; 2\} \in \{1; 2\}$; (8) $\{1; 2\} \subset \{1; 2\}$.

§ 57. Intersection of Sets

2.404. Find the intersection of the sets: $P = \{a; b; c; d; e; f\}$ and $E = \{a; g; h; f\}$.

2.405. Given sets: $A = \{26; 39; 5; 58; 17\}$; $B = \{5; 17; 37; 26; 80\}$; $C = \{80; 14; 37\}$. Find the intersection of each pair of the sets.

2.406. What elements does the intersection of the sets of letters in the words "mathematics" and "geometry" consist of?

2.407. Find the intersection of:
(1) the set Z of all integers and the set N of all natural numbers;
(2) the set N_0 of all nonnegative integers and the set L of all negative numbers.

2.408. Find the common divisors of the numbers:
(1) 72 and 54; (2) 36 and 24; (3) 15 and 135; (4) 11 and 49; (5) 61 and 29.

2.409. Find the intersection of the sets of solutions of two inequalities:
(1) $x > -2$ and $x > 0$; (2) $x > 3.7$ and $x \leqslant 4$;
(3) $x \geqslant 5$ and $x < -7 - \frac{1}{2}$; (4) $|x| \geqslant 3$ and $2 < x < 5$;
(5) $-2 < x < 4$ and $1 \leqslant x \leqslant 7.2$; (6) $-7 < x < 5$ and $-6.8 < x \leqslant 2.3$.

2.410. Find the intersection of the given sets:
(1) $(\infty, 12]$ and $[6, +\infty)$; (2) $(-\infty, 3)$ and $(-3, +\infty)$;
(3) $[-7, -5]$ and $[-6, 0]$; (4) $(-8, -1)$ and $(1, +\infty)$;
(5) $[0, 4]$ and $[1, 5]$; (6) $[0, 2]$ and $[3, 7]$.

2.411. Find the intersection of the sets of roots of the given equations:

(1) $x^2 - 4x + 3 = 0$ and $x^2 - 3x + 2 = 0$;
(2) $x^2 - 2x - 3 = 0$ and $(x + 1)(x - 8) = 0$;
(3) $x^2 - 6x + 9 = 0$ and $|3x| = 9$;
(4) $x^2 - 1 = 0$ and $|2x|/2x = 1$ $(x \neq 0)$.

2.412. Find the set of permissible values of x:

(a) (1) $\sqrt{25 - x^2} + \sqrt{x^2 - 9} = 5$; (2) $\sqrt{x^2 + x - 12} + \sqrt{x - 1} = 2$; (3) $\sqrt{3x - 5} + \sqrt{16 - x^2} = 7$;
(4) $\sqrt{\frac{16 - x^2}{25 - x^2}} + \sqrt{x} = 4$.

(b) On what set will the given equalities be identities:

(1) $|x| = x$; (2) $\frac{x^2 - 9}{x - 3} = x + 3$; (3) $\frac{x^2 - 1}{x + 1} = x - 1$;

(4) $\frac{4x^2 + 4x + 1}{2x + 1} = 2x + 1$; (5) $\frac{x^3 - 8}{x - 2} = x^2 + 2x + 4$;

(6) $\sqrt{x^2} = -x$; (7) $\sqrt{x^2} = x$; (8) $\sqrt{x^2} = |x|$?

2.413. Find the set of points of intersection of two circles: $x^2 + y^2 = 25$ and $(x - 13)^2 + y^2 = 144$.

2.414. Find the set of points of intersection of the straight line $y = -5x + 12$ and parabola $y = x^2 - 6x + 10$.

2.415. Find the set of points of intersection of the parabola $y = x^2$ and circle $x^2 + y^2 = 30$.

2.416. Find the set of points of intersection of the parabola $y = x^2 + 4$ and hyperbola $y = 5/x$.

2.417. Prove that if $A \subset B$, then $A \cap B = A$.

2.418. Given sets:

$P = \{x \mid x \in N,\ x < 20,\ x \text{ an odd number}\}$;
$O = \{x \mid x \in N,\ x < 20,\ x \text{ an even number}\}$;
$S = \{x \mid x \in N,\ x < 20,\ x \text{ divisible by } 3\}$;
$T = \{x \mid x \in N,\ x < 20,\ x \text{ divisible by } 4\}$.

List all the elements of the given sets and the sets:

$Q \cap T$; $Q \cap S$; $P \cap S$; $P \cap Q$; $Q \cap S \cap T$.

§ 58. Union of Sets

2.419. Find the union of the set of natural numbers and the set of odd natural numbers.

2.420. Find the union:

(a) of the sets M and P if $P = \{1; 2; 5; 8; 9\}$, and M is the set of natural numbers lesser than 7; (b) of the set P of natural numbers lesser than 15 and the set M of prime numbers lesser than 15.

2.421. Let A be the set of prime numbers which are less than 19; $B = \{7; 11; 13\}$; $C = \{1; 2; 3; 4\}$. Find $A \cup B$; $A \cup C$.

2.422. Write two sets such that their union is: (a) the set $M = \{1; 2; 3; 4; 5\}$; (b) the set of natural numbers.

2.423. Find the union of the sets A and B if: (1) $A = \{l; t; i\}$ and $B = \{t; d; o; l\}$; (2) $A = \{14, 21; 35; 42\}$ and $B = \{7; 14; 63; 70; 56\}$.

2.424. Find the union of the set of digits of the number 1 321 337 and the set of digits of the number 7 812 531.

2.425. The set D is the union of the set of two-digit natural numbers and the set of natural numbers from 1 to 20. Do the numbers 15, 99, 100, 5, 7 belong to the set D? Write the answer with the help of the symbols $\in$ and $\notin$.

2.426. Find the union of the set of letters in the word "bronze" and the set of letters in the word "barometer".

2.427. Write two sets A and B such that: (1) $A \cap B = \varnothing$; (2) $A \cap B \neq \varnothing$; (3) $A \subset B$. Find the union of the sets A and B in each of the cases.

2.428. Given sets: $A = \{21; 12; 11; 22\}$; $B = \{11; 12; 13; 14; 15\}$; $X = \{51; 15; 31\}$; $Y = \{11; 13; 15\}$.

Write the elements of the following sets: (1) $B \cup X$; (2) $Y \cup B$; (3) $A \cup Y$; (4) $A \cup B \cup X$; (5) $A \cup X \cup B$; (6) $A \cup B \cup X \cup Y$.

2.429. Show on the number line the union of the sets A and B if: (1) $A = \{x \mid x > -3\}$; $B = \{x \mid x < -5\}$; (2) $A = \{x \mid -2 \leqslant x \leqslant 0\}$; $B = \{x \mid -1 \leqslant x \leqslant 4\}$; (3) $A = \{x \mid 3.5 \leqslant x < 1\}$; $B = \{x \mid -7 < x < 412\}$.

2.430. Which of the two sets is a subset of the other set if: (1) A and $A \cup B$; (2) A and $A \cap B$?

2.431. M is the union of the sets B and C, and the set P is their intersection. Write the elements of the set B if: $M = \{m; q; a; e; l; n; t; u\}$; $P = \{a; n; t\}$; $C = \{m; a; l; t; n\}$.

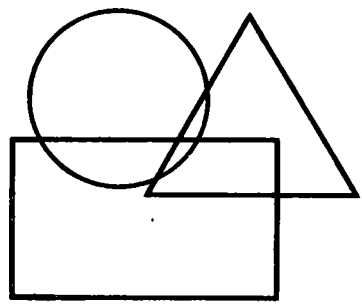

FIG. 2

2.432. Given sets: $A = \{-8; -6; -7; -5; -4\}$; $B = \{-6; -5; -4; -3; -2; -10\}$; $C = \{-4; -3; -2; -1; 0; 1; 2\}$; $D = \{-7; -6; -5; -4\}$.

Write the following sets: (1) $A \cup B \cup C \cup D$; (2) $A \cap B \cap C$; (3) $(A \cup B) \cap C$; (4) $(A \cap C) \cup (B \cap C)$; (5) $(A \cap B) \cup (C \cap D)$; (6) $(A \cup B) \cap (C \cup D)$. Find among them equal sets.

2.433. (a) Find the union of the given sets: (1) $(-\infty, 4]$ and $[-2, +\infty)$; (2) $[2, 4]$ and $[3, 6]$; (3) $(-1, 5)$ and $(1, 7)$; (4) $[-8, 8]$ and $[-6, 6]$.

(b) Find the union and intersection of the given sets: (1) N and Z; (2) Z and Q; (3) Q and N.

2.434. The set C consists of 11 elements, and the set D consists of 8 elements. How many elements does the set $C \cap D$ have if $C \cup D$ contains 15 elements?

2.435. Figure 2 represents three sets: A, B, and C. Hatch each of the following sets: (1) $A \cup B$; (2) $A \cap C$; (3) $A \cap (B \cap C)$; (4) $(A \cap B) \cap C$; (5) $A \cup (B \cap C)$; (6) $(A \cup B) \cap (A \cup C)$; (7) $(A \cup B) \cup C$; (8) $(A \cap C) \cup C$.

2.436. A, B, and C are subsets of the set $M = \{1; 2; 3; 4; 5; 6\}$. Write the elements of the set B if: (1) $A = \{3; 5\}$; $B \cap C = \{3\}$; $B \cup C = \{1; 2; 3; 4; 6\}$; (2) $A \cap A = \varnothing$, $C \cup A = \{6\}$; $B \cup C = M$, $B \cap C = \varnothing$.

2.437. (a) Given two sets A and B, where $A \subset B$. What can be said about the sets $A \cup B$ and $A \cap B$?

(b) Given a set $E = \{a; b; c; d; e; f\}$ and two of its subsets: $A = \{a; b; c; d\}$ and $B = \{c; d; e\}$:

(1) form the sets $A' = C_e A$ and $B' = C_e B$;

(2) form the sets $H = A \cap B$ and $H' = C_e H$; verify the equality $H' = A' \cap B'$;
(3) form the subsets: $K = A \cup B$ and $K' = C_e K$; verify the equality $K' = A \cap B'$;

(c) prove the equivalence: $A \cup B = A \to B \subset C$;

$$\left.\begin{array}{l} A \subset C \\ B \subset C \end{array}\right\} \Rightarrow (A \cup B) \subset C.$$

2.438. Let $a \in N$ and $b \in N$. Is it true that: (1) $a + b \in N$; (2) $a - b \in N$; (3) $ab \in N$; (4) $a/b \in N$; (5) $a^b \in N$?

2.439. From the set $\{1/2;\ 2/3;\ 4/3;\ 2/5;\ 5/2;\ 7/7\}$ single out the subset of: proper fractions; improper fractions, integers; mixed numbers.

2.440. Construct the set of points in the plane whose coordinates satisfy the following relations:

(1) $x + y > 0$; (2) $y - x > 1$; (3) $y \geqslant x^2$;
(4) $y < x^2$; (5) $x^2 + y^2 < 1$; (6) $x^2 + y^2 \geqslant 1$;
(7) $0 < x < 1$; (8) $-2 < y < 3$;

(9) $\begin{cases} -1 \leqslant x \leqslant 3, \\ 2 < y < 5; \end{cases}$ (10) $\begin{cases} x > 2, \\ y \leqslant 3; \end{cases}$ (11) $\begin{cases} x + y < 3, \\ x - y < 1; \end{cases}$

(12) $\begin{cases} 0 < x < 1, \\ y > -1. \end{cases}$

§ 59. Exponential and Logarithmic Functions

2.441. Construct the graphs of the following functions:

(1) $y = 2^x$; (2) $y = 2^{-x}$; (3) $y = -2^x$; (4) $y = 2^{x-1}$;
(5) $y = 2^x - 3$; (6) $y = 3^{|x|}$; (7) $y = (1/3)^{|x|}$;
(8) $y = (1/3)^{-|x|}$; (9) $y = (1/3)^x + 2$; (10) $y = |2^{|x|} - 1|$;
(11) $y = |3^{|x|} - 3|$.

2.442. Construct on the same coordinate axes the graphs of the functions $y = 3^x$ and $y = (1/3)^x$ for the following values of x:

x	-2	-1	$-1/2$	0	$1/2$	1	2
$y = 3^x$							
$y = (1/3)^x$							

Using the graphs, mention: (a) the common properties of both functions; (b) the different properties of the functions.

2.443. Which is greater:

(1) 2^{300} or 3^{200}; (2) 4^{300} or 2^{400}; (3) $\sqrt[3]{0.01}$ or $\sqrt[4]{0.001}$; (4) $\pi^{-\sqrt{3}}$ or $(1/\pi)^{-\sqrt{3}}$; (5) $(2/3)^{1+\sqrt{6}}$ or $(2/3)^{\sqrt{2}+\sqrt{3}}$; (6) $(\pi/4)^{1+\sqrt{3}}$ or $(\pi/4)^2$?

2.444. Compare with unity: $(2/3)^{3/4}$; $(5/4)^{5/6}$; $(2/3)^{7/3}$; $(2/3)^{-5/6}$; $(2.36)^0$; $(0.12)^0$; $3^{-\sqrt{2}}$; $0.8^{-2/3}$; $(0.(3))^{\sqrt{2}}$; $(\pi/7)^{-\sqrt{3}}$; $(\pi/2)^{\sqrt{2.7}}$; $e^{3/4}$; $\pi^{-1/3}$.

2.445. Determine which is greater $-x$ or y (in each given inequality):

(1) $(1/3)^x < (1/3)^y$; (2) $(1.5)^x < (1.5)^y$; (3) $(0.3)^x > (0.3)^y$; (4) $(8/3)^x > (8/3)^y$; (5) $(0.6)^x > (0.6)^y$; (6) $(5/4)^x < (5/4)^y$; (7) $(3.6)^x > (3.6)^y$; (8) $(2.3)^{-x} < (2.3)^{-y}$.

2.446. Find the domain of definition of each given function:

(1) $y = a^{2/x}$; (2) $y = a^{\sqrt{x-2}}$; (3) $y = a\sqrt[3]{x}$; (4) $y = a^{5/(x-3)}$; (5) $y = 2^{x^2-5x+6}$; (6) $y = 7^{5/(x^2-1)}$; (7) $y = 3^{4/(x^2-1)}$; (8) $y = e^{1/\sqrt{x}}$; (9) $y = 3^{\sqrt{x}} + 3^{\sqrt{1-x}}$; (10) $y = 2^{-\sqrt{-x}}$; (11) $y = 0.5^{\sqrt{4-x^2}} + \dfrac{1}{x-1}$; (12) $y = 2^{\sqrt{|x-3|-|8-x|}}$.

2.447. Write the following equalities with the help of logarithms:

(1) $2^3 = 8$; (2) $6^2 = 36$; (3) $2^4 = 16$; (4) $3^4 = 81$; (5) $4^3 = 64$; (6) $2^5 = 32$; (7) $2^{-1} = 1/2$; (8) $3^{-2} = 1/9$; (9) $27^{1/3} = 3$; (10) $729^{1/3} = 9$; (11) $(8/25)^{1/2} = (2\sqrt{2})/5$.

2.448. Verify the following equalities:

(1) $\log_3 9 = 2$; (2) $\log_4 16 = 2$; (3) $\log_5 125 = 3$; (4) $\log_{\sqrt{3}} 3 = 2$; (5) $\log_{\sqrt{2}} 4 = 4$; (6) $\log_2 (1/4) = -2$; (7) $\log_2 (1/8) = -3$; (8) $\log_{125} 5 = 1/3$; (9) $\log_a a^{1/2} = -2$; $a > 0$; $a \neq 1$; (10) $\log_{a^3} (1/a) = -1/3$; $a > 0$; $a \neq 1$.

2.449. Find x if:

(1) $x \log_2 0.125 = -2$; (2) $\log_x (1/3) = -2/3$;
(3) $\log_{0.32} (2\sqrt{2})/5 = x$; (4) $\log_{1/4} x = -1/2$;
(5) $\log_{3\sqrt{3}} x = -2/3$; (6) $\log_{0.04} 5 = x$; (7) $\log_{\sqrt[3]{9}} = 1.5$;
(8) $\log_3 (x - 2) = 2$; (9) $\log_x (1/25) = -2/3$;
(10) $\log_4 8\sqrt{2} = x$.

2.450. Compute:

(1) $2 \log_5 25 + 3 \log_2 64$; (2) $\log_2 \log_2 16$;
(3) $\log_6 36 + 5 \log_7 49 + 4 \log_{1/2} 4$;
(4) $2 \log_{1/3} 27 - 3 \log_{1/6} 6 + \log_{1/5} 125$.

2.451. Find x:

(1) $2 \log_3 x = 3 \log_3 x - 2$;
(2) $5 \log_2 x - 3 \log_7 49 = 2 \log_2 x$;
(3) $(\log_3 x)^2 - 3 \log_3 x + 2 = 0$;
(4) $(\log_2 x)^2 - \log_2 x - 2 = 0$.

2.452. Construct on the same coordinate axes the graphs of the two functions: $y = \log_3 x$ and $y = \log_{1/3} x$. Using the graphs, formulate: (a) the common properties of the functions; (b) different properties of the functions.

2.453. Graph the functions:

(1) $y = \log_2 x + 3$; (2) $y = \log_3 (x - 2)$;
(3) $y = \log_2 | x |$; (4) $y = | \log_2 x |$;
(5) $y = | \log_2 x - 1 |$; (6) $y = \log_{1/6} | x |$;
(7) $y = | \log_{1/2} | x | |$; (8) $y = | \log_{1/2} (x - 2) - 2 |$.
(9) $y = 2^{\log_2 (x+1)}$; (10) $| y | = \log_2 (-x)$;
(11) $y = 2^{\log_2 (1-x^2)}$; (12) $y = 2^{-\log_2 x}$.

2.454. Find the domain of definition for each of the given functions:

(1) $y = \log_a (-x)$; (2) $y = \log_a (1 - x^2)$;
(3) $y = \log_3 (1 - x)$; (4) $y = \log_a (1 + x^2)$;
(5) $y = \log_a \sqrt{x}$; (6) $y = \log_2 (x^2 - 5x + 6)$;
(7) $y = \log_3 \log_{1/2} x$; (8) $y = \log_2 \sin^2 x$;
(9) $y = \log_2 \cos x$; (10) $y = \log_2 (x^2 - 4x - 5)$;
(11) $y = \log_{1/3} (-3x^2 - 7x - 2)$; (12) $y = \log | 6x - 8 |$.

2.455. Find the domain of each function:

(1) $y=\log_2(x^2-1)$; (2) $y=\sqrt{x+1}\cdot\log x^3$; (3) $y=e^{-1/x^2}$;

(4) $\log_a(x^2+2x)$; (5) $y=\log_3\dfrac{x-3}{x+3}$;

(6) $y=\log_2\dfrac{4x-11}{2x^2-4x-6}$; (7) $y=\sqrt{\log\dfrac{1-2x}{x+3}}$;

(8) $y=\sqrt{\log_{0.3}\dfrac{x-1}{x+5}}$; (9) $y=\sqrt{\log^2_{0.5}(x-3)-1}$;

(10) $y=\log_3\log_{0.5}x$; (11) $y=\dfrac{\log_2 x}{\arcsin(x-3)}$.

2.456. In each of the given inequalities, determine what is greater x or y:

(1) $\log_3 x<\log_3 y$; (2) $\log_{0.5}x>\log_{0.5}y$;

(3) $\log_{0.1}x<\log_{0.1}y$; (4) $\log_{1/\pi}x<\log_{1/\pi}y$;

(5) $\log_\pi x>\log_\pi y$; (6) $\log_{1.5}(-x)<\log_{1.5}(-y)$.

2.457. What is greater:

(1) $\log_8 9$ or $\log_9 8$; (2) $\log_4 1$ or $\log_5 1$;

(3) $\log_5 5$ or $\log_6 6$; (4) $\log_{0.15}0.15$ or $\log_2 2$?

2.458. Compute:

(1) $2^{\log_2 8}$; (2) $36^{\log_6 2}$; (3) $81^{0.5\log_9 7}$; (4) $25^{1-0.5\log_5 4}$;

(5) $3.4^{1+\log_{3.4}10}$; (6) $10^{1-2\log 10}$; (7) $5^{\log_5 10-1}$;

(8) $3^{1-\log_3 7}$; (9) $3^{2-\log_3 10}$; (10) $2.4^{\log_{2.4}10+1}$;

(11) $2^{\log_{0.5}4}$; (12) $9^{\log_{729}512}$; (13) $0.25^{\log_2 0.25}$;

(14) $10\cdot100^{(1/2)\log 9-\log 2}$; (15) $100^{1/2-\log\sqrt[4]{4}}$;

(16) $100^{1-\log_{10}(5/2)}$; (17) $\log_7\log_7\sqrt{7\sqrt{7\sqrt{7}}}$;

(18) $\log_2 5\cdot\log_{25}27$.

2.459. Compute:

(1) $\sqrt{^{\log_6 5}\sqrt{25}+^{\log_6 7}\sqrt{49}}$; (2) $^{\log_9 3}\sqrt{81}+^{\log_9 3}\sqrt{27}+^{\log_7 3}\sqrt{9}$;

(3) $36^{\log_6 5}+10^{1-\log_{10}2}-3^{\log_9 36}$;

(4) $(81^{1/4-(1/2)\log_9 4}+25^{\log_{125}8})\cdot49^{\log_7 2}$.

2.460. Graph the functions:

(a) (1) $y = 2 \cdot 3^x$; (2) $y = 3 \cdot (1/2)^x$; (3) $y = 2^{x^2}$; (4) $y = (1/2) \log x$;

(b) (1) $y = \log (2 - x)$; (2) $y = \log \sin^2 x$; (3) $y = \log x + |\log x|$;

(4) $y = \log_2 (\sin x \cdot \cos x)$; (5) $y = \log_{1/2} (-x)$;

(6) $y = 3^{\log_3 x}$;

(7) $y = \log (x - 2)$; (8) $y = \log \cos^2 x$.

2.461. Find the points of intersection of the graph of the function $y = x^{\log x} - 10\,000x^3$ with the axis of abscissas.

2.462. For what value of k does the graph of the function $y = \log k_x - 2 \log (x + 1)$ intersect the axis of abscissas only at one point?

2.463. For what values of x is the graph of the function $y = 0.7^{\log x^2 - 8x + 8}$ situated not lower than the straight line $y = 1$?

2.464. Find the point of intersection of the graph of the function $y = [3.6^{1+\log_{3.6}(10+x)}]^{\log_6 5x}$ with the axis of ordinates.

2.465. Find the abscissa of the point of the graph of the function $y = \log_2 \log_6 (2^{\sqrt{x+1}} + 4)$, for which $y = 1$.

2.466. Show that the graph of the function $y = \frac{\log 5 + \log (x^2+1)}{\log (x-2)} - 2$ does not intersect the axis of abscissas.

2.467. For what values of x is the graph of the function $y = \log_{1/3} (x^2 - 8x) + 2$ not lower than the axis of abscissas?

§ 60. Taking the Logarithms and Antilogarithms of Algebraic Expressions

2.468. Take the logarithm of the following expressions:

(1) (a) $x = 2ab$; (b) $x = \frac{ab}{c}$; (c) $x = a^2bc^3$; (d) $x = a^5b^6/c^4$;

(e) $x = 2(a + b)$; (f) $x = 2b\sqrt{ac}$.

(2) (a) $x = \sqrt[3]{3a^3b/c^4}$; (b) $x = 5a \cdot \sqrt[3]{a^2(a-b)}$;

(c) $x = 1/(a^n \sqrt{b})$; (d) $x = (1/a)\sqrt{1/b}$.

(3) (a) $x=\sqrt{2\sqrt{6\sqrt{15}}}$; (b) $x=\sqrt[3]{3\sqrt[3]{21\sqrt[3]{6}}}$;

(c) $x=\sqrt[3]{\frac{a^3b}{\sqrt[5]{c^3}}}$; (d) $x=\sqrt[5]{\frac{a^3\sqrt[3]{b}}{b^2}}$.

(4) (a) $x=\sqrt{\frac{4a\sqrt{ab}}{5b\sqrt[3]{a^2b}}}$; (b) $x=\sqrt{\frac{10a^5\sqrt[4]{a^3b}}{7b\sqrt[5]{a^4b^3}}}$;

(c) $x=\sqrt{\frac{\sqrt[3]{a^3}\sqrt[4]{b}}{\sqrt[4]{a\sqrt[3]{a}}}}$; (d) $x=\sqrt{\frac{2a^2\sqrt[3]{a^2b}}{5b^2\sqrt{a}}}$;

(e) $x=\frac{6a\sqrt{2(a-b)c}}{5(a-b)^2}$.

(5) (a) $x=\sqrt[n]{b^{12}\sqrt[m]{a^3}}$; (b) $x=\sqrt[n+1]{a^n\sqrt[m]{b^{-1}}}$;

(c) $x=0.8^{\sqrt[4]{0.5}}$; (d) $x=0.5^{\sqrt[3]{0.4}}$.

(6) (a) $x=\log\left(1.6^{\sqrt[3]{1.2}}\right)$; (b) $x=\frac{5\log m}{\log (m)^3}$;

(c) $x=\log\sqrt{(a+b)^{\log(a+b)}}$; (d) $x=\log\sqrt{\log 3}$.

2.469. Find x:
(1) $\log x = \log 7 - \log 3 + \log 2$; (2) $\log x = \log 3 + \log 5 + \log 2$;
(3) $\log x = 2\log 13 - (2/5)\log 2 - (4/3)\log 7$;
(4) $\log x = (3/5)\log 11 - (2/7)\log 5$;
(5) $\log x = (1/3)\log 17 - (5/9)\log 3$;
(6) $\log x = 3\log a + 2\log b - 4\log c$;
(7) $\log x = \log a - 3\log b + 5\log c$.

2.470. (1) $\log x = (1/2)\log a + (1/3)\log b - (1/4)\log c$;
(2) $\log x = (2/5)\log(a+b) - (3/4)\log(a-b)$;
(3) $\log x = \log(a+b) - (2/3)(2\log a)$;
(4) $\log x = 2\log(a-b) + (3/4)(\log a - (2/3)\log b)$;
(5) $\log x = 3\log a + (1/3)[\log(a+b) + (2/5)\times \log(a-b) - \log b - (1/2)\log c]$.

2.471. (1) $\log x = -(2/3)\log b + (3/4)[\log a - 2\log c + (3/5)\log(a-b) + (3/5)\log(a+b)]$;
(2) $\log x = -(1/2)\log a + (1/4)[\log b - (2/3)\log a + (2/3)\log(a-b) - (1/2)\log(a+b)]$;
(3) $\log x = -\log(a+b) + (2/5)[2\log a + (1/2)\log b - (1/3)\log a - \log b - \log a]$;
(4) $\log x = 5\log a + (1/2)[\log(a+b) + (1/3)\log(a-b) - \log a - \log b]$;

(5) $\log x = (2/3)\,[\log a + (4/3)\log(a+b) - 2\log(a-b) - (1/2)\,b]$.

2.472. Prove the given equalities:

(a) $-\log_3 \log_3 \sqrt[3]{\sqrt[3]{3}} = 2$; (b) $-\log_2 \log_2 \sqrt{\sqrt[4]{2}} = 3$;

(c) $\log_b a \sqrt{a^2} - 2^{\log_b a \sqrt{a} \cdot \log_a b \sqrt{b} + (1/2)\log_a b \sqrt{b}} = (b-a)^2$.

2.473. (1) Given: $\log_5 4 = a$; $\log_5 3 = b$. Prove that $\log_{25} 12 = \frac{a+b}{2}$.

(2) Given: $\log_a 27 = b$. Find $\log_{\sqrt{3}} \sqrt[6]{a}$.

(3) Prove that $\log_{ab} c = \frac{\log_a c \cdot \log_b c}{\log_a c + \log_b c}$.

(4) Given: $\log_{10} 5 = a$; $\log_{10} 3 = b$. Find $\log_{30} 8$.

(5) Given: $\log_{10} 2 = a$; $\log_2 7 = b$. Find $\log_{10} 56$.

2.474. Compute A if

(1) $A = 5^{3\log_{125} 7} - (1/4)\log_{\sqrt{5}} 4$; (2) $A = 2^{2\log_4 9 + \log_{1/3} 3}$;

(3) $A = 27^{(\log_{0.5} 3)^{-1}} \cdot 9^{\log_3 2\sqrt{2}}$; (4) $A = 2^{B+6}$, where $B = 2/\log_{\sqrt{3}} 2$; $c = 1/\log_2 6$;

(5) $A = 4^B$, where $B = \log_5 2 + \log_{1/4} 10$;

(6) $A = 3^{(1/2)\log_{1/3}(1/25) - (1/6)\log_{\sqrt{3}} 8}$;

(7) $A = 9^B$, where $B = (1/2)\log_{1/5} 5 + (1/2)\log_3 2$;

(8) $5^{\log_5 A} = \log_3 9 + 4^{\log_2 3}$;

(9) $2\log_4 A = \log_2 12 - (1/4)\log_{\sqrt{2}} 9$;

(10) $A = 10^B$, where $B = 2\log_{0.01} 2 + \log_{100} \sqrt{3}$;

(11) $A = 4^B$, where $B = 2\log_{16} 10 - (1/2)\log_2 5$.

§ 61. Decimal Logarithms

2.475. Evaluate the following expressions;

(a) (1) $10^{\log 12}$; (2) $10^{-\log 5}$; (3) $10^{\log 6 + \log 30}$;
(4) $10^{\log 3 - \log 30}$;

(b) (1) $10^{1 - \log 2.5}$; (2) $10^{\log 80 - 1/2}$; (3) $100^{\log 7}$;
(4) $1000^{\log 20 - 1}$;

(c) (1) $0.1^{\log 6 - \log 2}$; (2) $(\sqrt{10}/10)^{\log 9 - 2}$;
(3) $(\sqrt{10})^{\log 25 + 2}$; (4) $0.01^{\log 0.2 - 1/2}$.

2.476. Write the given numbers in scientific notation ($a \cdot 10^n$, where $1 < a < 10$):
125; 14.36; 650; 8500; 13 750; 13.7; 3; 715; 19.14; 0.81; 0.305; 123 000 000; 0.000895; 0.0000001; 0.0007001; 0.001009; 0.035102; 3.0003; 0.001001.

2.477.

(a) Find the integral part of the number: 1.8; 263.4; 27; 0.343; -0.4; -0.97; -35.2; -10.7; $-\pi$; $2e$; $-0.5e$; $1/\pi$.

(b) Find the fractional part of the number: 3.6; 10; 0.64; -0.64; 6.27; -6.27; -100; 0.1; -14.5; -0.068; -1.05; -1.45; 9.99; 12.32.

(c) Find x:
(1) $[2.75 + x] = [2.75]$; (2) $\{2.75 + x\} = \{2.75\}$;
(3) $[0.21 + x] = [0.21]$; (4) $\{0.58 + x\} = 0.58$;
(5) $[y - 0.36] = [2.7]$; (6) $\{-0.39\} = 0.61$; (7) $[1.7 + z] = [-8.2]$; (8) $\{1.62 + x\} = 0.65$.

2.478. Find the characteristics of the logarithms of the given numbers using their scientific notation: 42.2; 50.5; 12 360; 2.84; 320 645; 3 163 250; 0.456; 0.0918; 0.00123; 3.00123; 0.3123; 0.0000511; 0.30100; 0.250015.

2.479. Find the characteristic and mantissa if $\log x$ equals:
(a) (1) -4.2129; (2) -0.5423; (3) -5.899; (4) -3.2173;
(b) (1) -1.6831; (2) -2.9543; (3) -3.5382; (4) -0.1612;
(c) (1) -0.02; (2) -4.0792; (3) -0.515; (4) -1.145.

2.480. Find the logarithms of the following numbers: 0.1; 0.01; 0.001; 100; 1 000 000; 0.000001; 8; 141; 954; 420; 6.40; 1235; 390.7; 1010; 18.43; 2.05; 900.1; 0.002891; 247.99; 0.00447; 3.372; 7.267; 0.111; 24 868; 57 384; 0.547963; 0.0152396.

2.481. Find the logarithms of the given numbers if it is known that $\log 2 \approx 0.3010$; $\log 3 \approx 0.4771$; $\log 5 \approx 0.6990$; $\log 7 \approx 0.8451$: 6; 4; 12; 15; 20; 16; 30; 32; $\frac{3}{5}$; $\frac{8}{15}$; $1\frac{2}{3}$; $4\frac{5}{4}$; 0.6; 0.12; 0.08; 343; 3.43; 0.049; 3.5; $1\frac{7}{8}$; 0.24; 0.56; 0.84.

2.482. Evaluate the following expressions:
(1) $\log 153 - \log 15.3$; (2) $\log 0.491 - \log 491$;
(3) $\log 12.1 - \log 1294$; (4) $\log 0.181 - \log 1.701 + \log 9$;
(5) $\log 14.24 - \log 85.44 + \log 60$;
(6) $\log 5 + \log 11 - \log 0.55$; (7) $\log 1.6 + \log 300 - \log 48$; (8) $\log 3.52 - \log 24.64 + \log 700$;
(9) $\log 0.08 - \log 272 + \log 3800$;
(10) $2 \log 35 - \log 3.5 - \log 0.35$.

2.483. Find x if its logarithm is equal to: 1.3010, 3.9030, $4 + 0.4771$, $-3 + 0.3010$, $2 - 5 + 0.4771$, 1.4771, 4.3010, -3.6990, -1.5229, $-2 + 0.6990$, -3.6990, $-1 + 0.8451$, 2.8451, -5.8451.

2.484. Compute x with the help of the table of logarithms:

(1) $x = \dfrac{103.8 \cdot 20.97}{5.174 \cdot 13.62}$; (2) $x = \dfrac{9.738 \cdot 21.09}{48.72 \cdot 0.8478}$;

(3) $x = \dfrac{92.17^2 \cdot 5.14^3}{2.184^4 \cdot 0.5836^2}$;

(4) $x = \dfrac{1.894^4 \cdot 23.4^3}{44.15^2 \cdot 0.9647}$; (5) $x = \sqrt[3]{5862^2}$;

(6) $x = \sqrt[5]{13.76^3}$;

(7) $x = \dfrac{8.15^2 \cdot \sqrt[3]{14.36}}{24.38 \cdot \sqrt{8.734}}$; (8) $x = \dfrac{12.48^3 \sqrt[4]{5.76}}{1.842 \cdot \sqrt[3]{673.8}}$;

(9) $x = \sqrt[5]{\dfrac{2.5914 \cdot \sqrt[3]{0.0836}}{1.472^2}}$; (10) $x = \dfrac{0.9562 \cdot \sqrt[3]{2.165}}{\sqrt[4]{0.582}}$;

(11) $x = \sqrt[3]{\dfrac{17.88}{0.032714 \cdot \sqrt[5]{2.7}}}$; (12) $x = (-5.32)^3 \cdot \sqrt[4]{0.0294}$;

(13) $x = \sqrt[5]{(-0.249)}$; (14) $x = \sqrt[3]{-18.34}$;

(15) $x = (0.461)^{0.461}$.

§ 62. Exponential and Logarithmic Equations and Systems

2.485. Solve the following equations:

(1) $5^x = 625$; (2) $3^x = 243$; (3) $3^{-x} = 81$; (4) $2^{-x} = 32$;
(5) $2^{2x-1} = 8$; (6) $(4/9)^x = (3/2)^3$; (7) $(3/4)^x = (4/3)^5$;
(8) $(3/7)^{3x+7} = (7/3)^{7x-2}$; (9) $(2/3)^x \cdot (9/8)^x = 27/64$;

(10) $\sqrt{3^x} = \sqrt[3]{9}$; (11) $\sqrt{2^x} \cdot \sqrt{3^x} = 36$;

(12) $2^x \cdot 5^x = 0.1\,(10^{x-1})^5$; (13) $4^{\sqrt{x+1}} = 64 \cdot 2^{\sqrt{x+1}}$;

(14) $27^{\sqrt{x+1}} = 729 \cdot 3^{\sqrt{x+1}}$; (15) $(0.25)^{2-x} = \dfrac{256}{2^{x+3}}$;

(16) $3125^{(x+1)/(x+2)} \div 15\,625^{(x+2)/(x+3)} = 0.2$;

(17) $\sqrt[x-1]{\sqrt[3]{2^{3x-1}}} - \sqrt[3x-7]{8^{x-3}} = 0$;

(18) $\sqrt{3} \cdot 3^{\frac{x}{1+\sqrt{x}}} \cdot \left(\dfrac{1}{3}\right)^{\frac{2+\sqrt{x}+x}{2(1+\sqrt{x})}} = 81$;

(19) $\sqrt{2^x \sqrt[3]{4^x \cdot 0.125^{1/x}}} = 4\sqrt[3]{2}$;

(20) $\sqrt{2} \cdot 0.5^{5/(4\sqrt{x}+10)} - 16^{1/2(\sqrt{x}+1)} = 0$;

(21) $0.2^{x^2+7x+4.5} = 5\sqrt{5}$.

2.486. Solve the following equations:

(1) $2^{x-2} = 1$; (2) $3^{3x-1} = 1$; (3) $a^{(x-2)/(x-3)} = 1$;

(4) $a^{x^2+3x+2} = 1$; (5) $0.17^{2x^2+3x-2} = 1$;

(6) $\sqrt{0.3^{4x^2-6x-4}} = 1$; (7) $\left(1\dfrac{2}{3}\right)^x \left(\dfrac{9}{25}\right)^x = 1$;

(8) $19^{3x^2+x-2} = 1$;

(9) $3^{x^2-5x+6} = 0.2^{x^2-5x+6}$.

2.487. Solve graphically:

(1) $2^x = x^2$; (2) $2^x = 4x$; (3) $(1/2)^x = 2x + 1$;

(4) $3^x = 12$; (5) $2^x = 6$; (6) $10^{4x} = 5\dfrac{3}{4}$

2.488. Solve the following equations:

(1) $3^x + 3^{x+1} = 108$; (2) $2^{x+2} - 2^x = 96$;

(3) $7^x - 7^{x-1} = 6$; (4) $2^x - 2^{x-2} = 3$;

(5) $3^{x+2} + 3^{x-1} = 28$; (6) $5^{x+1} - 5^{x-1} = 24$;

(7) $3^{2x-1} + 3^{2x-2} - 3^{2x-4} = 315$;

(8) $2^{x-1} + 2^{x-2} + 2^{x-3} = 448$;

(9) $5^x + 3 \cdot 5^{x-2} = 140$; (10) $7^{x+2} + 2 \cdot 7^{x-1} = 345$;

(11) $5 \cdot 3^{2x-1} - 9^{x-0.5} = 9^x + 4 \cdot 3^{2x+2}$;

(12) $2^{x+1} + 3 \cdot 2^{x-1} - 5 \cdot 2^x + 6 = 0$;

(13) $2^{x^2-1}-3^{x^2}=3^{x^2-1}-2^{x^2+2}$;

(14) $5^{2x-1}+2^{2x}-5^{2x}+2^{2x+2}=0$;

(15) $3\cdot 4^x+(1/3)\cdot 9^{x+2}=6\cdot 4^{x+1}-(1/2)\cdot 9^{x+1}$.

2.489. Solve the following equations:

(1) $3^{2x}-3^x=702$; (2) $7^{2x}-6\cdot 7^x+5=0$;

(3) $3^{2x}-5\cdot 3^x+6=0$; (4) $4^x+2^{x+1}=80$;

(5) $2^{2x+1}+2^{x+2}=16$; (6) $3^{x+2}+9^{x+1}-810=0$;

(7) $3^{4\sqrt{x}}-4\cdot 3^{2\sqrt{x}}+3=0$; (8) $(2/3)^x-(3/2)^x=65/36$;

(9) $4^{x+\sqrt{x^2-2}}-5\cdot 2^{x-1+\sqrt{x^2-2}}=6$;

(10) $49^{1+\sqrt{x-2}}-344\cdot 7^{\sqrt{x-2}}=-7$;

(11) $5^{1+x^2}-5^{1-x^2}=24$; (12) $3\cdot 5^{2x-1}-2\cdot 5^{x-1}=0.2$;

(13) $9^{x^2-1}-36\cdot 3^{x^2-3}+3=0$;

(14) $10^{1+x^2}-10^{1-x^2}=99$;

(15) $8^{2/x}-2^{(3x+3)/x}+12=0$;

(16) $(8^x+2^x)/(4^x-2)=5$; (17) $2\cdot 4^{2x}-17\cdot 4^x+8=0$;

(18) $3\sqrt[x]{81}-10\sqrt[x]{9}+3=0$;

(19) $(2^x+10)/4=9/2^{2x-2}$; (20) $5^{x-1}+5\cdot 0.2^{x-2}=26$.

2.490. Solve the following equations:

(1) $12^x+27^x=2\cdot 8^x$; (2) $8^x+18^x-2\cdot 27^x=0$;

(3) $4^x+10^x=2\cdot 25^x$; (4) $4\cdot 3^x-9\cdot 2^x=5\cdot 3^{x/2}\cdot 2^{x/2}$;

(5) $3^{2x+4}+45\cdot 6^x-9\cdot 2^{2x+2}=0$;

(6) $3\cdot 16^x+2\cdot 81^x=5\cdot 36^x$;

(7) $\sqrt[x]{100}+\sqrt[x]{25}=4.25\sqrt[x]{50}$;

(8) $6\cdot 4^x-13\cdot 6^x+6\cdot 9^x=0$.

2.491. Solve the following equations:

(1) $27^x-13\cdot 9^x+13\cdot 3^{x+1}-27=0$;

(2) $5^{2x-1}+5^{x+1}=250$;

(3) $3^{3x+1}-4\cdot 27^{x-1}+9^{1.5x-1}=80$;

(4) $3^2\cdot 3^5\cdot 3^8\ldots 3^{3n-1}=27^5$;

(5) $\sqrt[3]{27^{5\sqrt{x}}}=3^{x(\sqrt{x}-4)}$;

(6) $x^2\cdot 2^{-x}+16\cdot 2^{\sqrt{x+2}}=2^{4-x}+x^2\cdot 2^{\sqrt{x+2}}$;

(7) $x^2 \cdot 2^{2\sqrt{6-x}} - 16 \cdot 2^{2\sqrt{6-x}} = x^2 \cdot 2^{-2x} - 16 \cdot 2^{-2x}$;

(8) $(\sqrt{2-\sqrt{3}})^x + (\sqrt{2+\sqrt{3}})^x = 4$;

(9) $(\sqrt{7+\sqrt{48}})^x + (\sqrt{7-\sqrt{48}})^x = 14$;

(10) $5^{x/(\sqrt{x}+2)} \cdot 0.2^{4/(\sqrt{x}+2)} = 125^{x-4} 0.04^{x-2}$;

(11) $5^2 \cdot 5^4 \cdot 5^6 \ldots 5^{2x} = 0.04^{-28}$.

2.492. Solve the following equations:

(1) $(x-3)^{3x^2-10x+3} = 1$; (2) $(x-2)^{10x^2-3x-1} = 1$;

(3) $|x-3|^{3x^2-10x+3} = 1$; (4) $\sqrt[4]{(x-3)^{x+1}} = \sqrt[3]{(x-3)^{x-2}}$;

(5) $|x|^{x^2-2x} = 1$; (6) $|x-2|^{10x^2-3x-1} = 1$;

(7) $(x-3)^{x^2-x} = (x-3)^2$.

2.493. Solve the following systems of equations:

(1) $\begin{cases} 9^{x+y} = 729, \\ 3^{x-y} - 1 = 1; \end{cases}$ (2) $\begin{cases} 4^{x+y} = 128, \\ 5^{3x-2y-3} = 1: \end{cases}$ (3) $\begin{cases} 2^x + 3^y = 8\frac{1}{9}, \\ 2^x \cdot 3^y = \frac{8}{9}; \end{cases}$

(4) $\begin{cases} 5^x - 5^y = 100, \\ 5^{x-1} + 5^{y-1} = 200; \end{cases}$ (5) $\begin{cases} \sqrt[x-1]{49} = \sqrt[y-1]{343}, \\ 3^y = 9^{2-y}; \end{cases}$

(6) $\begin{cases} 2^x \cdot 3^y = 648, \\ 3^x \cdot 2^y = 432; \end{cases}$ (7) $\begin{cases} 8^{2x+1} = 32 \cdot 2^{4y-1} \\ 5 \cdot 5^{x-y} = \sqrt{25^{2y+1}}; \end{cases}$

(8) $\begin{cases} 2^{(x+y)/3} + 2^{(x+y)/6} = 6, \\ x^2 + 5y^2 = 6xy; \end{cases}$ (9) $\begin{cases} 3^{2x} - 2^y = 725, \\ 3^x - 2^{y/2} = 25; \end{cases}$

(10) $\log(x+6) - 2 = (1/2)\log(2x-3) - \log 25$;

(11) $\frac{\log(2x-5)}{\log(x^2-8)} = \frac{1}{2}$;

(12) $(1/2)\log\sqrt{x^2+x-5} = \log x + \log(1/x)$;

(13) $\log\sqrt{5x-4} - \log\sqrt{x-1} = 2 + \log 0.08$;

(14) $\log\sqrt{1-x^2} - 3\log\sqrt{1-x} = \log\sqrt{1+x} + 2$;

(15) $(1/12)\log^2 x = 1/3 - (1/4)\log x$;

(16) $\log^4(x-1) - 1.25\log^2(x-1)^2 + 4 = 0$;

(17) $\log^4(x^2+1)^2 - \log^2(x^2+1)^3 - 7 = 0$;

(18) $(4\log^2 x - 1) \cdot (\log^2 x^2 + 1) = 15$;

(19) $\log(152 + x^3) - 3\log(x+2) = 0$;

(20) $\log(5-x)-(1/3)\log(35-x^3)=0$;

(21) $\log\sqrt{8x+8}-(1/2)\log(x-13)=3\log 2$;

(22) $1-(1/2)\log(2x-1)=(1/2)\log(x-9)$;

(23) $\log(x-5)-\log(3x-20)^{1/2}=\log 2$;

(24) $(1/2)(\log x-\log 5)=\log 2-(1/2)\log(9-x)$;

(25) $(1/2)(2+\log(x+1)+\log 0.25)=\log 20-\log\sqrt{x+1}$.

2.494. Solve the following equations:

(1) $\log_3 x=2$; (2) $\log_x 2401=4$; (3) $\log_2(x^2-3x-2)=1$;

(4) $\log_x 0.125=-2$; (5) $\log_{x-1}(x^2-5+10)=2$;

(6) $\log_{x+1}(x^2+x+6)=2$;

(7) $\log_{1+x}(2x^3+2x^2-3x+1)=3$;

(8) $\log_3(3^{x^2-13x+28}+2/9)=\log_5 0.2$;

(9) $\log_x(2x^2-3x)=1$; (10) $\log_{x+2}(3x^2+4x-14)=2$;

(11) $\log_{\sqrt{2x^2-1}}(3x^4-3x^2+7)=4$.

2.495.

(1) $\log x=2-\log 5$; (2) $\log(1/2+x)=\log(1/2)-\log x$;

(3) $1-\log(x+3)=\log x$; (4) $1-\log 2=\log(6+x/2)$;

(5) $\log(x+1.5)=-\log x$; (6) $\log(x+3)=-\log 2$;

(7) $\dfrac{1}{5-\log x}+\dfrac{2}{1+\log x}=1$;

(8) $\log(3x-2)-2=(1/2)\log(x+2)-\log 50$;

(9) $\log(3x-5)-(1/2)\log(x+1)=1-\log 5$.

2.496. (a)

(1) $\log\log\log x=0$; (2) $\log_2\log_3\log_4 x=0$;

(3) $\log_7\log_2\log_{13} x=0$; (4) $\log_5\log_4\log_3 x=0$;

(5) $\log_\pi\log_2\log_7 x=0$; (6) $\log\log_2\log_4(x-15)=0$;

(7) $\log_3\log_{1/4}(2-x)^2=0$; (8) $\log\left(\log_2\log_3\sqrt{x}+1\right)=0$;

(b) (1) $x^{\log x}=10$; (2) $x^{\log x+2}=1000$;

(3) $x^{2-\log x/2}=100$; (4) $x^{3-\log(x/3)}=900$;

(5) $x^{2\log^2 x-1.5\log x}=\sqrt{10}$; (6) $x^{(\log x+7)/7}=10^{\log x+1}$;

(7) $x^{\log_2 x+4}=32$; (8) $\sqrt{x^{\log\sqrt{x}}}10$;

(9) $0.1^{-(\log(x+2)+2\log 20)}=2(x+201)$;

(10) $x^{\log^2 x-5\log x}=0.0001$, (11) $\sqrt{x^{\log_2\sqrt{x}}}=3$;

(12) $x^{\log_a x} = a^2 x$; (13) $x^{\log_4 x - 2} = 2^{3(\log_4 x - 1)}$;
(14) $x^{3\log^2 x} = 10x^2$; (15) $x^{\log x} = 1000x^2$;
(16) $2^{3\log x} \cdot 5^{\log x} = 1600$; (17) $x^{2\log^2 x} = 10x^3$;

(c) (1) $x(1 - \log 5) = \log(2^x + x - 1)$;
(2) $x(\log 5 - 1) = \log(2^x + 1) - \log 6$;
(3) $(1 + 1/(2x)) \log 3 + \log 2 = \log(27 - \sqrt[x]{3})$;
(4) $2(\log 2 - 1) + \log(5^{\sqrt{x}} + 1) = \log(5^{1-\sqrt{x}} + 5)$;
(5) $\log(4^{-1} \cdot 2^{\sqrt{x}} - 1) - 1 = \log(2^{\sqrt{x-2}} + 2) - 2\log 2$;
(6) $\log_3(3^x - 8) = 2 - x$; (7) $\log_2(9 - 2^x) = 3 - x$;
(8) $\log_2(4 \cdot 3^x - 6) - \log_2(9^x - 6) = 1$;
(9) $x + \log(1 + 2^x) = x \log 5 + \log 6$;
(10) $2x - \log(5^{2x} + 4^x - 16) = x \log 4$.

2.497. (a)

(1) $\log_{16} x + \log_4 x + \log_2 x = 7$;
(2) $\log_2 x + \log_4 x + \log_8 x = 11$;
(3) $\log_a x + \log_{a^2} x + \log_{a^3} x = 11$;
(4) $\log_a x - \log_{a^2} x + \log_{a^4} x = \frac{3}{4}$;
(5) $\log_4 \log_2 x + \log_2 \log_4 x = 2$;
(6) $\log_3 x + \log_{\sqrt{x}} x - \log_{1/3} x = 6$;

(b) (1) $3^{\log_3^2 x} + x^{\log_3 x} = 6$; (2) $3^{\log_3^2 x} + x^{\log_3 x} = 162$;
(3) $10^{\log^2 x} + x^{\log x} = 20$; (4) $6^{\log_6^2 x} + x^{\log_6 x} = 12$;

(c) (1) $\sqrt{2\log_8^{(-x)}} - \log_8 \sqrt{x^2} = 0$;
(2) $2\log x^2 - (\log(-x))^2 = 4$; (3) $4\log^2(-x) + 2\log_4 x^2 - 1$;
(4) $\log x^2 - \log^2(-x) = 9$.

2.498.

(1) $\log_x 25 - 3\log_{25} x = 1$; (2) $2\log_4 x + 2\log_x = 5$;
(3) $2\log_x 27 - 3\log_{27} x = 1$;
(4) $\log_3 x \log_9 x \log_{27} x \log_{81} x = \frac{2}{3}$;
(5) $5\log_{x/9} x + \log_{9/x} x^3 + 8\log_{9x^2} x^2 = 2$;
(6) $\log_{4x+1} 7 + \log_{9x} 7 = 0$;

(7) $\log_{x^2} 16 + \log_{2x} 64 = 3$; (8) $\log_x (125x) \log_{25}^2 x = 1$;

(9) $\log_x 2 - \log_4 x + \frac{7}{6} = 0$;

(10) $\log_{3x+7} (5x+3) + \log_{5x+3} (3x+7) = 2$.

2.499.

(1) $5^{\log x} = 50 - x^{\log 5}$;

(2) $4^{\log_3 x^2} - 4^{\log_3 x + 1} + 4^{\log_3 x - 1} - 1 = 0$;

(3) $\log (3x^2 + 12x + 19) - \log (3x+1) = 1$;

(4) $\sqrt{\log_a x} + \sqrt{\log_x a} = \frac{10}{3}$; (5) $\log_x 9 + \log_{x^2} 729 = 10$;

(6) $2.5^{\log_3 x} + 0.4^{\log_3 x} = 2.9$;

(7) $\log_a x + \log_{\sqrt{a}} x + \log_{\sqrt[3]{a^2}} x = 27$, $a > 0$, $a \neq 1$.

(8) $\log \left(625 \sqrt{5^{x^2 - 20x + 55}}\right) = 0$;

(9) $20 \log_{4x} \sqrt{x} + 7 \log_{16x} x^3 - 3 \log_{x/2} x^2 = 0$;

(10) $\frac{1 + 2 \log_9 2}{\log_9 x} - 1 = 2 \log_x 3 \log_9 (12 - x)$;

(11) $\log_{12} (4^{3x} + 3x - 9) = 3x - x \log_{12} 27$;

(12) $\log_2 \log_3 (x^2 - 16) - \log_{1/2} \log_{1/3} \frac{1}{x^2 - 16} = 2$;

(13) $|\log_2 (3x-1) - \log_2 3| = |\log_2 (5 - 2x) - 1|$;

(14) $\log_2 (4^{x-2} + 1) = \log_2 17 + x - 4$;

(15) $\log_{x+6} (2x - \sqrt{x+6}) = \frac{1}{2}$;

(16) $5^{\log x} - 3^{\log x - 1} = 3^{\log x + 1} - 5^{\log x - 1}$;

(17) $\frac{1}{2} \log (3^{\sqrt{0.5x}} + 73) + \log 10 = 2$;

(18) $\frac{1}{4} \log (9984 + 2^{\sqrt{0.5x}}) + \log 100 = 3$;

(19) $\log \sqrt{75 + 5 \sqrt[3]{x-1}} = 1$;

(20) $\log \sqrt{5^{x(13-x)}} + 11 \log 2 = 11$;

(21) $3 \cdot 4^{\log_x 2} - 46 \cdot 2^{\log_x 2 - 1} = 8$;

(22) $\log \left(64 \sqrt[24]{2^{x^2 - 40x}}\right) = 0$;

(23) $|\log_{\sqrt{3}} x - 2| - |\log_3 x - 2| = 2$;

(24) $\log 81 \sqrt[3]{3^{x^2 - 8x}} = 0$; (25) $\log \sqrt[3]{271 + 3^{\sqrt{3x}}} = 1$.

2.500. Solve the following systems of equations:

(1) $\begin{cases} x+y=34, \\ \log_2 x+\log_2 y=6; \end{cases}$ (2) $\begin{cases} \log x+\log y=5, \\ \log x-\log y=3; \end{cases}$

(3) $\begin{cases} xy=40, \\ x_{\log y}=4; \end{cases}$ (4) $\begin{cases} x^{\log y}=100, \\ xy=1000; \end{cases}$

(5) $\begin{cases} \log x+\log y=\log 11+\log 9, \\ \log (x+y)-\log (x-y)=1; \end{cases}$

(6) $\begin{cases} 2^{\sqrt{x}+\sqrt{y}}=512, \\ \log \sqrt{xy}=1+\log 2; \end{cases}$ (7) $\begin{cases} \log_y x+\log_x y=2, \\ x^2-y=20; \end{cases}$

(8) $\begin{cases} 3^y \cdot 9^x=81, \\ \log (x+y)^2-\log x=2\log 3; \end{cases}$

(9) $\begin{cases} \log_y x+\log_x y=\frac{5}{2}, \\ xy=27; \end{cases}$ (10) $\begin{cases} x-y=90, \\ \log x+\log y=3. \end{cases}$

2.501.

(1) $\begin{cases} \log_a x+\log_{a^2} y=\frac{3}{2}, \\ \log_{b^2} x+\log_b y=\frac{3}{2}; \end{cases}$ (2) $\begin{cases} 3^{\log_3 x}-2^{\log_4 y^2}=77, \\ 3^{\log_3 \sqrt{x}}-2^{\log_{16} y^2}=7; \end{cases}$

(3) $\begin{cases} \log_x 10+\log_y 10=5, \\ \log_{10} x+\log_{10} y=5/4; \end{cases}$ (4) $\begin{cases} \log^2 x+\log^2 y=10\log^2 2, \\ xy=4; \end{cases}$

(5) $\begin{cases} (x^2-y^2)^{\log (x-y)}=0.1, \\ (x^2+2xy+y^2)^{\log (x^2-y^2)}=10^4; \end{cases}$

(6) $\begin{cases} x^{2y^2-1}=5, \\ x^{y^2+2}=125; \end{cases}$ (7) $\begin{cases} x^{\log_y x}=2, \\ y^{\log_x y}=16; \end{cases}$

(8) $\begin{cases} \log_4 x-\log_2 y=0, \\ x^2-2y^2-8=0; \end{cases}$ (9) $\begin{cases} y-\log_3 x=1, \\ x^y=3^{12}; \end{cases}$

(10) $\begin{cases} \log_4 x+\log_4 y=1+\log_4 9, \\ x+y-20=0. \end{cases}$

2.502.

(1) $\begin{cases} \log (x^2+y^2)=2, \\ \log_2 x-4=\log_2 3-\log_2 y; \end{cases}$ (2) $\begin{cases} x^{x-2y}=9y^2, \\ 9y^{x-2y}=x^2; \end{cases}$

(3) $\begin{cases} \log(x^2+y^2)=1+\log 8, \\ \log(x+y)-\log(x-y)=\log 3; \end{cases}$

(4) $\begin{cases} 3^x\cdot 2^y=972, \\ \log_{\sqrt{3}}(x-y)=2; \end{cases}$ (5) $\begin{cases} \log_4 x-\log_2 y=0, \\ x^2-5y^2+4=0; \end{cases}$

(6) $\begin{cases} \log x\cdot\log(xy)=2, \\ \log(x/y)=3; \end{cases}$ (7) $\begin{cases} \log_a x+\log_a y=2, \\ \log_b x-\log_b y=4; \end{cases}$

(8) $\begin{cases} \log_{xy}(x-y)=1, \\ \log_{xy}(x+y)=0; \end{cases}$ (9) $\begin{cases} y=1+\log_4 x, \\ x^y=4^6; \end{cases}$

(10) $\begin{cases} \log(x^2+y^2)=2, \\ \log_2 x-4=\log_2 3-\log_2 y. \end{cases}$

§ 63. Exponential and Logarithmic Inequalities

2.503. Solve the following inequalities:

(a) (1) $3^x>81$; (2) $0.5^x>8$; (3) $(1/3)^x<27$;
(4) $0.5^{x^2-4x}<8$; (5) $0.25^{x^2-3x}\geqslant 4$; (6) $3^{x^2-8x+6}\leqslant\sqrt{3}$;
(7) $(1/2)^{x^2-3x+4}\leqslant(1/2)^{x+9}$; (8) $(1/2)^{x^2-4x}<1/4$;
(9) $10^{2x-4}>10^{x^2-3x}$; (10) $3^{(x-3)/(x-2)}<1/3$;
(11) $(10/\sqrt{3})^{x^2-45}\geqslant 0.81^x$; (12) $0.8^{(3-2x)/(1-x)}>16/25$;
(13) $0.2^{(x^2+2)/(x^2-1)}>25$;

(b) (1) $0.5^x>3$; (2) $2^x>3$; (3) $\sqrt[x^2]{(3/7)^{x^2-2x}}\geqslant 1$;
(4) $0.5\sqrt[x]{2}<0.00625$; (5) $0.3^{2x^2-3x+6}<0.00243$;
(6) $3^{(x-3)/(x-2)+1}>1/3$; (7) $(1/2)^{\sqrt{x^2+2x+1}}<(1/2)^{1-x}$;
(8) $\dfrac{0.2^{x-1.5}}{\sqrt{5}}<5\cdot 0.04^{x-1}$; (9) $7^{x^2-4x-2}>\dfrac{1}{49}$;
(10) $0.5^{(x^2+x-2)(3-x)}>1$.

2.504.

(1) $4^x<2^{x+1}+3$; (2) $3^{2x+5}\leqslant 3^{x+2}+2$;
(3) $2^{1/x}+4^{1/x+2}<62$; (4) $3^{2x}-6\cdot 3^x+9>0$;
(5) $2^x-0.5^x>3$; (6) $5\cdot 2^{3x}-24\cdot 2^{5-3x}+56\leqslant 0$;
(7) $4^{1/x-1}-2^{1/x-2}-3\leqslant 0$; (8) $5^{2x}-6\cdot 5^x+5<0$;

(9) $5^{2x+1} > 5^x + 4$; (10) $10 \cdot 0.3^{2x} \leqslant 1.3 \cdot 0.3^{x+1} - 0.027$;
(11) $9^x < 8 \cdot 3^x + 9$; (12) $9^{x+1} - 5 \cdot 3^{x+2} < 27$;
(13) $12^{2x} - 6 \cdot 12^x - 72 > 0$; (14) $4^{\sqrt{x+1}} - 2^{\sqrt{x+1}} + 2 < 0$;
(15) $\frac{0.2^{x-0.5}}{\sqrt{5}} < 5 \cdot 004^{x-1}$; (16) $25^x > 6 \cdot 5^x + 55$;
(17) $3^{2\sqrt{x}-1} - 10 \cdot 3^{\sqrt{x}-1} + 3 < 0$; (18) $5^{2x} + 2 \cdot 5^x - 35 > 0$.

2.505.

(1) $(1/2)^{(x^3-2x^2+1)^{1/2}} < (1/2)^{1-x}$; (2) $\frac{1}{2^x-1} > \frac{1}{1-2^{x-1}}$;
(3) $\frac{1}{3^x+5} < \frac{1}{3^{x+1}-1}$; (4) $x^3 \cdot 3^x - 3^{x+3} \leqslant 0$;
(5) $0.5^{(x^3-2x^2+1)1/2} < 0.5^{1-x}$;
(6) $|x|^{x^2-x-2} < 1$; (7) $(x-3)^{x^2-7x} > 1$;
(8) $(x^2-8x+8)^{x-0.2} < 1$; (9) $(3-x)^{(3x-5)/(3-x)} < 1$.

2.506.

(a) (1) $\log_2(3x+1) > \log_2(x-1)$;
(2) $\log_{0.5}(3x+1) \geqslant \log_{0.5}(x-1)$;
(3) $\log_5(2x-1) > \log_5(3x+2)$;
(4) $\log_{0.5}(2x+3) \leqslant \log_{0.5}(4x-1)$;
(5) $\log_{0.5}(2x+6) > \log_{0.5}(x+8)$;
(6) $\log_3(2x+1) < \log_3 5$;
(7) $\log_8(x-2) - \log_8(x-3) \leqslant 2/3$;
(8) $\log_{0.25}(x-1) + \log_{0.25}(x+1) > \log_{0.25} 3$;
(9) $\log_{0.3}(3x^2-8) < \log_{0.3}(10x-11)$;
(10) $\log_5(x-1) > \log_5(3x+2)$;
(11) $\log_5(x^2+1) < \log_5(3x-1)$; (12) $\log_{1/2}(2x+3) > 0$;

(b) (1) $\log_{0.3}(3x-8) > \log_{0.3}(x^2+4)$;
(2) $\log(6/x) > \log(x+5)$;
(3) $\log_{0.5}(x+3) < \log_{0.25}(x+15)$, $x > 1$;
(4) $\log_{1/3}(x^2+x+2) < \log_{1/3}(2x^2-18)$;
(5) $\log_4(x+7) > \log_2(x+1)$; (6) $\log_{x-1} 4 > 1$;
(7) $\log_{x+1}(x-2) < 0$; (8) $\log_{x-4}(2x^2-9x+4) > 1$;
(9) $\log_{2x}(x^2-5x+6) < 1$; (10) $\log_8(x^2-4x+3) < 1$;

(11) $\log_{1/3}(x^2-5x+7)<0$;
(12) $(x-1)\log_2(x^2-4x+3)<0$;
(13) $\log_{0.3}(1-x-2x^2)>0$; (14) $\log_{1/3}(x-4/x)>-1$;
(15) $\log_{0.5}(1/x+0.5)>0$; (16) $\log_{0.3}\left(x+\frac{2}{x-3}\right)<0$;
(17) $\log_{1/3}(x^2+5x+6)>\log_{1/3}2$.

2.507. (a)

(1) $\log_{1/3}\frac{x-1}{x+1}>-1$; (2) $\log_{1/3}\frac{x^2+4x}{2x-3}<1$;
(3) $\log_{1/3}\frac{x+1}{4x-1}<1$; (4) $\frac{1-2x}{\log_{1/3}x}>0$;
(5) $\log_2\frac{3x-1}{2-x}<0$; (6) $\log_6\frac{3}{2-x}<0$;
(7) $\frac{\log^2 x-1}{x-5}<0$; (8) $\frac{\log_2(3-x)}{\log_2(5-x)}<\frac{1}{2}$;
(9) $\log_4\frac{2x-1}{x+4}>\frac{1}{2}$; (10) $\log_{0.3}\frac{x-2}{2x+3}<0$;
(11) $\frac{\log_3(x+2)}{\log_3(x-3)}>0$; (12) $\log_{0.3}\left(x+\frac{2}{x-3}\right)>0$;
(13) $\frac{1-\log_4 x}{1-\log_2 x}\leqslant\frac{1}{2}$;

(b) (1) $\log_{(x-1)/(x+5)}0.3>0$; (2) $\log^2_{x-3}729>3$;
(3) $\log_x\frac{4x+5}{6-5x}<1$; (4) $\log_x\frac{x+3}{x-1}>1$;
(5) $\log_2\log_2\frac{x-1}{2-x}>0$;
(6) $\log_{0.3}\log_6\frac{x^2+x}{x+4}<0$;
(7) $\log_{3/4}(\sqrt{x+3}-x)>0$;
(8) $\log_2\log_4\frac{2x-1}{x+4}>-1$; (9) $\left(\frac{1}{2}\right)^{\log_{1/3}\frac{1-x}{2-x}}>1$;
(10) $0.2^{\log_{0.7}\frac{2x+1}{2x-3}}<1$; (11) $5^{\log_x\frac{8-12x}{x-6}}>25$;
(12) $\log_{|x-1|}0.5>0.5$; (13) $\frac{1}{\log_3 x}\leqslant\frac{1}{\log_3\sqrt{x+2}}$;
(14) $x^{\log_a}\geqslant a^2x$; (15) $3^{\log_3\frac{3-x}{x}}<1$.

§ 64. Sequences of Numbers. Finite and Infinite Sequences

2.508. Write the formula of the general term for the sequences of: (1) even numbers; (2) odd numbers; (3) numbers multiple of 3; (4) numbers multiple of 7; (5) numbers inverse to the number n, $n \in N$; (6) squares of natural numbers; (7) cubes of natural numbers.

2.509. Write the first five terms of the sequence whose general term is expressed by the formula:

(1) $a_n = 2n$; (2) $a_n = 5n$; (3) $a_n = n/3^n$;
(4) $a_n = 2^n + 1/2^n$; (5) $a_n = 2n - 1$; (6) $a_n = n^2$;
(7) $a_n = 1/n$; (8) $a_n = (-1)^{n-1}/n$; (9) $a_n = -n^2 + 1$;
(10) $a_n = -2^n$; (11) $a_n = n/(n+1)$; (12) $a_n = 2n^2 - n + 1$;
(13) $a_n = (-1)^n$; (14) $a_n = 2^{-n}$; (15) $a_n = (2n-1)/(3+n)$;
(16) $a_n = (2n + (-1)^n)/(3n)$; (17) $a_n = 2 + 3n$;
(18) $a_n = (\sin(\pi n/3))$; (19) $a_n = (1 + (-1)^n)/2$;
(20) $a_n = \tan \frac{\pi}{n+2}$; (21) $a_n = \frac{5^n - 1}{n}$.

2.510. Write the formula of the general term for each of the given sequences:

(1) 3; −3; 3; −3; ...; (2) 1/3; 1/9; 1/27; ...;
(3) tan 45°; tan 22° 30′; tan 11° 15′; ...;
(4) 1; −1/2; 1/4; −1/8; 1/16; ...;
(5) 1/2; 2/3; 3/4; 4/5; 5/6; ...; (6) 1; 9; 25; 49; 81; ...;
(7) $\frac{1 \cdot 3}{2 \cdot 4}$; $\frac{3 \cdot 5}{4 \cdot 6}$; $\frac{5 \cdot 7}{6 \cdot 8}$; $\frac{7 \cdot 9}{8 \cdot 10}$; ...;
(8) 1; 1·2; 1·2·3; 1·2·3·4; ...;
(9) 3/5; 7/11; 11/17; 15/23; ...;
(10) 1; 1/2; 1/6; 1/24; 1/120; 1/720; ...;
(11) 1/3; 1/4; 1/5; 1/6; 1/7; ...;
(12) $\sqrt{2}/2$; 1; $\sqrt{2}/2$; 0, $-\sqrt{2}/2$; −1; $-\sqrt{2}/2$; 0; ...;
(13) 1; 2; 1/3; 4; 1/5; 6; 1/7; 8; ...

2.511. Is the sequence of all positive roots of the given equation finite:

(1) $\sin x = -1$; (2) $\tan x = x$; (3) $\sin x = ax + b$;

(4) $|x| = x$; (5) $\sin x = \sqrt{2}/2$; (6) $\sin^2 x + \cos^2 x = 1$;
(7) $\frac{2x}{x} = 2$?

§ 65. Monotonic Sequences. Bounded and Unbounded Sequences

2.512. Determine which of the given sequences are increasing, which are decreasing, and which are unsteady:

(a) (1) 1; 3; 5; 7; . . .; $2n-1$; . . .; (2) 1; 1/2; 1/3; 1/4; . . .; $1/n$; (3) 1; 4; 9; . . .; n^2; . . .; (4) 1; -0.1; 0.01; . . .; $(-0.1)^{n-1}$; (5) 1; 1/4; 1/9; . . .; (6) 1; 2/101; 4/201; 8/301; . . .;

(b) (1) $a_n = 2n^2$; (2) $b_n = 7^{-n} - 2$;
(3) $b_n = -(1/2)^n$; (4) $b_n = (-1)^n \cdot n$.

2.513. Determine which of the given sequences are monotonic if:

(1) $y_n = \frac{2-9n}{4}$; (2) $y_n = \frac{3n+7}{2n+5}$; (3) $y_n = n^2 - 12n$;
(4) $y_n = 4n - n^2$; (5) $\frac{c_{n+1}}{c_n} < 1$; (6) $\frac{c_{n+1}}{c_n} \geqslant 1$;
(7) $\frac{c_{n+1}}{c_n} \leqslant 1$; (8) $\frac{c_{n+1}}{c_n} > 1$; (9) $a_n = n^2 + 2n$;
(10) $a_n = 1/(n^2+n)$; (11) $a_n = \cos n$;
(12) $a_n = \sqrt{n^2+n} - n$; (13) $a_n = 2n^2 - 2n$;
(14) $a_n = [10 - 2n^2]$; (15) $a_n = \sqrt[3]{n+1} - \sqrt[3]{n}$.

2.514. Prove that a sequence is increasing if the formula of its general term has the form:

(1) $a_n = 2n/(2n+1)$; (2) $a_n = 5 \cdot (3/2)^{n-1}$;
(3) $y_n = (n-1)/(2n)$; (4) $x_n = (6n-5)/(2n)$;
(5) $x_n = \frac{3n-1}{5n+2}$; (6) $x_n = 2 - 1/2^{n-1}$;
(7) $a_n = 1 + 1/2^2 + 1/3^2 + 1/4^2 + \ldots + 1/n^2$;
(8) $a_n = \left(\frac{a+1}{2a}\right)$, $a > 0$; $a \neq 1$.

2.515. Prove that a sequence is decreasing if the formula of its general term has the form:

(1) $y_n = (6-n)/(5n-1)$; (2) $a_n = 2 \cdot (3/5)^{2n-1}$;

(3) $y_n = 1/(n^2 - 2n + 3)$; (4) $u_1 = 3$; $u_{n+1} = (2 + u_n^2)/(2u_n)$.

2.516. Given an infinite sequence of numbers: 1; 4; 7; 10; ...

(a) Write the formula for the general term of this sequence;

(b) determine whether each of the following numbers is a term of this sequence, and if it is, then what ordinal number does it have: (1) 298; (2) 4891; (3) 10 536; (4) 24 850?

2.517. Find out which of the given sequences are bounded and which are unbounded:

(a) (1) 1/2; 1/3; 1/4; . . .; $1/n$; (2) 1/2; 3/4; 5/6; . . .; $(2n - 1)/(2n)$; (3) 2; -3; 4; -5; . . .; $-1^{n+1} \cdot n + 1$; . . .;

(4) $\frac{\sin 1}{1}$; $-\frac{\sin 2}{2}$; $\frac{\sin 3}{3}$; . . .; $\frac{(-1)^{n-1} \sin n}{n}$; . . .;

(b) (1) $u_n = (3n + 8)/(2n)$; (2) $a_n = (-1)^n/2$;

(3) $b_n = 2(-1)^n$; (4) $c_n = (3 + n^2)/(1 + n^2)$;

(5) $a_n = (3n + 2)/(2n - 1)$;

(6) $b_n = [(n - 2)(7 - n)]/(2n^2 + 1)$;

(7) $b_n = (n - 1)/(2 + 3n) - (n^2 + 1)/(3n^2 + 2n)$.

2.518. (a) Determine whether the sequence 1; -1; 2; -2; 3; -3; 4; -4; . . . is: (1) bounded below; (2) bounded above;

(b) determine for what values of a and b the sequence $y_n = (an + 2)/(bn + 1)$ is: increasing, decreasing, nonincreasing, nondecreasing.

§ 66. Arithmetic Progression

2.519. Write the first five terms of the arithmetic progression (a_n) if:

(1) $a_1 = 10$, $d = 4$; (2) $a_1 = 29$, $d = -5$; (3) $a_1 = 1.7$, $d = -0.2$; (4) $a_1 = m$, $d = 3m$; (5) $a_1 = 12m$, $d = -10m$; (6) $a_1 = x + 1$, $d = 3$; (7) $a_1 = x - 2$, $d = 5$; (8) $a_1 = x - 4$, $d = 6$.

2.520. Find the difference and the eighth term of the arithmetic progression (a_n) if:

(1) $a_1 = 70$, $a_2 = 100$; (2) $a_1 = -20$, $a_2 = -15$;
(3) $a_1 = 240$, $a_2 = 190$; (4) $a_1 = 1/3$, $a_2 = 1$;
(5) $a_1 = x + 1$, $a_2 = x + 3$; (6) $a_1 = 2m$, $a_2 = m$;
(7) $a_1 = m$, $a_2 = m - n$; (8) $a_1 = 36$, $a_2 = 4b + 1$.

2.521. Find the first two terms of an arithmetic progression:

(1) a_1; a_2; 7; 9; 11; . . . ; (2) a_1; a_2; $x + 5$; $x + 7$; $x + 9$; . . . ; (3) a_1; a_2; 16; 12; 8; . . . ; (4) a_1; a_2; 7; 10; 13; . . . ; (5) a_1; a_2; $7m$; $10m$; $13m$; . . .

2.522. Express the difference (d) of an arithmetic progression in terms of: (1) a_{10} and a_{11}; (2) a_6 and a_8; (3) a_{23} and a_{25}; (4) a_{12} and a_{16}; (5) a_{33} and a_{35}; (6) a_{36} and a_{38}.

2.523.
(1) $a_6 = 17$, $a_8 = 23$. Find a_5, a_7, a_{10}; (2) $a_{11} = 25$, $a_{13} = 29$. Find a_9, a_{12}, a_{14}; (3) $a_{12} = x + 23$, $a_{14} = x + 27$. Find a_8, a_{11}, a_{18}; (4) $a_3 = 4n$, $a_5 = 0$. Find a_9, a_{11}, a_{14}.

2.524. Find the nth term of the arithmetic progression (a_n) in which:
(1) $a_1 = 8, d = 5, n = 15$; (2) $a_1 = 110, d = -10, n = 11$; (3) $a_1 = -1.6, d = -0.2, n = 23$; (4) $a_1 = 4, d = -1/4, n = 15$; (5) $a_1 = x + 1, d = 3, n = 12$; (6) $a_1 = 2x + 7, d = 2, n = 14$; (7) $a_1 = 3, d = -m, n = 10$; (8) $a_1 = 3m + 1, d = -m, n = 11$.

2.525. In an arithmetic progression find a_{11} if:
(1) $a_{10} = 20, d = 2$; (2) $a_{100} = 0, d = -3$; (3) $a_{29} = 11, d = 0.2$; (4) $a_{31} = 11, d = 0.5$; (5) $a_{41} = -17, d = 1/5$; (6) $a_{51} = 16.4, d = 0.4$.

2.526. Find the first term and difference of an arithmetic progression if:

(1) $a_5 = 19$, $a_{27} = 107$; (2) $a_6 = 6.2$, $a_{31} = 21.2$;
(3) $a_{14} = 26.5$, $a_{27} = 52.5$; (4) $a_{20} = 0$, $a_{66} = -92$;
(5) $a_8 = 1$, $a_{25} = 9.5$; (6) $a_{11} = 26$, $a_{41} = 44$.

2.527. (1) Between the numbers 7 and 35, write six numbers to form an arithmetic progression.

(2) Between the numbers 1 and 25, write five numbers to form an arithmetic progression.
(3) Between the numbers 2 and 14; 4 and 26, write five numbers (between each pair) which are arithmetic means with the given numbers.
(4) Between the numbers a and b, write m numbers to form an arithmetic progression. Write the first three terms of this progression.

2.528. Find the first negative term of the arithmetic progression 5.3; 5.12, . . .

2.529. Find the first term and difference of an arithmetic progression if:

(1) $\begin{cases} a_1 + a_7 = 22, \\ a_{10} - a_3 = 21; \end{cases}$ (2) $\begin{cases} a_5 + a_{11} = 42, \\ a_4 + a_{10} = 38; \end{cases}$

(3) $\begin{cases} a_2 + a_5 - a_3 = 10, \\ a_1 + a_6 = 17; \end{cases}$ (4) $\begin{cases} a_1 + a_6 - a_5 = 10, \\ 2a_1 + a_5 = 34; \end{cases}$

(5) $\begin{cases} a_1 + a_5 = 24, \\ a_2 \cdot a_3 = 60; \end{cases}$ (6) $\begin{cases} a_1 + a_5 = 5/3, \\ a_3 \cdot a_4 = 65/72; \end{cases}$

(7) $\begin{cases} a_7 - a_3 = 8, \\ a_2 \cdot a_7 = 75; \end{cases}$ (8) $\begin{cases} a_4 \div a_6 = -1, \\ a_2 \cdot a_8 = -1. \end{cases}$

2.530. Find a_1 and d of an arithmetic progression if:

(1) $\begin{cases} a_1 + a_3 + a_5 = -12, \\ a_1 \cdot a_3 \cdot a_5 = 80; \end{cases}$ (2) $\begin{cases} a_1 + a_2 + a_3 = 66, \\ a_2 \cdot a_3 = 528; \end{cases}$

(3) $\begin{cases} a_{17} + a_8 = 27, \\ a_2 \cdot a_7 = 24; \end{cases}$ (4) $\begin{cases} a_4^2 + a_{12}^2 = 1170, \\ a_7 + a_{15} = 60; \end{cases}$

(5) $\begin{cases} S_2 - S_4 + a_2 = 14, \\ S_3 + a_3 = 17; \end{cases}$ (6) $\begin{cases} 5a_1 + 10a_5 = 0, \\ S_4 = 14; \end{cases}$

(7) $\begin{cases} a_2 + a_5 - a_3 = 10, \\ a_1 \cdot d = 3; \end{cases}$ (8) $\begin{cases} 5a_1 + 10a_5 = 0, \\ a_1 \cdot a_4 = 8. \end{cases}$

2.531. (a) Determine the sum of all the natural numbers: (1) from 1 to 50; (2) from 1 to 100; (3) from 1 to 120.

(b) Find the sum of all the even natural numbers up to 100 inclusive.

(c) Determine the sum of all the odd natural numbers from 15 to 83.

(d) Find the sum of all the three-digit numbers which are divisible by 7.

(e) Find the sum of all the even three-digit natural numbers which are divisible by 3.

2.532. Determine the sum and difference of the terms of the arithmetic progression in which:

(1) $a_1 = 5$, $a_n = 105$, $n = 26$; (2) $a_1 = -10$, $a_n = -20$, $n = 6$; (3) $a_1 = \frac{3}{4}$, $a_n = 3\frac{7}{8}$, $n = 26$; (4) $a_1 = a$, $a_n = 9a + 8b$, $n = 9$.

2.533. Determine the first term and the sum of the terms of the arithmetic progression in which:

(1) $d = 10$, $n = 45$, $a_n = 459$; (2) $d = 2$, $n = 15$, $a_n = -10$; (3) $d = -1/4$, $n = 13$, $a_n = 1$; (4) $d = 1 + q$, $n = 28$, $a_n = 28 + 27q$.

2.534. Determine the number of terms and the sum of terms of the arithmetic progression in which:

(1) $a_1 = 0$, $d = 1/2$, $a_n = 5$; (2) $a_1 = -37\frac{1}{2}$, $d = 4$, $a_n = 46\frac{1}{2}$; (3) $a_1 = 14.5$, $d = 0.7$, $a_n = 32$;
(4) $a_1 = -4.5$, $d = 5.5$, $a_n = 100$.

2.535. Find d_1 and a_1 if:

(1) $a_4 = 10$, $a_7 = 19$;
(2) $a_8 = 17$, $a_6 = 13$; (3) $a_7 = 23$, $a_9 = 29$;
(4) $a_5 = 8$, $a_{10} = -12$.

2.536. Find the first three terms and the difference of an arithmetic progression if the sum of its terms is expressed by the formula:

(1) $S_n = 5n^2$; (2) $S_n = 4n^2 - 3n$;
(3) $S_n = (3n^2 - 3n) \cdot 0.5$; (4) $S_n = 5n^2 + 3n$;
(5) $S_n = 3n^2$; (6) $S_n = 7n^2 - 5n$;
(7) $S_n = (n^2 + 3n + 2)/6$; (8) $S_n = mn^2$;
(9) $S_n = 2n^2 - 3n$.

2.537. Solve the following equations:

(1) $1 + 4 + 7 + \ldots + n = 117$;

(2) $1 + 7 + 13 + \ldots + n = 280$;

(3) $(n + 1) + (n + 4) + (n + 7) + \ldots + (n + 28) = 155$;

(4) $2 + 4 + 6 + \ldots + 2n = 56$;

(5) $(1+3+5+\ldots+(2n+1))$
$+\left(3.5+5+6.5+\ldots+\left(\frac{7+3n}{2}\right)\right)=105$;

(6) $(3+6+9+\ldots+3(n-1))$
$+\left(4+5.5+7+\ldots+\frac{8+3n}{2}\right)=137.$

2.538. Find the arithmetic progression consisting of natural numbers if the products of its first three numbers and its first four numbers are equal to 6 and 24, respectively.

2.539. (1) Find the sum of the first nineteen terms of the arithmetic progression a_1, a_2, a_3, ... if it is known that $a_4 + a_8 + a_{12} + a_{16} = 224$.

(2) The sum of the third and ninth terms of an arithmetic progression is equal to 8. Find the sum of the first eleven terms of the arithmetic progression.

2.540. (1) The sum of three numbers forming an arithmetic progression is equal to 2, and the sum of the squares of the same numbers equals $1\frac{5}{9}$. Find these numbers.

(2) The first term of an arithmetic progression is equal to 429, its difference being equal to -22. Find the number of terms of this progression if their sum is equal to 3069.

(3) Find the sum of twenty terms of the arithmetic progression a_1, a_2, a_3, ... if $a_6 + a_9 + a_{12} + a_{15} = 20$.

(4) Find the arithmetic progression a_1, a_2, a_3, ... if $a_1 + a_2 + a_3 = 9$ and $a_1 \cdot a_2 \cdot a_3 = 15$.

2.541. Solve the following equations:

(1) $0.3^{2+4+6+\ldots+2x} = 0.3^{72}$; (2) $3^3 \cdot 3^5 \cdot 3^8 \cdot \ldots \cdot 3^{2n-1} = 27^5$;

(3) $3^{\log_3 x+\log_3 x^2+\ldots+\log_3 x^8+\log_3 x^8} = 27x^{30}$;

(4) $\log_{10} x + \log_{\sqrt{10}} x + \log_{\sqrt[3]{10}} x + \ldots + \log_{\sqrt[10]{10}} x = 5.5$;

(5) $\log_{\sqrt{3}} x + \log_{\sqrt[4]{3}} x + \log_{\sqrt[6]{3}} x + \ldots + \log_{\sqrt[16]{3}} x = 36$;

(6) $2^2 \cdot 2^4 \cdot 2^6 \cdot \ldots \cdot 2^{2n} = 0.25^{-28}$;

(7) $\log_a x + \log_{\sqrt{a}} x + \ldots + \log_{\sqrt[n]{a}} x = \frac{a+1}{2}$.

2.542. (1) Prove that any term of an arithmetic progression, beginning with the second one, is an arithmetic mean between any two terms equidistant from it.
(2) Prove that if a, b, and c are three successive terms of an arithmetic progression, then $a^2 + 8bc = (2b + c)^2$.
(3) Given $\frac{S_m}{S_n} = \frac{m^2}{n^2}$. Prove that $\frac{a_m}{a_n} = \frac{2m-1}{2n-1}$.
(4) Prove that if a, b, and c form an arithmetic progression, then the numbers $a^2 + ab + b^2$, $a^2 + ac + c^2$, and $b^2 + bc + c^2$ also form an arithmetic progression.
(5) Prove that if the numbers $\frac{1}{b+c}$; $\frac{1}{c+a}$; $\frac{1}{b+a}$ form an arithmetic progression, then the numbers a^2, b^2, and c^2 also form an arithmetic progression.

2.543. Prove that if in an arithmetic progression $S_m = S_n$, then $S_{m+n} = 0$.

2.544. Prove that for the terms of an arithmetic progression the following equalities are valid:

(1) $a_p + a_m = a_{p+k} + a_{m-k}$;

(2) $a_m = \frac{a_{m+k} + a_{m-k}}{2}$,

where $p = 1; 2; 3; \ldots$; $m = 2; 3; 4; \ldots$; $k = 1; 2; \ldots; -1$.
(3) The numbers $a_1, a_2, \ldots, a_n, a_{n+1}$ form an arithmetic progression. Prove that $\frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} + \ldots + \frac{1}{a_n a_{n+1}} = \frac{n}{a_1 a_{n+1}}$.

§ 67. Geometric Progression

2.545. Find the fifth term of the geometric progression: (1) 6; 18; 54; . . . ; (2) 256; 128; 64; . . . ; (3) 96; −48; 24; . . . ; (4) 2; 1; 1/2; . . . ; (5) $4\sqrt{2}$; −4; $2\sqrt{2}$; . . . ; (6) 0.003; $0.003\sqrt{10}$; 0.03, . . . ; (7) 0.125; $-0.125\sqrt{2}$; 0.25, . . .

2.546. Find the common ratio of the geometric progression:

(1) $b_1 = 5$, $b_9 = 1280$; (2) $b_1 = 2/3$, $b_6 = -5\frac{1}{16}$;

(3) $b_6 = 25$, $b_8 = 9$; (4) $b_4 = -12$, $b_7 = 23\frac{7}{16}$;

(5) $b_n = \sqrt{3/2}$, $b_{n+1} = \sqrt{2/3}$; (6) $b_2 = 1/2$, $b_4 = 9/32$.

2.547. Find the first term of the geometric progression in which:

(1) $b_8 = 384$, $q = 2$; (2) $b_9 = 4/9$, $q = -1/3$; (3) $b_6 = 32/81$, $q = -2/3$; (4) $b_8 = 256$, $q = 4$; (5) $b_7 = 1024$, $q = 4$; (6) $b_5 = 1\frac{115}{128}$, $q = \frac{3}{4}$.

2.548. Express the ratio q of a geometric progression in terms of:

(1) b_1 and b_3; (2) b_4 and b_8; (3) b_5 and b_9; (4) b_8 and b_{11}; (5) b_k and b_{k+3}.

2.549. Find the sum of the terms of the geometric progression in which:

(1) $b_1 = 3$, $q = 3$, $n = 6$; (2) $b_1 = 1/2$, $q = -1/3$, $n = 6$; (3) $b_1 = 2.5$, $q = 1.5$, $n = 5$; (4) $b_1 = 8$, $q = 1/2$, $n = 5$.

2.550. Determine the first term and the sum of the terms of a geometric progression in which:
(1) $q = 4$, $n = 7$, $b_7 = 1024$; (2) $q = 1/2$, $n = 10$, $b_{10} = 7$; (3) $q = 3/4$, $n = 5$, $b_5 = 1\frac{115}{128}$; (4) $q = 2\frac{1}{2}$, $n = 6$, $b_6 = 3125$.

2.551. (a) Find the number of terms and the sum of the terms of the geometric progression in which:

(1) $b_1 = 9$, $b_n = 32/27$, $q = 2/3$; (2) $b_1 = 3/8$, $b_n = 96$ $q = -4$; (3) $b_1 = 1/3$, $b_n = 1/6561$, $q = 1/3$; (4) $b_1 = 5$ $b_n = 405$, $q = 3$.

(b) Find the number of the geometric progression in which:

(1) $b_1 = 3$, $b_n = 96$, $S_n = 189$; (2) $b_1 = 1$, $b_n = -512$, $S_n = 341$; (3) $b_1 = 2$, $b_n = \frac{1}{8}$, $S_n = 3\frac{7}{8}$; (4) $q = 2$, $b_n = 96$, $S_n = 189$.

(c) Find the first and the last terms of the geometric progression in which:

(1) $n = 8$, $q = 2$, $S_8 = 765$; (2) $n = 5$, $q = \frac{1}{2}$, $S_5 = 3\frac{7}{8}$;

(3) $n = 4$, $q = \frac{2}{3}$, $S_4 = 65$; (4) $n = 12$, $q = 2$, $S_{12} = 4095$.

2.552. (1) Between the numbers 9 and 243, write two numbers which, together with the given numbers, form a geometric progression.
(2) Between the numbers 160 and 5, write four numbers which, together with the given numbers, form a geometric progression.
(3) Between the numbers 1 and 7, write six numbers which together with the given numbers, form a geometric progression.
(4) Between the numbers 1 and 16, insert three numbers such that, together with the given numbers, they form a geometric progression.
(5) Between the numbers 60 and 15/16, insert five numbers such that, together with the given numbers, they form a geometric progression.
(6) Between the numbers 47 and 1269, insert two numbers which, together with the given numbers, will form a geometric progression.
(7) Between the numbers 31 and 496, insert three numbers such that, together with the given numbers, they form a geometric progression.
(8) Between the numbers a/b^2 and b/a^2, insert five numbers such that, together with the given numbers, they form a geometric progression.

2.553. (1) Find the ordinal number of the term of the geometric progression 0.1, 0.3, . . ., which is equal to 218.7.
(2) Find the ordinal number of the term of the geometric progression 1280, 640, . . ., which equals 20.

2.554. Find the first term and common ratio of the geometric progression in which:

(1) $b_5 - b_1 = 15$; $b_4 - b_2 = 6$; (2) $b_2 + b_5 - b_4 = 10$; $b_3 + b_6 - b_5 = 20$; (3) $b_7 - b_5 = 48$; $b_6 + b_5 = 48$; $S_n = 1023$; (4) $b_6 - b_4 = 216$; $b_3 - b_1 = 8$; $S_n = 40$.

2.555. Find the geometric progression which consists of 6 terms if the sum of its first three terms is equal to 168, and the sum of the last three terms is equal to 21.

2.556. Find four numbers forming a geometric progression in which the second term is less than the first by 35 and the third term is less than the fourth by 560.

2.557. (1) Find four numbers forming a geometric progression in which the sum of the extremes is equal to —49, and the sum of the means equals 14.

(2) Find four numbers forming a geometric progression in which the sum of the extremes is equal to 35, and the sum of the means equals 30.

(3) Find four numbers forming a decreasing geometric progression in which the sum of the extremes is equal to 27, and the sum of the means to 18.

(4) Find four numbers forming a geometric progression in which the sum of the extremes is equal to 27, and the sum of the means equals to 72.

2.558. (1) Find the first and fifth terms of a geometric progression if its ratio is equal to 3, and the sum of the first six terms is equal to 1820.

(2) Find four numbers which form a geometric progression if their sum is equal to —40, and the sum of their squares is equal to 3280.

2.559. (1) The product of the first three terms of a geometric progression is equal to 1728, their sum being equal to 63. Find the first term and common ratio of this progression.

(2) Find three numbers which form a geometric progression if their sum is equal to 126, and their product equals 13 824.

2.560. Find three numbers forming a geometric progression if their product is equal to 64, and the arithmetic mean equals 14/3.

2.561. Find the sum of an infinitely decreasing geometric progression if:

(1) $1\frac{1}{2}; \frac{3}{4}; \frac{3}{8}; \ldots$; (2) $3\sqrt{2}; \sqrt{2}; \frac{\sqrt{2}}{3}; \ldots$;

(3) $\sqrt{3/2}; 2/3; 2/3\sqrt{2/3}; \ldots$;

(4) $\sqrt{5}; \sqrt{1/5}; (1/25)\sqrt{5}; \ldots$;

(5) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$; 1; $\frac{\sqrt{3}-1}{\sqrt{3}+1}$; ...;

(6) 1; x; x^2; ...; for $|x|<1$; (7) $\frac{15}{64}(3-\sqrt{3})$;

$\frac{15}{64}(\sqrt{3}-1)$; ...; (8) $\frac{\sqrt{2}+1}{\sqrt{2}-1}$; $\frac{1}{2-\sqrt{2}}$; $\frac{1}{2}$; ...;

(9) $\sqrt{3}$; $\frac{354}{33+121\sqrt{3}}$; ...

2.562. The sum of an infinitely decreasing geometric progression is equal to 4, the sum of the cubes of its terms being equal to 64/7. Find b_1 and q.

2.563. The sum of an infinitely decreasing geometric progression is equal to 4, and the sum of the cubes of its terms equals 192. Find the first term and common ratio.

2.564. Find the common ratio of an infinitely decreasing geometric progression in which each term is four times the sum of all subsequent terms.

2.565. In an infinitely decreasing geometric progression with positive terms the sum of the first three terms is 10.5, the sum of the progression being equal to 12. Find the progression.

2.566. Find the first term of an infinitely decreasing geometric progression in which:

(1) the sum of the four terms is equal to $33\frac{3}{4}$, the sum of the progression being equal to 36;
(2) the sum of the progression is equal to 729, and the second term to 162;
(3) the sum of the progression is equal to 14.4, the ratio of the progression being equal to 3/8;
(4) the sum of the progression is equal to 10.8, the ratio of the progression being equal to 2/9.

2.567. (a) Solve the equation:

$$1 + a + a^2 + \ldots + a^x = (1 + a)(1 + a^2)(1 + a^4)(1 + a^8).$$

(b) Compute the sum:

(1) $S_n = a + 2a^2 + 3a^3 + 4a^4 + \ldots + na^n$;

(2) $S_n = 1 + 2x + 3x^2 + \ldots + (n + 1)x^n$.

2.568. Solve the following equations:

(1) $1/x + x + x^2 + \ldots + x^n + \ldots = 7/2$ if $|x| < 1$;

(2) $2x + 1 + x^2 - x^3 + x^4 - x^5 + \ldots = 13/6$ if $|x| < 1$;

(3) $(3\,(1 - 1/2 + 1/4 - 1/8 + \ldots))^{\log_a x} =$
$(20\,(1 - 1/4 + 1/16 - 1/64 + \ldots))^{\log_{xa}}$.

2.569. Find four numbers which form a geometric progression if the first number is greater than the second by 36, and the third number exceeds the fourth one by 4.

2.570. The sum of the first three ter ms of a geometric progression is equal to 91. If we add 25 to the first term, 27 to the second and 1 to the third, thenw e get three numbers which form an arithmetic progression. Find the seventh term of the given geometric progression.

2.571. Three numbers form a geometric progression. If the second number is increased by 2, then the given progression will become an arithmetic one. If the third term of the obtained arithmetic progression is increased by 9, then the progression will again become geometric. Find these numbers.

2.572. Find four numbers the first three of which form a geometric progression, and the last three an arithmetic progression. The sum of the extremes equals 21, the sum of the means being 18.

2.573. Three numbers form a geometric progression, the third one being equal to 12. If we replace 12 by 9, then the three numbers will form an arithmetic progression. Find these numbers.

2.574. (a) Prove that a sequence of numbers forms a geometric progression if its general term is expressed by the formula:

(1) $b_n = 3/5 \cdot 2^n$; (2) $b_n = 5/2^n$; (3) $b_n = (-1/2)^n$.

(b) The sum of the first n terms of the geometric progression (b_n) can be computed by the formula $S_n = 2\,(5^n - 1)$. Find S_4, b_1, and b_4.

(c) The sum of the first n terms of the geometric progression can be computed by the formula $S_n = 3.5\,(4^n - 1)$. Find b_1, q, S_5.

(d) Prove that if the sum of the first n terms of the sequence (b_n) can be found by the formula $S_n = 3^n - 1$, then (b_n) is a geometric progression.

(e) Is the sequence the sum of whose first n terms can be found by the formula: (1) $S_n = n^2 - 1$; (2) $S_n = 2^n - 1$; (3) $S_n = 3^n + 1$ a geometric progression?

2.575. (1) Find the sum of n terms of the series if:

(a) $1 + 3/2 + 5/4 + 9/8 + \ldots$; (b) $3 + 33 + 333 + \ldots$; (c) $7 + 77 + 777 + \ldots$; (d) $9 + 99 + 999 + \ldots$

(2) Prove that if three terms a, b, c form a geometric progression, then $(a + b + c)(a - b + c) = a^2 + b^2 + c^2$.
(3) The sum of terms of an infinitely decreasing geometric progression is equal to 56, and the sum of the squares of its terms is equal to 448. Find b_1 and q.
(4) Find the sum of the infinitely decreasing geometric progression $1/\sqrt{2}$, $1/(1 + \sqrt{2})$, $\ldots$
(5) Find the fourth term of the geometric progression in which the sum of the first three odd terms is equal to 84, and the sum of the first three even terms is equal to 168.
(6) Solve the equation:

$$2^{x-1} + 2^{x-4} + 2^{x-2} = 6.5 + 3.25 + 1.625 + \ldots$$

§ 68. The Limit of an Infinite Sequence of Numbers

2.576. By the definition of the limit of a sequence of numbers, prove the validity of the following equalities:

(a) (1) $\lim\limits_{n\to\infty} \left(\frac{1}{2}\right)^n = 0$; (2) $\lim\limits_{n\to\infty} \frac{n}{2n-1} = \frac{1}{2}$;

(3) $\lim\limits_{n\to\infty} \frac{n-3}{n} = 1$; (4) $\lim\limits_{n\to\infty} \frac{(1/2)\,n-2}{n+1} = \frac{1}{2}$;

(b) (1) $\lim\limits_{n\to\infty} \frac{3n-1}{n} = 3$; (2) $\lim\limits_{n\to\infty} \frac{5}{n^2} = 0$;

(3) $\lim\limits_{n\to\infty} \frac{1-n}{n} = -1$; (4) $\lim\limits_{n\to\infty} \frac{1-(3/4)\,n}{n-2} = -\frac{3}{4}$;

(c) (1) $\lim\limits_{n\to\infty} \frac{n}{1+n^2} = 0$; (2) $\lim\limits_{n\to\infty} \frac{5n}{2n-n^3} = 0$;

(3) $\lim\limits_{n\to\infty} \frac{10n}{n-2n^2} = 0$;

(d) (1) $\lim\limits_{n\to\infty}\frac{n}{n-2}=1$; (2) $\lim\limits_{n\to\infty}\frac{n-1}{2n}=\frac{1}{2}$;

(3) $\lim\limits_{n\to\infty}\frac{5n^2-3}{4n^2+1}=\frac{5}{4}$; (4) $\lim\limits_{n\to\infty}\frac{(3/5)\,n^2}{1+n^2}=\frac{3}{5}$;

(e) (1) $\lim\limits_{n\to\infty}\frac{n+1}{n}=1$; (2) $\lim\limits_{n\to\infty}\frac{1}{n^3+1}=0$;

(3) $\lim\limits_{n\to\infty}\frac{3n-2}{2n}=1.5$.

2.577. What limit has the given sequence? Prove:

(1) $a_n=(5n-1)/(2n)$; (2) $b_n=(3n^2+1)/n^2$;
(3) $x_n=(6n-3)/(3n)$; (4) $x_n=2+(-1)^n/n$;
(5) $x_n=(-1/3)^n$; (6) $a_n=(2n+7)/n$.

2.578. Which of the given sequences are convergent and which are divergent? Give the proof:

(1) $x_n=2n-1$; (2) $x_n=(-1)^n/(2n)$; (3) $x_n=(-1)^n+1$; (4) $x_n=(-1)^n/(3n+1)$; (5) $x_n=(2n+11)/(2n)$; (6) $x_n=n^2$; (7) $x_n=1/(2n-1)$; (8) $x_n=(-1)^n\cdot n$.

2.579. Evaluate the following limits:

(a) (1) $\lim\limits_{n\to\infty}\frac{5}{n+1}$; (2) $\lim\limits_{n\to\infty}\frac{n^2-3n+4}{n+4}$;

(3) $\lim\limits_{n\to\infty}\frac{2n^2+6n-1}{5n+3}$; (4) $\lim\limits_{n\to\infty}\left(\frac{2n-1}{3n+1}-\frac{2n^2+3}{3n^2-1}\right)$;

(b) (1) $\lim\limits_{n\to\infty}\frac{n^4+3n^2}{n^5+n^3+2n^2}$; (2) $\lim\limits_{n\to\infty}\frac{n^2-4}{n-2}$;

(3) $\lim\limits_{n\to\infty}\frac{n^2-1}{n+1}$; (4) $\lim\limits_{n\to\infty}\left(\frac{2n^2+5n-1}{3n^3-2n^2}+\frac{3+5n}{3n-1}\right)$;

(c) (1) $\lim\limits_{n\to\infty}\frac{n^2-5n+4}{n^2+n-2}$; (2) $\lim\limits_{n\to\infty}\frac{n^2+2n-15}{n^2-9}$;

(3) $\lim\limits_{n\to\infty}\frac{n^3-a^3}{n^2-a^2}$; (4) $\lim\limits_{n\to\infty}\left(\frac{3n}{5n-1}\cdot\frac{n^2+1}{n^2+2n-1}\right)$;

(d) (1) $\lim\limits_{n\to\infty}\frac{3n+1}{n+1}$; (2) $\lim\limits_{n\to\infty}\frac{5n^2-1}{4n^2+3}$;

(3) $\lim\limits_{n\to\infty}\frac{an^3+b}{cn^3+d}$; (4) $\lim\limits_{n\to\infty}\frac{2-3n+n^2}{3n^2+2}\cdot\frac{4-2n^4+3n^9}{n^9-2n+0.7}$;

(e) (1) $\lim\limits_{n\to\infty}\dfrac{3n^2+2n-5}{7n^2-n+3}$; (2) $\lim\limits_{n\to\infty}\dfrac{-3-n}{n^2+2n-1}$;

(3) $\lim\limits_{n\to\infty}\dfrac{5n+1}{3-2n}$; (4) $\lim\limits_{n\to\infty}\dfrac{2\sqrt{2n^4-n^3+7n}}{1-3n-2n^2}$;

(f) (1) $\lim\limits_{n\to\infty}\dfrac{2n-9}{3n^2+n-1}$; (2) $\lim\limits_{n\to\infty}\dfrac{3-n+2n^2}{2+n-4n^2}$;

(3) $\lim\limits_{n\to\infty}\dfrac{(n+3)(n+5)}{n^2+1}$; (4) $\lim\limits_{n\to\infty}\dfrac{2\sqrt{1-n^2+n^4}}{2n-1+3n^2}$.

2.580. Evaluate the following limits:

(1) $\lim\limits_{n\to\infty}\dfrac{1+1/2+1/4+\ldots+1/2^n}{1+1/3+1/9+\ldots+1/3^n}$;

(2) $\lim\limits_{n\to\infty}(1/n^2)(1+2+3+\ldots+n)$;

(3) $\lim\limits_{n\to\infty}\left(\dfrac{1+2+3+\ldots+n}{n+2}-\dfrac{n}{2}\right)$;

(4) $\lim\limits_{n\to\infty}\left(\dfrac{1+3+5+\ldots+2n+1}{n+1}-\dfrac{2n+1}{2}\right)$.

§ 69. The Limit of a Function

2.581. Using the definition of the limit and conditions for the limit of a function to exist, prove:

(1) $\lim\limits_{x\to 2}(5x+3)=13$; (2) $\lim\limits_{x\to -1}\dfrac{2x+1}{3+x}=-\dfrac{1}{2}$;

(3) $\lim\limits_{x\to 5}(x^2+2)=27$; (4) $\lim\limits_{x\to\infty}\dfrac{x+1}{x-1}=1$;

(5) $\lim\limits_{x\to+\infty}\dfrac{x-1}{3x+2}=\dfrac{1}{3}$; (6) $\lim\limits_{x\to 0.5}\dfrac{x^2-0.25}{x-0.5}=1$;

(7) $\lim\limits_{x\to\infty}\left(2+\dfrac{1}{x}\right)=2$; (8) $\lim\limits_{x\to\infty}\dfrac{x+3}{2x}=\dfrac{1}{2}$;

(9) $\lim\limits_{x\to\infty}\left(\dfrac{x+2}{x}+1\right)=2$.

2.582. Evaluate the limits of the given functions:

(1) $\lim\limits_{x\to -1}(x^3+2x^2-3x-4)$; (2) $\lim\limits_{x\to 1}\dfrac{3x^2+2x+1}{x^3-2x^2+3x-1}$;

(3) $\lim\limits_{x\to -2}\dfrac{x^4+2}{3x^3-1}$; (4) $\lim\limits_{x\to 1}(4x^2-2x+1)$;

(5) $\lim\limits_{x\to -1}\dfrac{5x^3+4x^2+3x+2}{x^3+x^2-2x}$;

(6) $\lim\limits_{x\to -2}\dfrac{x^2+3x+2}{2x^2+x-6}$; (7) $\lim\limits_{x\to -1}\dfrac{3x^2+2x-1}{2x^2+x-1}$;

(8) $\lim\limits_{x\to 0}\dfrac{\sqrt{x+1}-1}{x}$; (9) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-\sqrt{1-x}}{4x}$;

(10) $\lim\limits_{x\to 0}\dfrac{\sqrt{a+x}+\sqrt{a-x}}{x}$; (11) $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{1+x^2}-1}{x}$;

(12) $\lim\limits_{x\to 5}\dfrac{\sqrt{x-1}-2}{x-5}$; (13) $\lim\limits_{x\to -3}\dfrac{x+3}{\sqrt{x+4}-1}$;

(14) $\lim\limits_{x\to -1}\dfrac{x^3+1}{x+1}$; (15) $\lim\limits_{x\to 2}\dfrac{x^3-8}{x-2}$;

(16) $\lim\limits_{x\to 0}\dfrac{\sqrt{x}-x}{\sqrt{x}+x}$; (17) $\lim\limits_{x\to -3}\dfrac{x+3}{\sqrt{x+4}-1}$;

(18) $\lim\limits_{x\to 8}\dfrac{\sqrt[3]{x}-2}{x-8}$; (19) $\lim\limits_{x\to 1}\dfrac{x-1}{\sqrt[3]{x}-1}$.

2.583. Evaluate the following limits:

(1) $\lim\limits_{x\to 0}\dfrac{x^3-1}{2x^2-x-1}$; (2) $\lim\limits_{x\to 1}\dfrac{x^2-3x+2}{x^2-4x+3}$;

(3) $\lim\limits_{x\to -2}\dfrac{3x^2+5x-2}{5x^2+12x+4}$; (4) $\lim\limits_{x\to 1/2}\dfrac{2x^2+5x-3}{x^2-6x-7}$;

(5) $\lim\limits_{x\to -2}\dfrac{2+x}{1-\sqrt{x+3}}$; (6) $\lim\limits_{x\to 0}\dfrac{\sqrt{a+x}-\sqrt{a}}{x}$;

(7) $\lim\limits_{x\to 1}\dfrac{x-1}{1-\sqrt{x}}$; (8) $\lim\limits_{x\to 16}\dfrac{\sqrt[4]{x}-2}{\sqrt{x}-4}$;

(9) $\lim\limits_{x\to 0}\dfrac{2\sqrt{x}-3x}{3\sqrt{x}-2x}$; (10) $\lim\limits_{x\to 0}\dfrac{x^2}{\sqrt{x^2+4}-2}$;

(11) $\lim\limits_{x\to 3}\dfrac{x-3}{\sqrt{2x+10}-4}$; (12) $\lim\limits_{x\to 1}\dfrac{\sqrt{x+3}-2}{x-1}$;

(13) $\lim\limits_{x\to 0} \dfrac{\sqrt{2}-\sqrt{2-x}}{2x}$; (14) $\lim\limits_{x\to 0} \dfrac{\sqrt{a+x}-\sqrt{a-x}}{x}$;

(15) $\lim\limits_{x\to 0} \dfrac{\sqrt[3]{1+x}-\sqrt[3]{1-x}}{x}$;

(16) $\lim\limits_{x\to -1} \dfrac{x^3+x^2-x-1}{x^3+x^2+x+1}$;

(17) $\lim\limits_{x\to 0.4} \dfrac{5x^3-2x^2+5x-2}{5x^4-2x^3-5x^2+2x}$;

(18) $\lim\limits_{x\to 4} \dfrac{x+\sqrt{x}-6}{x-5\sqrt{x}+6}$; (19) $\lim\limits_{x\to 0} \dfrac{\sqrt{1+3x^4}-\sqrt{1-2x}}{x+x^2+2x^3}$;

(20) $\lim\limits_{x\to 1} \dfrac{\sqrt[3]{x}-1}{\sqrt[3]{x^2}+2\sqrt[3]{x}-3}$;

(21) $\lim\limits_{x\to -1} \dfrac{\sqrt{2x+3}-1}{\sqrt{5+x}-2}$; (22) $\lim\limits_{x\to 0} \dfrac{1-\sqrt{2x+1}}{\sin x}$;

(23) $\lim\limits_{x\to \pi/4} \dfrac{\sin x+\sin 5x}{\cos x+\cos 5x}$;

(24) $\lim\limits_{x\to 0} \dfrac{(x^2+3x-1)\tan x}{x^2+2x}$;

(25) $\lim\limits_{x\to -1} \dfrac{\sqrt{x^2+3}+2x}{x+1}$; (26) $\lim\limits_{x\to 1} \dfrac{x^4-2x^2+1}{x^2-1}$;

(27) $\lim\limits_{x\to 2} \dfrac{\sqrt{3x-5}-1}{x-2}$; (28) $\lim\limits_{x\to 0} \dfrac{x^2}{\sqrt[3]{1+x^2}-1}$;

(29) $\lim\limits_{x\to 1} \dfrac{x^3-1}{x^2+5x-6}$; (30) $\lim\limits_{x\to 5} \dfrac{5-x}{\sqrt{5}-\sqrt{x}}$.

2.584. Evaluate the following limits:

(1) $\lim\limits_{x\to 0} \dfrac{\sin 4x}{2x}$; (2) $\lim\limits_{x\to 0} \dfrac{\sin 5x}{6x}$; (3) $\lim\limits_{x\to 0} \dfrac{3x}{\sin (x/2)}$;

(4) $\lim\limits_{x\to 0} \dfrac{\tan x}{x}$; (5) $\lim\limits_{x\to \pi/3} \dfrac{\sin (x-\pi/3)}{x-\pi/3}$;

(6) $\lim\limits_{x\to 0} \dfrac{\tan ax}{x}$; (7) $\lim\limits_{x\to 0} \dfrac{\cos 6x-\cos 4x}{x^2}$;

(8) $\lim\limits_{x\to 0} \dfrac{x-\sin 3x}{x}$; (9) $\lim\limits_{x\to 0} \dfrac{\sin 3x-\sin 12x}{10x}$;

(10) $\lim\limits_{x\to 0}\dfrac{\sin 5x+\sin 7x}{x/4}$; (11) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+\cos x}-\sqrt{2}}{x^2}$;

(12) $\lim\limits_{x\to 0}\dfrac{\sqrt{5+\sin x}-\sqrt{5-\sin x}}{x}$;

(13) $\lim\limits_{x\to 0}\dfrac{\cos 10x-\cos x}{4x^2}$; (14) $\lim\limits_{x\to 0}\dfrac{\sin 11x-6x}{5x}$;

(15) $\lim\limits_{x\to 0}\dfrac{1-\sin x}{1+\cos 2x}$; (16) $\lim\limits_{x\to 0}\dfrac{\cos(x/2)-\sin(x/2)}{x}$;

(17) $\lim\limits_{x\to 0}\dfrac{(\sqrt{x}-1)\sin x}{x^2}$; (18) $\lim\limits_{x\to 0}\dfrac{\sin ax}{\sin bx}$;

(19) $\lim\limits_{x\to 0}\dfrac{\tan ax}{\tan bx}$.

2.585. Evaluate the following limits and confirm or reject the given statements:

(1) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x+x^2}-1}{x}-\lim\limits_{x\to\pi/4}\dfrac{\sqrt{2}-2\cos x}{\sin(x-\pi/4)}>\log 0.005$;

(2) $\lim\limits_{x\to 1}\dfrac{x^4-2x^2+1}{x^2-1}\neq\lim\limits_{x\to 0}\dfrac{\sin 2x-\tan 2x}{x^2\sin x}$;

(3) $\lim\limits_{x\to 2}\dfrac{\sqrt{x^2+5}-3}{x^3-8}\lim\limits_{x\to 0}\dfrac{\tan 2x-\tan x}{\sin 2x-\sin x}\geqslant 0$;

(4) $\lim\limits_{x\to 0}\dfrac{\sin(x/2)\sin 4x}{2x^2}=0.5\lim\limits_{x\to -2}\dfrac{x^3+x^2-4x-4}{2x+4}$;

(5) $\lim\limits_{x\to 2\pi}\dfrac{\sin 2x-2\tan x}{\cos x-1}=\lim\limits_{x\to 2/3}\dfrac{9x^2-12x+4}{27x^3-8}$;

(6) $\lim\limits_{x\to 2}\dfrac{5x^3-40}{12+4x-3x^2-x^3}\lim\limits_{x\to 0}\dfrac{\cos^2 x-\sin^2 x-1}{\sqrt{x^2+1}-1}=12$;

(7) $\lim\limits_{x\to 0}\dfrac{\tan 3x-\sin 3x}{27x^3}\cdot\lim\limits_{x\to 0}\dfrac{\sqrt{x^2+1}-1}{\sqrt{x^2+4}-2}=1$.

2.586. Evaluate the given limits:

(1) $\lim\limits_{x\to\infty}(1+2x)^{1/x}$; (2) $\lim\limits_{n\to\infty}\left(\dfrac{n-3}{n}\right)^{n/3}$;

(3) $\lim\limits_{n\to\infty}\left(1+\dfrac{2}{n}\right)^{2n}$; (4) $\lim\limits_{x\to\infty}\left(\dfrac{2x-3}{2x+1}\right)^{x+1}$;

(5) $\lim\limits_{x\to\infty}\left(\frac{x+1}{x}\right)^{x}$; (6) $\lim\limits_{x\to\infty}\left(\frac{x^2-1}{x^2}\right)^{3x}$;

(7) $\lim\limits_{x\to\infty}\left(\frac{x-2}{x+1}\right)^{2x}$; (8) $\lim\limits_{x\to\infty}\left(\frac{3x-2}{3x+1}\right)^{2x+1}$;

(9) $\lim\limits_{x\to\infty}\left(\frac{4x-3}{4x+3}\right)^{x+1}$.

§ 70. Continuity of a Function. The Derivative

2.587. Prove that the following functions are continuous:

(a) (1) $y=2x$; (2) $y=x^2-3$; (3) $y=x-2x^2$;

(4) $y=x^3$; (5) $y=\sin x$; (6) $y=\cos x$;

(b) (1) $y=\frac{3x^2-2}{2x^2+7}$ at the point $x_0=3$;

(2) $y=\frac{x^4+2}{3x^2-1}$ at the point $x_0=2$;

(3) $y=\frac{x^2-1}{x-1}$ at the point $x_0=2$; (4) $y=\frac{x^2+3x+2}{2x^2+x-6}$ at the point $x_0=2$; (5) $y=\frac{x^2-5x+4}{x^2+x-2}$ at the point $x_0=2$.

2.588. Determine the points of discontinuity and intervals of continuity for the following functions:

(a) (1) $y=\frac{1}{1-x}$; (2) $y=\frac{x}{x-2}$; (3) $y=\frac{2x}{x+1}$;

(4) $y=\frac{6}{x^2-2x-3}$;

(b) (1) $y=\tan x$; (2) $y=\cot x$; (3) $y=\frac{5}{\cos x+1}$;

(4) $y=\frac{1}{\tan x-1}$.

2.589. Find the derivative of the given functions by definition:

(a) (1) $y=3x^2$; (2) $y=5-4x^2$; (3) $y=ax+b$;

(4) $z=5t^2-3t^3$; (5) $y=1-x^2$; (6) $y=x^3-5x^4+x^5$;

(b) (1) $y=3x/2$; (2) $y=(2/3)\,x^3-x$;

(3) $y=3x^2+5x-6$; (4) $y=(x+1)^2$;

(5) $y=1-4x^2+x^{3n-2}$; (6) $y=x^{n-11}+x^{5-2n}-7$.

The Derivative of a Power Function

2.590. Find the derivatives:

(a) (1) $y=x^7$; (2) $y=1/x^2$; (3) $y=x^5$; (4) $y=x\sqrt{x}$;
(5) $y=4/x$; (6) $y=-2x^{\sqrt{2}}$; (7) $y=x^{4\sqrt{5}}$;

(b) (1) $y=\sqrt[3]{x}-2\sqrt{x^{-1}}+3x^{-2}-(5x)^{-3}+4$;
(2) $y=\sqrt{x\sqrt{x}}$; (3) $y=3x^2\sqrt[3]{x}$; (4) $y=x^5\sqrt[3]{x}$;
(5) $y=\sqrt{2x\sqrt{2x}}$; (6) $y=2x^2\sqrt[3]{x^{-1}}$; (7) $y=\sqrt{x}\sqrt[3]{x}$;

(c) (1) $y=\sqrt{3x\sqrt{5x}}$; (2) $y=2\sqrt{\sqrt{x}x^{-3}}$;
(3) $y=x^{-4}\sqrt{x}$;
(4) $y=1/(x\sqrt{2x})$; (5) $y=5\sqrt[5]{x^3}$; (6) $y=\sqrt[4]{x^{-3}}$;

(d) (1) $y=1/x^3$; (2) $y=x^3\sqrt[3]{x}$; (3) $y=x\sqrt[3]{x}$;
(4) $y=\dfrac{5x^2-x-1}{x}$; (5) $y=\dfrac{7-3x+x^4}{x-x^2}$;
(6) $y=3\sqrt[3]{x^2}+2x^3\sqrt{x}+1/x^3$.

2.591. Find the derivatives of the given functions:

(a) (1) $y=1-x$; (2) $y=x-x^2$;
(3) $y=2x+x^2-7$; (4) $y=1-3x+6x^2$;
(5) $y=x-(\sqrt{2}/2)x^2$;

(b) (1) $y=2x^3-x$; (2) $y=3x^7-6x^6-4x^3+5x^2+17$; (3) $y=-x^3-3x^2+6x-100$;
(4) $y=3x^2+5x-6$; (5) $y=5-6x+17x^4$;
(6) $y=(2x-3)^4$; (7) $y=x^2+ax^2$;
(8) $y=a^2+bx-3x^2+mx^3/a^2$;

(c) (1) $y=3x^3-2x^2$; (2) $y=6x^3-4x^2+10x$;
(3) $y=(3/4)x^4-(1/2)x^2+2x$;
(4) $y=3x^4+4x^3+1$; (5) $y=x^3-x^2-1$;
(6) $y=x^3/3-3/x$; (7) $y=x/6+2/x$.

The Derivative of a Product of Functions

2.592. Find the derivatives of the given functions:

(1) $y = (x^2 + 1)(3 - 5x^2)$; (2) $y = 5x^2(x - x^2)$;
(3) $y = (x^2 + x + 1)(x^2 - x + 1)$;
(4) $y = (1 - 3x + 7x^2)(-5x^2 - 1)$;
(5) $y = x^2(1 - x^2)$; (6) $y = (x^2 - a)(x^3 + b)$;
(7) $y = (1/6)(x - 2)(x^2 - 3)$.

The Derivative of a Quotient of Functions

2.593. Find the derivative for each of the given functions:

(a) (1) $y = \frac{1-x}{1+x}$; (2) $y = \frac{x}{x^2-1}$; (3) $y = \frac{5}{6x-4}$;

(4) $y = \frac{2x-3}{5-4x}$; (5) $y = \frac{2x^2}{1-7x}$; (6) $y = \frac{3}{x-x^2+3}$;

(7) $y = \frac{5}{6-x}$; (8) $y = \frac{-7}{3-10x}$;

(b) (1) $y = \frac{ax+b}{cx+d}$; (2) $y = \frac{x^3}{-2x^2+3x-5}$;

(3) $y = \frac{x^2+3}{x-1}$; (4) $y = \frac{2x}{x^2+2}$; (5) $y = \frac{x^2-2}{x^2+2}$;

(6) $y = \frac{2x^4}{2x^2-5x+2}$.

The Derivative of Trigonometrical Functions

2.594. Find the derivatives of the following functions:

(a) (1) $y = \sin x + \cos x$; (2) $y = \sin x - \cos x$;
(3) $y = 3 \sin x + 2 \cos x$; (4) $y = 2 \sin x - 3 \cos x$;
(5) $y = -\sin x + 7 \cos x$; (6) $y = -6 \sin x - 6 \cos x$;
(7) $y = \sin x \cos x$; (8) $y = x \sin x$; (9) $y = x^2 \cos x$;

(b) (1) $y = (\cos x)/x$; (2) $y = x^3 \cos x$;
(3) $y = (1 - 2 \sin x)(1 - 3 \cos x)$;
(4) $y = (a + bx) \sin x + (c + dx) \cos x$;
(5) $y = \sin^3 x + \cos^3 x$; (6) $y = 3x - 2/(\cos^2 2x)$.

The Derivative of a Composite Function

2.595. Find the derivatives of the following functions:

(a) (1) $y=\sqrt{a^2+bx}$; (2) $y=(x^4-ax^3+b^2x^2)^3$;

(3) $y=\sqrt[3]{1+x^3}$; (4) $y=\sqrt{1+x^2}$;

(5) $y=3\sqrt[3]{1-x}$; (6) $y=\sqrt{(a+bx)^3}$;

(b) (1) $y=a\sqrt{1+x}$; (2) $y=3(x-5)^3+2(1-x)^4$;

(3) $y=1/(3+2x)^4$; (4) $y=4/(x+3)^2$;

(5) $y=(2x+3)^3$; (6) $y=(x+7)^3$;

(c) (1) $y=\sin 2x$; (2) $y=\sin x+\pi/4$;

(3) $y=1/\sin^2 x$; (4) $y=(2\sin 3x)/\cos^2 x$;

(5) $y=\sqrt{\sin x}$; (6) $y=(1/3)\tan^3(1/x)$;

(d) (1) $y=\sqrt[3]{\sin 2x}$; (2) $y=4\tan 2\sqrt{x}$;

(3) $y=(1/5)\tan^5 x+(2/3)\tan^3 x+\tan x$;

(4) $y=\cos(x-\pi/3)$; (5) $y=\sin(a+bx)$;

(e) (1) $y=\cos 2x$; (2) $y=2\sin(x/5)+3\cos 6x$;

(3) $y=5-\sin 7x$; (4) $y=4+3\cos(x/7)$;

(5) $y=3x\sin 2x+2x\cos 3x$; (6) $y=A\sin(\omega x+\varphi)$;

(7) $y=A\cos(\varphi-\omega x)$;

(f) (1) $y=(x+1)\sqrt[3]{x^2}$; (2) $y=x^2\cdot\cos(1/x)$;

(3) $y=(2/3)(x^3-\sqrt{(x^2-1)^3}-x)$; (4) $y=(x^3+1)\cos 2x$;

(5) $y=\sin 2x\tan x$.

The Derivative of a Logarithmic Function

2.596. Find the derivative of the given function:

(a) (1) $y=x\ln x$; (2) $y=\dfrac{\ln x}{x}$; (3) $y=2\ln(x+1)$;

(4) $y=\ln\dfrac{1}{x}$; (5) $y=\ln\dfrac{a+x}{a-x}$; (6) $y=\ln\dfrac{x^2}{1-x^2}$;

(7) $y=\ln^2\sqrt{x}$;

(b) (1) $y = \ln \sin x - \frac{1}{2} \sin^2 x$; (2) $y = \ln \sqrt{\frac{1+x}{1-x}}$;
(3) $y = \frac{1}{2} \tan^2 x + \ln \cos x$; (4) $y = \ln \sqrt{\frac{1+2x}{1-2x}}$;

(c) (1) $y = a \log x$; (2) $y = x^m \ln x$; (3) $y = 1/\log x$;
(4) $y = (\ln x + 1)/(x + 1)$; (5) $y = x + \ln x$;
(6) $y = 1/x + \log x$; (7) $y = -\ln \cos x$;

(d) (1) $y = \ln\left(1 + \frac{a}{x}\right)$; (2) $y = 10 \ln \frac{4}{4+x}$;
(3) $y = \ln \frac{x^2}{1-x^2}$; (4) $y = \ln \sin \frac{x}{3}$;
(5) $y = \ln (x + 1 + x^2)$; (6) $y = \ln (1 + x + \sqrt{2x + x^2})$;

(e) (1) $y = \log (\sin 3x + 2^x)$; (2) $y = \log_{11} (x^2 + 4\sqrt{x} + 5)$;
(3) $y = x^3 \ln x$; (4) $y = \frac{\sin x}{\ln 7x}$; (5) $y = \frac{\ln 2x}{\sqrt{x+3}}$.

The Derivative of an Exponential Function

2.597. Find the derivative of the given function:

(a) (1) $y = 2^x$; (2) $y = a^{2x}$; (3) $y = a^{x^2}$; (4) $y = 3^{x+2}$;
(5) $y = a^{-x}$;

(b) (1) $y = xe^x - e^x$; (2) $y = a^{\sin x}$; (3) $y = xe^{2x}$;
(4) $y = e^{2 \sin x}$;

(c) (1) $y = a^{1/x}$; (2) $y = x^n n^x$; (3) $y = \ln e^{\cos x}$;
(4) $y = xa^x$;

(d) (1) $y = x/e^x$; (2) $y = e^{x \ln x}$; (3) $y = e^{\cos x} \sin x$;
(4) $y = -2e^{\sin x}$;

(e) (1) $y = \cos e^x$; (2) $y = e^{\ln^2 x}$; (3) $y = e^{\tan x}$;
(4) $y = -2e^{\sin^2 x} + \cos 2e^{x^2}$.

(f) (1) $y = \frac{0.3^x}{\log x + 3}$; (2) $y = 5^{-4x}$;
(3) $y = \frac{3}{7^{3x}}$; (4) $y = \frac{2^x}{\cos x}$;

(g) (1) $y = 5^x \cdot \sin x$; (2) $y = e^{\sin x} + x$; (3) $y = 3^x \ln (5x)$;
(4) $y = 0.3^{7-2x}$.

2.598. Write the equation of the tangent to the graph of the given function at the point with the abscissa x_0:

(a) (1) $y = 10^x$, $x_0 = 1$; (2) $y = e^{2x}$, $x_0 = 0$; (3) $y = x/e^3$, $x_0 = 0$; (4) $y = 3^{-x}$, $x_0 = 1$;

(b) (1) $y = \ln(2x)$, $x_0 = 0.5$; (2) $y = \log(3x)$, $x_0 = 1/3$; (3) $y = \ln x$, $x_0 = 1$; (4) $y = \ln x$, $x_0 = e$; (5) $y = \log x$, $x_0 = 1$; (6) $y = \log x$, $x_0 = 10$.

§ 71. Applying the Derivative to Investigating Functions for Increase and Decrease. Maximum and Minimum of a Function. Investigating Quadratic Trinomials

2.599. Determine the intervals of increase or decrease and the points of maximum or minimum of the following functions:

(a) (1) $y = 3 + 4x - x^2$; (2) $y = x^3 - 3x + 1$; (3) $y = x^4 - 2x^2 - 3$; (4) $y = x^2 - 5 - 2x - x^3$; (5) $y = \frac{x^2+1}{x}$; (6) $y = \frac{2x+1}{1-3x}$; (7) $y = \frac{2x-1}{2-4x}$; (8) $y = \frac{3-2x}{x+1}$; (9) $y = \frac{1}{x^2+x+1}$; (10) $y = \frac{(x-2)(x+3)}{(x-5)^2}$;

(b) (1) $y = \sin x - (1/2)x$; (2) $y = \sin x - (\pi/2)x$; (3) $y = \sin^2 x - x$; (4) $y = -\cos(x/3)$; (5) $y = 3\sin x - 4\cos x$; (6) $y = 2\sin x + 3\cos x$; (7) $y = -2\cos x + \cos 2x$; (8) $y = x + \cos^3 x$;

(c) (1) $y = x - \ln x$; (2) $y = x \ln x$; (3) $y = 2^{x^2-4x}$; (4) $y = xe^x$; (5) $y = (\ln x)/x$; (6) $y = e^x/(x+1)$; (7) $y = x/\ln x$; (8) $y = (\ln x + 2)/x$; (9) $y = x^2 - \ln(1+2x)$; (10) $y = e^{-x} - e^{-2x}$; (11) $y = e^{-x}\sin x$; (12) $y = x - \ln(1+x)$.

2.600. Find the extrema of the following functions:

(a) (1) $y = 4x^2 - 6x - 7$; (2) $y = -3x^2 - 12x + 100$; (3) $y = (a - x)(a - 2x)$; (4) $y = x^5$; (5) $y = 2x^3 + 6x^2 - 18x + 120$.

(6) $y = 2x^3 - 6x^2 - 48x - 17$; (7) $y = 3x^4 - 4x^3$;
(8) $y = (x-1)^3 - 3(x-1)$;

(b) (1) $y = \sqrt{x^3 - 3x}$; (2) $y = (x+1)/x$; (3) $y = x^3 + 3/x$;
(4) $y = (x^2+1)/x$; (5) $y = 4/\sqrt{x^2+16}$;

(c) (1) $y = x + \ln(1-2x)$; (2) $y = 3\ln(x+1/2) + 1$;
(3) $y = x/\sin x$; (4) $y = (\ln x + 2)/x$; (5) $y = 2^{\cos x}$;
(6) $y = 5^{1-\sin x}$; (7) $y = 3 + 3^{(\sin x)}$; (8) $y = 10/5^{(\cos x)}$;

(d) (1) $y = \sin x + \cos x$; (2) $y = \sqrt{3}\sin x - \cos x$;
(3) $y = \sin^2 x - \cos^2 x$; (4) $y = 2|\sin x|$; (5) $y = 3\cos x^{-1}$;
(6) $y = x/\sqrt{2} + \cos x$; (7) $y = x^2/2 + \cos x$;
(8) $y = \sin^2 x - x$; (9) $y = \sin x - \pi/2x$.

2.601. Prove that the given function increases on the entire domain of definition:

(1) $y = x^3 - 3x^2 + 3x + 21$;
(2) $y = 0.8x^5 - x^4 + 3x^3 - 2x^2 + 4x$;
(3) $y = x^3 + 4x$; (4) $y = 2x + \sin x$;
(5) $y = x + 1/(1 + x^2)$.

2.602. Find abscissa and ordinate of the point of maximum of the given function:

(1) $f(x) = 8x^4 - 16x^2 + 5$; (2) $f(x) = 3x^3 - 4.5x^2 + 3$;
(3) $f(x) = x^3 - 3x^2 + 1$; (4) $f(x) = 3x^4 - 4x^3 + 2x - 1$.

2.603. Find the abscissa and ordinate of the point of minimum of the given function:

(1) $f(x) = x^3 + 1.5x^2 + 6x + 5$; (2) $f(x) = 6x - 2x^3$;
3) $f(x) = 3x^3 - 4.5x^2 + 1.5$; (4) $f(x) = 2x^3 - 3x^2 + 1$;
(5) $f(x) = (1 - x^2)(1 + 2x^2) - 2$;
(6) $f(x) = 3x^3 - 4.5x^2 + 2$.

Find the greatest and least values of the following functions on the given intervals:

2.604.

(1) $f(x) = x^3 - 3x$; $x \in [-0.5, 0.5]$;
(2) $f(x) = x^3 - 3x^2 + 3x + 2$, $x \in [-2, 2]$;

(3) $f(x) = x^4 - 8x^2 - 9$, $x \in [-1.1, 0; 3]$;
(4) $f(x) = 3x^4 + 4x^3 + 1$, $x \in [-2, 1]$;
(5) $f(x) = x^5 - x^3 + x + 2$, $x \in [-1, 1]$;
(6) $f(x) = x/3 + 3/x$, $x \in [-5, -1]$;
(7) $f(x) = x/8 + 2/x$, $x \in [1, 6]$.

2.605.

(1) $y = x - 2\sqrt{x-2} - 3$, $x \in [2, 11]$;
(2) $y = 9x - 3x^2 + 1$, $x \in [0, 3]$;
(3) $y = 2x^3 - 3x^2 + 2$, $x \in [-1, 2]$;
(4) $y = x^2 - 6\sqrt[3]{x}$, $x \in [0, 8]$;
(5) $y = (x-2)^3 (x+1)^2$, $x \in [-1, 2]$;
(6) $y = (2 - \sqrt{x})^3 (\sqrt{x} + 1)^2$, $x \in [0, 4]$;
(7) $y = 2x^2 - \ln x$, $x \in [1/e, e]$;
(8) $y = 4/\sqrt{x^2 + 16}$; $x \in [-1, 3]$;
(9) $y = 2(x^2 + 1)$, $x \in [-1, 3]$;
(10) $y = x/3 + 4/x$, $x \in [-5, -1]$;
(11) $y = x^3 - 5x^3 + 10x + 1$, $x \in [-1, 2]$;
(12) $y = 2x^3 + 3x^2 - 1$, $x \in [-2, -0.5]$; $[-0, 1]$.

2.606.

(1) $y = \sin x + \cos x$, $x \in [\pi/3, \pi]$; $[\pi/3, 5\pi/6]$;
(2) $y = \sin x - \sqrt{3}\cos x$, $x \in [-\pi, 0]$;
(3) $y = \cos^2 x/2 \cdot \sin x$, $x \in [0, \pi]$;
(4) $y = x + \cos^2 x$, $x \in [0, \pi/2]$;
(5) $y = \tan x + \cot 2x$, $x \in [\pi/6, \pi/3]$.

Investigate the given functions and sketch their graphs:

2.607.

(a) (1) $y = x^2 + 2x + 1$; (2) $y = x^2 - x + 1/4$;
(3) $y = -x^2 - 8x - 16$; (4) $y = -3x^2 - 3x - 6$;
(5) $y = -(1/2)x^2 - 2x - 2$; (6) $y = 3x^2 - 5x + 2$;
(7) $y = -3x^2 + 4x - 5$; (8) $y = 3x^2 + 10x - 3$;
(9) $y = 2x^2 - 3x - 2$;

(b) (1) $y = 2x^3 + 3x^2 - 1$; (2) $y = x^3 - 3x^2 + 2$;
(3) $y = 2x^3 - 5x^2 + 36x$; (4) $y = x^4 - 10x^2 + 9$;
(5) $y = x^4 - 4x^2 + 3$; (6) $y = 8 + 2x - x^4$;
(7) $y = 0.25x^4 - 2x^3 - 9/4$; (8) $y = 0.2x^5 - 4x^2$;

(c) (1) $y=\frac{x}{x^2+1}$; (2) $y=-\frac{5}{(x-2)(x^2+1)}$;
(3) $y=\frac{x+4}{x^2}$; (4) $y=\frac{x+2}{x^2-9}$; (5) $y=\frac{x^2}{x^3-1}$;
(6) $x=\frac{1}{(x-1)(x-4)}$;

(d) (1) $y=xe^x$; (2) $y=\log\frac{1+x}{1-x}$; (3) $y=\frac{1-x}{(x-2)^3}$;
(4) $y = 0.5x + \sin x$.

2.608. Find the equation of the tangent line to the graph of the given function with abscissa at the point x_0:
(1) $f(x) = x^2$, $x_0 = 2$; (2) $f(x) = x^2 + 4x + 3$, $x_0 = 1$;
(3) $f(x) = (x^2 - 3x + 5)/4$, $x_0 = 2$;
(4) $f(x) = e^x$, $x_0 = 0$; (5) $f(x) = 3^x$, $x_0 = 1$;
(6) $f(x) = e^x$, $x_0 = -1$; (7) $f(x) = (0.7)^x$, $x_0 = -2$;
(8) $f(x) = x^2e^{-x}$, $x_0 = 1$; (9) $f(x) = x^2 + x^{-2}$, $x_0 = 1$;
(10) $f(x) = x/(x-1)$, $x_0 = 0$;
(11) $f(x) = 2(x - \sqrt{x^2-5})^{-1}$, $x_0 = 3$.

2.609. Find the coordinates of the point at which the tangent line to the graph of the function $f(x)$ is parallel to the straight line y:
(1) $f(x) = x^2 - x - 3$; $y = 5x + 2$;
(2) $f(x) = \sqrt{4x}$; $y = x$;
(3) $f(x) = x^3/3 - x^2 - x + 1$; $y = 2x - 1$;
(4) $f(x) = 4/x$; $y = -4x$;
(5) $f(x) = x^2 - x - 2$; $y = 0$;
(6) $f(x) = x^2 - 2x$; $x = 0$.

2.610. Write the equations of the tangents to the curves $y = 2x^2 - 5$ and $y = x^2 - 3x + 5$ drawn through the points of intersection of these curves.

2.611. Write the equations of the tangent to the curve $y = 3x^3 + 2x + 5$ at the point of intersection of this curve with the axis of ordinates.

2.612. Write the equations of the tangent to the curve $y = 8x^3 - 1$ at the point of intersection of this curve with the axis of abscissas.

2.613. At what points does the tangent to the graph of the function $y = x^3/3 - 5x^2/2 + 7x - 4$ make an angle of 45° with the y-axis?

2.614. At what angle to the x-axis is the tangent inclined if it is drawn to the curve $y = 2x^3 - x$ at the point of intersection of this curve with the y-axis?

2.615. At what angle to the x-axis is the tangent inclined if it is drawn to the curve $y = x^3 - x^2 - 7x + 6$ at the point M_0 (2, —4)?

2.616. Let $S(t)$ be the law of rectilinear motion of a point, where S is the path covered and t the time of motion. (1) $S(t) = 0.5t^4 - 5t^3 + 12t^2 - 1$. At what instants of time is the acceleration of the moving point equal to zero? (2) $S(t) = 8 - 2t + 24t^2 - 0.3t^5$. At what instant of time does the point have the greatest speed? Find this speed. (3) The motions of two bodies along one and the same straight line are given by the equations: $S_1 = 4t^2 + 2$ (m); $S_2 = 3t^2 + 4t - 1$ (m). Find the speeds of the bodies at the instants of their meeting.
(4) Rectilinear motions of two points are given by the equations $S_1 = 2t^3 - 5t^2 = 3t$ (m) and $S_2 = 2t^3 - 3t^2 - 11t + 7$ (m). Find the accelerations of the moving points at the instant when their speeds are equal.

§ 72. The Antiderivative (Primitive). The Indefinite Integral

Find the antiderivative $F(x)$ if:

2.617.

(1) $f(x) = 7 - 2x$; (2) $f(x) = 3 + 5x$;
(3) $f(x) = kx + b$; (4) $f(x) = 2x - 3x^2$;
(5) $f(x) = 4 - x^3$; (6) $f(x) = x^2 + 4x - 7$;
(7) $f(x) = ax^2 + bx + c$; (8) $f(x) = 1/(3 + 2x)^4$;
(9) $f(x) = 5 - \sin 7x$; (10) $f(x) = x + 1/x$;
(11) $f(x) = \sqrt{2x}$; (12) $f(x) = 2 \sin x + \cos 3x$;
(13) $f(x) = x^{-5} + x^{-2}$;
(14) $f(x) = (x^3 - 3x^2 + 3x - 1)/(x - 1)$;
(15) $f(x) = 3/(x + 4)$; (16) $f(x) = 2/(3 \sin^2 2x)$.

2.618.

(1) $f(x)=3/(\cos^2 2x)$; (2) $f(x)=\sin 2x+3x^2$;
(3) $f(x)=x^{3+\sqrt{2}}$; (4) $f(x)=\ln x$; (5) $f(x)=\log_3(x+2)$;
(6) $f(x)=2\ln x-x^2$; (7) $f(x)=4-3\cos(x/7)$;
(8) $f(x)=2\sin(x/5)+3\cos 6x$; (9) $f(x)=\frac{4}{(x+3)^2}+\frac{7}{\sin^2 3x}$;
(10) $f(x)=e^{3x}$; (11) $f(x)=1/e^{2x}$; (12) $f(x)=2^{x/4}-\sin 3x$;
(13) $f(x)=3^{x/2}-5$; (14) $f(x)=(0.7)^{4x}$;
(15) $f(x)=5^{1-3x}+(0.6)^{x/2}$; (16) $f(x)=\sqrt{e^{3x}}$;
(17) $f(x)=10/\sqrt{8^{3x}}$.

2.619. Find the antiderivative $F(x)$ of the function $y=f(x)$ whose graph passes through the point M:

(1) $f(x)=e^{-3x}$; M $(0, -2)$; (2) $f(x)=1/2^x$; M $(1, 5/\ln 2)$;
(3) $f(x)=e^{x/2}$; M $(0, 3)$;
(4) $f(x)=5^{-3x}$; M $(-1/3, -7/\ln 5)$;
(5) $f(x)=7^{x/4}$; M $(8, 1/\ln 7)$;
(6) $f(x)=x^4$; M $(-1, 2)$;
(7) $f(x)=\sin 2x$; M $(0, 1)$;
(8) $f(x)=1/(\sin^2 3x)$; M $(\pi/12, -1)$;
(9) $f(x)=x^{-4}$; M $(2, -3)$.

2.620. (1) The graph of one of the antiderivatives of the functions 2^x passes through the point M $(0, 5/\ln 2)$, and that of the other through the point M $(2, 7/\ln 2)$. Which graph is situated higher?
(2) The graph of one of the antiderivatives of the functions 3^x passes through the point M $(1, 8/\ln 3)$, and that of the other through the point M $(3, 20/\ln 3)$. Which graph is situated higher?
(3) Find the function $F(x)$ if it is known that $F'(x)=4x^3-3x^2$ and that $F(1)=3$.
(4) For the function $f(x)=\cos 4x$ find the antiderivative $F(x)$ if $F(\pi/24)=-1$.

2.621. Compute the given integrals:

(1) $\int x^5\,dx$; (2) $\int m\sqrt{x}\,dx$; (3) $\int 4x^3\,dx$; (4) $\int \frac{m}{\sqrt{x}}\,dx$;

(5) $\int 6x^2\,dx$; (6) $\int \frac{2}{\sqrt[4]{x^3}}\,dx$; (7) $\int 7x^4\,dx$; (8) $\int 2\sin x\,dx$;

(9) $\int \cos^4 x\,dx$; (10) $\int \cot x\,dx$; (11) $\int \cos 3x\,dx$;

(12) $\int x^{3/2}\,dx$; (13) $\int \sin\frac{x}{2}\,dx$; (14) $\int (1+x+2x^2+3x^2)\,dx$;

(15) $\int \frac{dx}{\sqrt[3]{x^2}}$; (16) $\int \frac{3}{x^2}\,dx$; (17) $\int \tan x\,dx$.

The Definite Integral

Compute the given definite integrals using the Newton Leibniz formula:

2.622.

(1) $\int_3^6 dx$; (2) $\int_1^2 6x^5\,dx$; (3) $\int_4^5 \sqrt{x}\,dx$; (4) $\int_2^5 4x\,dx$;

(5) $\int_1^2 x^3\,dx$; (6) $\int_1^2 \frac{6}{x^3}\,dx$; (7) $\int_0^5 \frac{dx}{\sqrt{x}}$; (8) $\int_0^{\pi} \sin x\,dx$;

(9) $\int_0^{\pi/2} \sin^2 x\,dx$; (10) $\int_0^{\pi} \cos^2 x\,dx$.

2.623.

(1) $\int_0^1 \sqrt{1-x}\,dx$; (2) $\int_0^1 \frac{dx}{\sqrt{3x+1}}$; (3) $\int_{-2}^2 (1-x)^3\,dx$;

(4) $\int_{-2}^0 \frac{dx}{(1-2x)^3}$; (5) $\int_2^7 \frac{dx}{\sqrt[3]{(8-x)^2}}$;

(6) $\int_{-1}^0 \frac{dx}{\sqrt{1-x}}$; (7) $\int_0^{3\pi/2} \cos\frac{x}{3}\,dx$; (8) $\int_{\pi/6}^{\pi/4} \sin 2x\,dx$;

(9) $\int_{\pi/6}^{\pi/3} \cos\frac{\pi}{3}\,x\,dx$; (10) $\int_{\pi/8}^{\pi/6} \frac{dx}{\cos^2 2x}$; (11) $\int_{\pi/12}^{\pi/9} \frac{2\,dx}{3\cos^2 3x}$.

2.624.

(1) $\int_1^2 \frac{x\,dx}{1+x^2}$; (2) $\int_0^{\sqrt{3}} \frac{x\,dx}{2\sqrt{1+x^2}}$;

(3) $\int_0^3 \frac{2x\,dx}{\sqrt{16+x^2}}$; (4) $\int_{-1}^1 \frac{x^2\,dx}{3+2x^3}$;

(5) $\int_{-1}^2 \frac{2x\,dx}{(2x^2+1)}$; (6) $\int_0^1 \frac{6x^2\,dx}{1+2x^3}$; (7) $\int_{-5}^9 \frac{x\,dx}{x^2+144}$;

(8) $\int_2^4 \frac{15x\,dx}{(x^2-1)^3}$; (9) $\int_{2\sqrt{2}}^4 x\sqrt{x^2-7}\,dx$;

(10) $\int_1^2 (x^2-1)^3 x\,dx$; (11) $\int_0^{\pi/2} \sin x \cos^2 x\,dx$; (12) $\int_0^1 \frac{e^x\,dx}{e^x+5}$;

(13) $\int_1^{0.4} \frac{5\,dx}{4+25x^2}$; (14) $\int_0^{\sqrt{2}} \frac{dx}{\sqrt{4-x^2}}$; (15) $\int_1^5 x\sqrt{x-1}\,dx$.

2.625.

(1) $\int_{-2}^{14} \sqrt[3]{(1+x/2)^2}\,dx$; (2) $\int_0^{12} \frac{dx}{\sqrt{1+x/4}}$;

(3) $\int_2^8 \frac{dx}{x \ln 2}$; (4) $\int_{-4}^2 \frac{dx}{\sqrt{2-x/2}}$; (5) $\int_0^{16} \frac{dx}{\sqrt{x+9}-\sqrt{x}}$;

(6) $\int_1^8 \frac{x-1}{\sqrt[3]{x^2}}\,dx$; (7) $\int_1^{64} \frac{2+3\sqrt[3]{x^2}+5\sqrt{x^3}}{\sqrt{x^3}}\,dx$;

(8) $\int_2^2 \frac{x^2-3x+2}{x-1}\,dx$; (9) $\int_1^2 x(x+1)(x+2)\,dx$;

(10) $\int_1^8 \frac{\sqrt[3]{x^2}-\sqrt[3]{x}}{\sqrt[3]{x}}\,dx$; (11) $\int_0^3 |x-1|\,dx$;

(12) $\int_0^9 \frac{x-4}{\sqrt{x}-2}\,dx$; (13) $\int_2^4 \frac{x^3-1}{x^2+x+1}\,dx$;

(14) $\int_0^1 \frac{x+8}{\sqrt[3]{x}+2}\,dx$.

2.626.

(1) $\int_0^{\pi/6} \frac{\cos^3 x}{1+\cos 2x}\,dx$; (2) $\int_{\pi/6}^{\pi/3} \frac{\cos 2x}{\sin^2 x \cos^2 x}\,dx$;

(3) $\int_0^{\pi/3} \frac{\sin x - \sin 3x}{\cos 2x}\,dx$; (4) $\int_0^{2\pi} \sin 3x \cos 5x\,dx$;

(5) $\int_{\pi/6}^{\pi/2} \frac{1+\sin 2x+\cos 2x}{\sin x+\cos x}\,dx$;

(6) $\int_0^{\pi/2} \cos^3 4x\,dx$; (7) $\int_0^{\pi/2} \frac{2\sin^2 x-1}{\sin x+\cos x}\,dx$;

(8) $\int_{\pi/12}^{\pi/4} \sin\left(\frac{\pi}{4}+x\right)\sin\left(\frac{\pi}{4}-x\right)dx$;

(9) $\int_{\pi/24}^{\pi/4} 8\sin x \cos x \cos 2x \cos 4x\,dx$;

(10) $\int_{\pi/3}^{\pi} \left(\cot\frac{x}{4}-\tan\frac{x}{4}\right)\sin^2\frac{x}{2}\,dx$;

(11) $\int_0^{\pi} \sin x \cos 3x \cos 4x\,dx$;

(12) $\int_{\pi/3}^{\pi} (\sin 2x \cos 3x + 2\sin^2 x \sin 3x)\,dx$;

(13) $\int_{\pi/6}^{\pi/2} \left(1-2\cos^2\frac{x}{2}+\sqrt{3}\sin x\right)dx$;

(14) $\int\limits_{\pi/3}^{\pi/2} \frac{\sin(40^\circ+x)-\sin(40^\circ-x)}{\cos 40^\circ}\,dx;$

(15) $\int\limits_{0}^{\pi} (2\cos 6x+\cos x+2\sin 4x\sin x)\,dx;$

(16) $\int\limits_{\pi/2}^{3\pi/2} 2\cos^3 x\sin x\,dx.$

§ 73. Application of the Definite Integral to Computing Areas

Compute the area of the figure bounded by the indicated lines:

2.627.

(1) $y=4x-5$; $x=-3$; $x=2$; $y=0$; (2) $y=2x^2$; $y=0$; $x=2$; $x=4$; (3) $y^2=9x$; $x=1$; $x=9$; (4) $y=x^2-x$; $y=0$; $x=0$; $x=2$; (5) $y=x^2-x+2$; $y=0$; $x=0$; $x=3$; (6) $y=2x-x^2$; $y=0$; (7) $y=9-x^2$; $y=0$.

2.628.

(1) $y=x^2+4x$; $y=x+4$; (2) $y=x^2$; $y=2-x^2$; (3) $y=x^2+4x+5$; $y=0$; $x=0$; $x=-2$; (4) $y=0.5x^2-3x+2$; $y=x-4$; (5) $y=x^2-5x+4$; $y=2x-2$; (6) $y=8-0.5x^2$; $y=3.5$; (7) $y=x^2-3x+4$; $y=x+1$.

2.629.

(1) $y=x^2-2x+2$; $y=2+4x-x^2$; (2) $y=4x-x^2$; $y=4-x$; (3) $y=0.5x^2+2$; $y=x^2+2.5$; $x=1$; (4) $y=x^{2n}$; $y=1$; (5) $y=\sqrt{x}$; $y=0$; $x=9$; (6) $y=1/\sqrt{x}$; $y=0$; $x=1$; $x=4$; (7) $y=4/x^2$; $y=6-2x$.

2.630.

(1) $y=5/x$; $y=6-x$; (2) $y=3/x$; $y=3$; $x=2$; (3) $y=1/x$; $y=0$; $x=2$; $x=10$; (4) $y=2/x$;

$y = x + 1$; $x = 3$; (5) $y = 3/x$; $x + y = 4$; (6) $y = 1/x$; $y = 0$; $x = 1$; $x = 2$.

2.631.

(1) $y = e^x$; $y = 0$; $x = 0$; $x = 3$; (2) $y = e^x$; $y = 0$; $x = 1$; $x = 2$; (3) $y = e^{-x}$; $y = 0$; $x = -1$; $x = 2$; (4) $y = 2^x$; $y = 2$; $x + 1 = 0$; (5) $y = e^x$; $y = x + 1$; $x = 2$; (6) $y = 3^x$; $y = 0.7^x$; $x = 1$; (7) $y = e^x$; $y = \cos(\pi x/2)$; $x = 1$.

2.632. (1) Compute the area of the figure bounded by the parabola $y = x^2 - 2x + 2$, tangent line to it at the point (3, 5), and straight line $x = 0$.

(2) Compute the area of the figure bounded by the parabola $y = -x^2 - 4x - 3$ and the tangents to it at the points (0, −3) and (−3, 0).

(3) Compute the area of the figure (to within a unity) bounded by the curve $y = x^4 - 2x^3 + x^2 + 3$, straight line $y = 0$, and two ordinates corresponding to the points at which the given figure has a minimum.

(4) Determine for what positive value of the parameter a the area of the figure bounded by the lines $y = \sin ax$, $y = 0$, $x = \pi/a$, and $x = \pi/(3a)$ is equal to 3.

§ 74. The Method of Mathematical Induction

2.633. Prove the given equality using the method of mathematical induction:

$$1 + 3 + 5 + 7 + \ldots + (2n - 1) = n^2.$$

2.634. Find the sum $1 + 2 + 3 + 4 + 5 + \ldots + n$ for any natural value of n.

2.635. Find the following sums:

(a) $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{5 \cdot 7} + \ldots + \frac{1}{n(n+1)}$;

(b) $\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \ldots + \frac{1}{(2n-1)(2n+1)}$;

(c) $1^3 + 2^3 + 3^3 + 4^3 + \ldots + n^3$.

2.636. Prove the formulas of the general terms and sums of arithmetic and geometric progressions using the method of mathematical induction.

2.637. Prove that for any integral n the following expressions are divisible by 6: (1) $n^3 + 11$; (2) $n^3 + 5n$.

2.638. Prove that the expression $n^5 - n$ is divisible by 30 for any integral n.

2.639. Prove that $(1 + a)^n \geqslant 1 + na$ for any natural n and $a > -1$.

2.640. Prove that for $n \geqslant 3$ $2^n > 2n + 1$.

2.641. Determine for what natural values of n the inequality $2^n > n^2$ is valid.

2.642. Prove the following inequalities:

(a) $\frac{n}{2} < 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{2^n - 1} < n$, where $n = 2, 3, \ldots$;

(b) $\frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots + \frac{1}{n^2} < \frac{n-1}{n}$;

(c) $\sin^{2n} a + \cos^{2n} a \leqslant 1$; (d) $|\sin nx| \leqslant n |\sin x|$.

2.643. Prove that $p^5 - p$ is divisible by 5 for any natural p.

2.644. Prove that the sum of cubes of three successive natural numbers is divisible by 9.

2.645. Prove that for any natural n the difference $9^{2n+2} - 64^{n+1}$ is divisible by 17.

2.646. Prove that for any natural n the sum $9^{n+1} + 2^{6n+1}$ is divisible by 11.

2.647. Prove that for any natural n the difference $n^7 - n$ is divisible by 42.

2.648. Prove that the expression $7 \cdot 5^{2n-1} + 2^{3n+1}$ is divisible by 17 for any natural n.

2.649. Prove the following equalities:

(1) $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$;

(2) $1 + 2 + 6 + \ldots + \frac{n(n+1)}{2} = \frac{n(n+1)(n+2)}{6}$;

(3) $1 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$;

(4) $1 + 3^3 + 5^3 + \ldots + (2n-1)^3 = n^2(2n^2 - 1)$;

(5) $\frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \ldots + \frac{1}{n^2 + 3n + 2} = n/2n + 4$.

§ 75. Combinations and Binomial Formula

2.650. Find:

(1) A_6^2; A_7^4; A_8^5; A_6^3/A_5^2; $(A_8^5+A_7^4)/A_6^3$;

(2) $(A_{10}^6-A_{10}^5)/(A_9^5-A_5^4)$; $(A_m^5+A_m^4)/A_m^3$;

$(A_m^{n+2}+A_m^{n+1})/A_m^n$; $(A_{m+n}^{n+2}+A_{m+n}^{n+1})/A_{m+n}^n$;

(3) P_4; P_6; P_9; P_8/P_6; $(P_5+P_4)/P_3$; $(P_6-P_4)/P_3$;

$(P_8-P_7)/(7P_7)$;

(4) C_6^2; C_8^3; C_{11}^4; C_{15}^4; C_{12}^7; C_{20}^{17}; C_{40}^{38}; C_{54}^{52}; C_m^n; C_{m+1}^{n-1}; C_{m+1}^{n+1}; C_{m-1}^{n+1}; C_{m-n}^{n+1}; C_{m+2}^{n-2}; C_{m+2}^n; C_m^{n-1}; C_{n+2}^{k+1}; C_{n+2}^{k+1}.

2.651. Solve the following equations:

(a) (1) $A_x^2=42$; (2) $A_x^3=56x$; (3) $A_{x+1}^2=30$;

(4) $(A_x^5+A_x^3)/A_x^3=43$; (5) $(A_x^7-A_x^5)/A_x^5=89$;

(b) (1) $C_x^3=C_{x+2}^4$; (2) $C_{x-3}^2=21$; (3) $C_x^3=\frac{5x(x-3)}{4}$;

(4) $C_x^3+C_x^2=15(x-1)$; (5) $12C_x^1+C_{x+4}^2=162$; $C_x^2=28$;

(6) $30C_{x-3}^{x-9}=19A_{x-4}^4$; (7) $C_{2x+3}^{2(x-1)}=16x(x+1)(2x+1)$;

(8) $C_{x+1}^{x-4}=\frac{7}{15}x(x+1)(x-1)$;

(9) $C_{x+8}^{x+3}=5(x+6)(x+5)(x+4)$; (10) $12C_{x+3}^{x-1}=55A_{(x+1)}^2$;

(c) (1) $\frac{A_x^4\cdot P_{x-4}}{P_{x-2}}=42$; (2) $\frac{A_{x+1}^{n+1}\cdot P_{x-n}}{P_{x-1}}=90$;

(3) $\frac{P_{x+2}}{A_x^n\cdot P_{x-n}}=132$; (4) $\frac{A_{x+2}^{n+2}\cdot P_{x-n}}{P_x}=110$.

2.652. Simplify:

(a) (1) $\frac{A_{10}\cdot P_{10-n}}{P_9}$; (2) $\frac{A_{m-1}^{n-1}\cdot P_{m-n}}{10P_{m-1}}$;

(3) $\frac{P_{2x+1}}{A_{2x-1}^{n-1}\cdot P_{2x-n}}$; (4) $\frac{P_{m-n}\cdot A_m^n}{P_{m+1}}$.

(b) Solve the given systems of equations:

(1) $\begin{cases} A_x^y\div A_x^{y-1}=10, \\ C_x^y\div C_x^{y-1}=\frac{5}{3}; \end{cases}$ (2) $\begin{cases} C_x^{y+1}=2.5x, \\ C_{x-1}^y=10; \end{cases}$

(3) $\begin{cases} C_x^y = C_x^{y+2}, \\ C_x^2 = 153; \end{cases}$ (4) $\begin{cases} A_x^y \div A_x^{y-1} = 8, \\ C_x^y \div C_x^{y-1} = 1.6; \end{cases}$

(5) $\begin{cases} A_2^{3x} \div A_2^{3x-1} = 8, \\ C_{2n}^{3x} \div C_{2n}^{3x-1} = \frac{8}{9}; \end{cases}$ (6) $\begin{cases} A_x^{n-3} \div A_x^{n-2} = 1 \div 8, \\ C_x^{n-3} \div C_x^{n-2} = 5 \div 8; \end{cases}$

(7) $\begin{cases} A_{2x}^{n-2} \div A_{2x}^{n-3} = 8, \\ C_{2x}^{-2} \div C_{2x}^{-3} = 2\frac{2}{3}; \end{cases}$ (8) $\begin{cases} A_m^n \div A_m^{n-1} = 9, \\ C_m^n \div C_m^{n-1} = \frac{3}{2}. \end{cases}$

2.653. Verify the following equalities:

(a) (1) $C_{m+1}^7 = C_{m+1}^{-6}$; (2) $C_m^9 + C_m^8 = C_{m+1}^9$;

(3) $C_m^n + C_m^{n-1} = C_{m+1}^n$; (4) $C_{m+1}^4 - C_m^3 = C_m^4$;

(b) (1) $C_m^8 - C_{m+1}^8 + C_m^7 = 0$;

(2) $\frac{C_n^1 + 2C_n^2 + 3C_n^3 + \ldots + nC_n^n}{n} = 2^{n-1}$;

(3) $C_n^m + C_{n-1}^m + \ldots + C_{n-10}^m = C_{n+1}^{m+1} - C_{n-10}^{m+1}$.

2.654. Find the product of binomials:

(1) $(x+1)(x+2)(x+3)(x+4)$;

(2) $(a-1)(a-2)(a-3)(a-4)(a-5)$;

(3) $(x-1)(x-2)(x-3)(x-4)$;

(4) $(a-1)(a-2)(a+3)(a+4)(a-5)$;

(5) $(y+1)(y+3)(y-4)(y-2)$.

2.655. Find the expansion of each binomial:

(1) $(x+a)^6$; (2) $(x+c)^9$; (3) $(2a+3b)^5$;

(4) $(\sqrt[3]{m} - \sqrt[3]{n})^7$; (5) $(\sqrt[3]{x} - \sqrt{x})^4$;

(6) $(2m-3n)^6$; (7) $(\sqrt{3} - 3\sqrt{2})^4$;

(8) $(\sqrt{a/b} + \sqrt{b/a})^6$; (9) $(\sqrt{a+1} - \sqrt{a-1})^4$;

(10) $(\sqrt{x} + \sqrt{y})^5 + (\sqrt{x} - \sqrt{y})^5$.

2.656. Find:

(1) the eighth term of the expansion of the binomial $(x+2)^{12}$;

(2) the fourth term of the expansion of the binomial $(x+2)^9$;

(3) the middle term of the expansion of the binomial $(1/x - \sqrt{x})^{16}$.

2.657. Find:

(1) the term of the expansion of $(x + 1)^{12}$ containing x^6;
(2) the term of the expansion of $(\sqrt{x} + \sqrt{2})^{15}$ containing x^5;
(3) the term of the expansion of $(\sqrt{x} + 1/\sqrt[4]{x})^9$ not containing x;
(4) the rational terms in the expansion of each binomial:

$(3^{1/2} + 2^{1/2})^5$; $(\sqrt[5]{3} + \sqrt[7]{2})^{24}$; $(\sqrt{5} - \sqrt{2})^{10}$.

2.658. Find the term of the expansion of the binomial $(z\sqrt{z} + 1/\sqrt[3]{z})^m$, which contains z^5 if the sum of the binomial coefficients of this expansion is equal to 128.

2.659. Find the term of the expansion of the given binomial which does not contain a:

(a) (1) $(1/\sqrt[3]{a^2} + \sqrt[4]{a^3})^{17}$; (2) $(\sqrt[3]{a} - 1/\sqrt{a})^{15}$;
(3) $(\sqrt[9]{1/a^8} + \sqrt[3]{a^2})^7$;

(b) (1) $(a\sqrt[3]{a^{-1}} + 1/\sqrt[7]{a^2})^{10}$;
(2) $(2a/b - \sqrt{b}/(2a))^{12}$; (3) $(\sqrt[-3]{a^2} + \sqrt[5]{a^3})^{20}$.

2.660. Find the exponent of the binomial if:
(1) in the expansion of $(1 + x)^n$ the coefficient of the fifth term of the expansion is equal to that of the ninth term;
(2) in the expansion $(ax^{-1/2} + xa^{-1/2})^n$ the ratio of the coefficient of the third term of the expansion to that of the second term is equal to 11/2;
(3) in the binomial $(9x - x^{-1/2}/\sqrt{3})^n$ the binomial coefficient of the third term is equal to 105;
(4) the sum of the coefficients of the first, second, and third terms of the expansion of $(x^{-1} + x^2)^n$ is equal to 46.

2.661. Determine x from the condition that the third term of the expansion of the binomial $(x + x^{\log x})^5$ is equal to 1 000 000.

2.662. Find the fifth term of the expansion of the binomial $(\sqrt[3]{2} - \sqrt{2^{-1}})^n$ if the last term of the expansion is equal to $(1/3\sqrt[3]{9})^{\log_3 8}$.

§ 76. Complex Numbers. Addition and Subtraction of Complex Numbers

2.663. Transform the following expressions using the symbol i:

$\sqrt{-4}$; $\sqrt{-1/25}$; $\sqrt{-3}$; $\sqrt{-32}$; $\sqrt{-16}$; $\sqrt{-36a^2}$; $\sqrt{-(36/21)b^4}$; $\sqrt{-(49/625)m^2n^2}$; $\sqrt{-0.225\,b^8}$.

2.664. Transform the following expressions using the symbol i:

(1) $\sqrt{-25}+\sqrt{-49}-\sqrt{-64}+\sqrt{-1}$; (2) $\sqrt{-144}-\sqrt{-81}-\sqrt{-1}+\sqrt{-1}$; (3) $3\sqrt{-4}+5\sqrt{-27}-3\sqrt{-16}-5\sqrt{-3}$; (4) $\sqrt{-25}-5\sqrt{-8}+\sqrt{-49}-2\sqrt{-2}$; (5) $\sqrt{-72}+\sqrt{-98}-\sqrt{-18}+\sqrt{-121}$.

2.665. (a) Find the sum:

(1) $(5+4i)+(3-7i)$; (2) $(2-8i)+(5-i)$;
(3) $(2+5i)+(-2-2i)$; (4) $(4+3i)+(-4+3i)$;
(5) $(a+bi)+(c+di)$;
(6) $(3x-4yi)+(-x+2yi)$; (7) $\left(1\frac{3}{4}+\frac{2}{3}i\right)+\left(1\frac{1}{2}-\frac{5}{6}i\right)+\left(-\frac{3}{4}-2i\right)$; (8) $(0.12-1.4i)+(1.08+0.4i)+(2.5-0.2i)$;

(b) Find the difference:

(1) $(5+4i)-(2-3i)$; (2) $(8-5i)-(5+13i)$;
(3) $(a+bi)-(2a-3bi)$; (4) $(2+i)+(3-6i)-(1-i)$;
(5) $\left(\frac{1}{2}-\frac{1}{4}i\right)-\left(\frac{3}{5}+\frac{2}{3}i\right)+\left(\frac{3}{4}-\frac{5}{6}i\right)$;
(6) $(0.8-0.2i)+(0.1-1.3i)-(1.5+0.7i)-(2.3-0.6i)$.

2.666. Perform the indicated operations:

(1) $(2a-3bi)+(-a-bi)+(4a+2bi)-(2a-5bi)$;
(2) $(5x+3yi)+(-2x+8yi)-(2x-yi)-(7x-2yi)$;
(3) $(2c-8di)-(5c-2di)+(c-di)-(4c-3di)$;
(4) $(m-ni)+(3m-2ni)-(-m-ni)-(5m+10ni)$.

§ 77. Raising the Imaginary Unit to a Power. Multiplication and Division of Complex Numbers

2.667. Raise to the indicated power:

(1) $(\sqrt{-1})^6$; (2) $(\sqrt{-1})^{21}$; (3) $(\sqrt{-1})^7$; (4) $(\sqrt{-1})^{56}$; (5) i^{40}; (6) i^{18}; (7) i^{4n+2}; (8) i^{4n-1}; (9) i^{6n+3}; (10) i^{14n+1}.

2.668. Compute:

(1) $1/i^{11}-1/i^{41}+1/i^{75}-1/i^{1023}$; (2) $i^6+i^{16}+i^{26}+i^{36}+i^{46}+i^{56}$; (3) $i^3+i^{13}+i^{23}+i^{33}+i^{43}+i^{53}$; (4) $i+i^2+i^3+i^4+\ldots+i^n$, where $n>4$; (5) $i\cdot i^2\cdot i^3\cdot i^4\cdot\ldots\cdot i^{100}$;

(6) $\dfrac{(1+i)\cdot 100}{(1-i)^{96}-i(1+i)^{98}}$.

2.669. Multiply:

(1) $2i\cdot 3i$; (2) $4i\cdot 9i$; (3) $5i\cdot(-4i)$; (4) $2.5i\cdot 4i$; (5) $\sqrt{-16}\cdot\sqrt{-9}$; (6) $\sqrt{-8}\cdot\sqrt{-2}$; (7) $\sqrt{-a}\cdot\sqrt{-b}$; (8) $\sqrt{a-b}\cdot\sqrt{b-a}$; (9) $(5+2\sqrt{-7})(6-5\sqrt{-7})$; (10) $(2-5i)\cdot(8-3i)$; (11) $(a+\sqrt{-b})(a+3\sqrt{-b})$; (12) $(2+3i)(3-2i)$; (13) $(5+i\sqrt{3})(5-i\sqrt{3})$.

2.670. Factor:

(1) a^2+b^2; (2) a^2+4b^2; (3) $4m^2+9n^2$; (4) $a+2$; (5) a^2+1; (6) $5+7$; (7) a^2+b^4; (8) $a^2+b^2/9$; (9) $4a^2+25b^2$; (10) $a+b$.

2.671. Factor the number 40 into the product of two complex factors whose sum is equal to 10.

2.672. Write two complex numbers whose sum and product are real numbers.

2.673. Reduce the given fractions:

(1) $\dfrac{a^2+b^2}{a-bi}$; (2) $\dfrac{a^2+b^2}{a+bi}$; (3) $\dfrac{4}{1-i\sqrt{3}}$;

(4) $\dfrac{4}{1+i\sqrt{3}}$; (5) $\dfrac{28}{\sqrt{15}+i\sqrt{13}}$; (6) $\dfrac{40}{5-i\sqrt{15}}$;

(7) $\dfrac{65}{45+i\sqrt{35}}$.

2.674. Divide:

(1) $5 \div \sqrt{-5}$; (2) $5i \div \sqrt{-5}$; (3) $a \div \sqrt{-a}$; (4) $\frac{3}{5i}$;

(5) $\frac{2}{1-i}$; (6) $\frac{5i}{2-3i}$; (7) $\frac{1+i}{1-i}$; (8) $\frac{1+i\sqrt{3}}{1-i\sqrt{3}}$;

(9) $\frac{6-i}{\sqrt{6}-2i}$; (10) $\frac{7-3i}{1+3i}$; (11) $\frac{6-i\sqrt{2}}{1+i\sqrt{2}}$;

(12) $\frac{5-2i}{1-2i}$; (13) $\frac{4-2i}{1+3i}$.

2.675. Perform the indicated operations:

(1) $(1+i)^2$; (2) $(1+i)^3$; (3) $(1+i)^4$; (4) $(1+i)^8$;

(5) $(1+i)^{50}$; (6) $(1+i)^4+(1-i)^4$; (7) $(1/2+i\sqrt{3}/2)^2$;

(8) $(\sqrt{3}+i)^{10}-(\sqrt{3}-i)^{10}$;

(9) $\left(\frac{\sqrt{3}+i}{2}\right)^6+\left(\frac{i-\sqrt{3}}{2}\right)^6$;

(10) $\left(\frac{1+i\sqrt{7}}{2}\right)^4-\left(\frac{1-i\sqrt{7}}{2}\right)^4$;

(11) $(3+i\sqrt{2})^8-(3-i\sqrt{2})^8$;

(12) $\frac{1+i}{1-i}+\frac{1-i}{1+i}$; (13) $\frac{a+bi}{c+di}-\frac{a+bi}{c-di}$;

The Condition of Equality of Two Complex Numbers

2.676. Proceeding from the condition of equality of two complex numbers, find the real numbers x and y if:

(a) (1) $2 + 5ix - 3iy = 14i + 3x - 5y$;
(2) $(1 + i)x + (1 - i)y = 3 - i$; (3) $8i/x + iy - 2 = 7i - 10/x + y$; (4) $i/x + i/y + 1/6 = 1/x - 1/y + 5i/y$;

(b) (1) $aix + biy - a = i - a^2x - b^2y$;

(2) $\frac{x-1}{3+i}+\frac{y-1}{3-i}=i$; (3) $x^2+iy^2+6+i=2x+ix$;

(4) $\frac{3x-iy}{2y-5ix}=\frac{7+5i}{12-8i}$;

(c) (1) $\sqrt{x^2-2x+8}+(x+4)i=2+i$;

(2) $(x-i)^2-(2y+i)^2=4(\sqrt{3}-1)i-2y^2-x$;

(3) $\frac{x-2}{1-i}+\frac{y-3}{1+i}=1-3i$; (4) $(x+iy)^2=7+24i$;
(5) $(x+4i)(1+yi)=11-2i$;
(6) $(2+yi)(x-5i)=21-i$.

§ 78. The Trigonometric Form of a Complex Number

2.677. Represent the following complex numbers in trigonometric form:

(1) i; (2) -1; (3) $-i$; (4) $-\sqrt{3}+i$; (5) $-\sqrt{3}-i$;
(6) $-3+i$; (7) $3-i$; (8) $12i-5$; (9) $3i-4$;
(10) $1+\cos\alpha+i\sin\alpha$; (11) $\sin\alpha+i\cos\alpha$.

2.678. Multiply:

(1) $2(\cos 60^\circ+i\sin 60^\circ)\cdot 3(\cos 45^\circ+i\sin 45^\circ)$;
(2) $\sqrt{2}(\cos 30^\circ+i\sin 30^\circ)\cdot 2\sqrt{2}(\cos 60^\circ+i\sin 60^\circ)$;
(3) $3(\cos\pi/6+i\sin(\pi/6)\cdot 2(\cos(\pi/3)+i\sin(\pi/3))$;
(4) $\sqrt{3}(\cos 120^\circ+i\sin 120^\circ)\cdot(\sqrt{3}/2)(\cos 150^\circ+i\sin 150^\circ)$.

2.679. Compute making use of De Moivre's formula:

(1) $(3(\cos 45^\circ+i\sin 45^\circ))^6$; (2) $(2(\cos 15^\circ+i\sin 15^\circ))^4$;
(3) $(\cos 30^\circ-i\sin 30^\circ)^8$; (4) $(\cos(\pi/4)+i\sin(\pi/4))^{10}$;
(5) $(2(\cos 60^\circ+i\sin 60^\circ))^6$; (6) $(\cos 150^\circ+t\sin 150^\circ)^{10}$.

2.680. Prove the following equalities:

(1) $(\cos 60^\circ+i\sin 60^\circ)^6=1$; (2) $(2(\cos 45^\circ+i\sin 45^\circ))^4=-16$;
(3) $(3(\cos 45^\circ+i\sin 45^\circ))^6=-27$;
(4) $(2(\cos 56^\circ 15'+i\sin 56^\circ 15'))^8=16i$.

2.681. Divide:

(1) $4(\cos 75^\circ+i\sin 75^\circ)\div 0.4(\cos 30^\circ+i\sin 30^\circ)$;
(2) $3(\cos 45^\circ+i\sin 45^\circ)\div 1.5(\cos 135^\circ+i\sin 135^\circ)$;
(3) $4(\cos 240^\circ+i\sin 240^\circ)\div 2(\cos 60^\circ+i\sin 60^\circ)$;
(4) $(\cos 130^\circ+i\sin 130^\circ)\div(\cos 40^\circ+i\sin 40^\circ)$;
(5) $(\cos 130^\circ-i\sin 130^\circ)\div(\cos 40^\circ+i\sin 40^\circ)$;
(6) $(-\cos 100^\circ+i\sin 100^\circ)\div(\cos 40^\circ-i\sin 40^\circ)$;
(7) $2(\cos 107^\circ+i\sin 107^\circ)\div 5(\cos 47^\circ+i\sin 47^\circ)$.

2.682. Applying De Moivre's formula, express:
(1) $\cos 3\varphi$; $\cos 4\varphi$ in terms of $\cos \varphi$;
(2) $\sin 3\varphi$; $\sin 4\varphi$ in terms of $\sin \varphi$.

2.683. Find all the values of the given root:

(a) (1) $\sqrt[3]{1}$; (2) $\sqrt[3]{-1}$; (3) $\sqrt[3]{-27}$; (4) $\sqrt[4]{1}$;
(5) $\sqrt[4]{-1}$;

(b) (1) $\sqrt[4]{-625}$; (2) $\sqrt[4]{i}$; (3) $\sqrt{-1}$; (4) $\sqrt[6]{1}$;
(5) $\sqrt[3]{2-2i}$; (6) $\sqrt[4]{-24-8i\sqrt{3}}$;
(7) $\sqrt[4]{-1/2-i\sqrt{3}/2}$; (8) $\sqrt[n]{1}$.

§ 79. Solving Binomial and Trinomial Equations

2.684. Solve the following equations:

(a) (1) $x^3 + 1 = 0$; (2) $x^3 - 1 = 0$; (3) $x^3 + 27 = 0$;
(4) $8x^3 + 1 = 0$; (5) $27x^3 - 125 = 0$; (6) $x^3 - 27 = 0$;

(b) (1) $x^4 - 16 = 0$; (2) $x^4 - 81 = 0$; (3) $2x^3 + 3 = 0$;
(4) $0.5x^3 - 32 = 0$; (5) $x^4 + 1 = 0$; (6) $x^6 - 1 = 0$;

(c) (1) $2x^3 + 3 = 0$; (2) $x^4 + 81 = 0$;
(3) $x^4 + 625 = 0$; (4) $x^6 + 64 = 0$; (5) $5x^3 - 4 = 0$;
(6) $0.75x^3 - 18 = 0$.

2.685. Solve the following equations:

(a) (1) $x^6 - 3x^3 + 2 = 0$; (2) $x^6 + 4x^3 + 3 = 0$;
(3) $x^6 - 28x^3 + 27 = 0$; (4) $x^6 - 9x^3 + 8 = 0$;

(b) (1) $x^6 - 19x^3 - 216 = 4$; (2) $4 = -(x^{-6} + 3)/x^{-3}$;
(3) $(x^2 + 2x)^2 - (x + 1)^2 = 55$; (4) $x^6 - 2x^3 + 1 = 0$;

(c) (1) $\frac{8-x^{-2}}{x^{-4}} = \frac{x^{-2}-(3x+5x^2)}{5}$; (2) $x^{6/5} + 8 = 9\sqrt[5]{x^3}$;

(d) (1) $x^{10} - 10x^5 + 9 = 0$; (2) $x^6 + 2x^3 + 1 = 0$.

2.686. Set up a quadratic equation with real coefficients if one of its roots is equal to $28/(5 + i\sqrt{3})$.

2.687. Solve the quadratic equation $9x^2 - 18x + 13 = 0$ and verify the property of the sum and product of the roots of this equation.

2.688. Simplify the given expressions:

(1) $\frac{(1+i)^8}{(1-i)^4}+\frac{3+2i}{i(6-8i)}\cdot(1+i)^2$;

(2) $\frac{(1+i)^8}{0.4(1-i)^8}\cdot\left(\frac{2-i}{4+3i}-\frac{1-2i}{3+4i}\right)$;

(3) $\left(\sqrt{-4}+\frac{1}{\sqrt{-9}}\right)^2-\left(\frac{1}{\sqrt{-4}}+\sqrt{-9}\right)^2$;

(4) $\left(\frac{1-\sqrt{-1}}{2+\sqrt{-1}}-\frac{4+\sqrt{-4}}{2-\sqrt{-4}}+\frac{9-\sqrt{-9}}{3+\sqrt{-9}}\right)\cdot(1.4+8.2i)$.

2.689. Determine the real value of a for which the number $3i^3-2ai^2+(1-a)i+5$ will be: (1) real; (2) pure imaginary; (3) equal to zero.

2.690. Evaluate the given expressions:

(1) $6x^3-8x^2+11x-2$ for $x=(1-i\sqrt{5})/2$;
(2) $3x^3-8x^2+5x-1$ for $x=(1-i\sqrt{2})/2$.

2.691. Perform the indicated operations:

(1) $\sqrt{3+4i}$; (2) $\sqrt{-3-4i}$; (3) $\sqrt{4-3i}$;
(4) $\sqrt{1+i\sqrt{3}}$; (5) $\sqrt{7+i\sqrt{15}}$; (6) $\sqrt{5-i\sqrt{11}}$;
(7) $\sqrt{1-i}$.

2.692. Simplify:

(1) $\sqrt{8+6i}+\sqrt{8-6i}$; (2) $\sqrt{8+6i}-\sqrt{8-6i}$;
(3) $\sqrt{40+9i}+\sqrt{40-9i}$; (4) $\sqrt{-15+8i}+\sqrt{-15-8i}$.

2.693. Given an equation: $x^3-1=0$; x_1, x_2, and x_3 are roots of this equation. Prove that: (1) $x_1+x_2+x_3=0$; (2) $x_1\cdot x_2\cdot x_3=1$.

2.694. Given a complex number $z=x+iy$. Determine the set of points in the plane if: (1) $|z|=1$; (2) $|z|<1$; (3) $|z|>1$; (4) $1<|z|<2$; (5) $|z|\geqslant 3$; (6) $|z+i|\leqslant 1$; (7) $|z+2i|=|z-(-2i)|\leqslant 4$.

2.695. Given a complex number $z=x+iy$. Find the points in the plane which satisfy the given equation: (1) $|z+i|=|z+2|$; (2) $|z+i|=|z-2|$; (3) $|z-1|=|z-3|=|z-i|$.

2.696. Given a complex number $z = x + iy$. Find the points in the plane which satisfy the given inequalities:

(1) $\left|\frac{z-1}{z+1}\right| < 1$; (2) $|z-1| > |z-i|$; (3) $|z-2i| < |z-1|$;
(4) $|z+i| = |z+2|$; (5) $|z+2i| = |z+1|$.

§ 80. Bezout's Theorem and Its Application to Solving Higher-degree Equations

2.697. Find the remainder without performing the indicated division:

(1) $(3x^4 - 2x^3 + 5x^2 - x + 2) \div (x - 1)$;
(2) $(x^5 + x^4 - 3x^2 + 4x^2 - 5x - 6) \div (x + 1)$;
(3) $(2x^4 + x^3 - 3x^2 + 7x - 5) \div (x - 2)$;
(4) $(x^6 - x^5 - x^4 + 3x^3 - 2x^2 + 5x - 4) \div (x + 2)$;
(5) $(x^4 + 2x^3 + x^2 - 4x + 6) \div (x - 1/2)$.

2.698. Find the value of a for which the polynomial $2x^5 - 3x^3 + 11x^2 - x + a$, when divided by $x + 2$, yields a remainder equal to 3.

2.699. For what value of a does the polynomial $x^3 - 3x^2 + 5x + a$, when divided by $x - 1$, yields a remainder equal to 4? 5?

2.700. Find a and b if the polynomial $2x^3 - 3x^2 - ax + b$, when divided by $x + 1$, yields a remainder 7, and when divided by $x - 1$, yields a remainder 5.

2.701. Prove that the polynomial $x^6 - 2x^5 - 4x^4 + 3x^3 - 3x^2 + 11x - 6$ is divisible (without a remainder) by the product $(x + 1)(x + 2)(x - 3)$.

2.702. Can the fraction $\frac{x^4+2x^2-3}{x^3-2x^2-x+2}$ be reduced by: (a) $x+1$; (b) $x+2$; (c) $x-2$; (d) $x-3$?

2.703. Can the fraction $\frac{3x^4-4x^3+1}{x^3-3x-2}$ be reduced by: (a) $x-2$; (b) $x+1$; (c) $x-1$; (d) $x+2$?

2.704. Can the fraction $\frac{x^5+x^4-5x^3+5x^2+4x+4}{x^4+5x^3+5x^2-5x-6}$ be reduced by: (a) $x-1$; (b) $x+1$; (c) $x+2$; (d) $x-2$; (e) $x+3$; (f) $x-3$?

2.705. Find a and b such that the polynomial $x^4 - 10x^3 + ax^2 + bx - 24$ is divisible by $x^2 - x - 6$.

2.706. Find m and n for which the polynomial $x^4 + 7x^3 + mx^2 + nx + 20$ is divisible by $x^2 + 3x - 10$.

2.707. Factor the following polynomials:

(a) $x^2 - 7x^2 + 16x - 12$; (b) $x^3 - 4x^2 + x - 6$;
(c) $x^3 - 2x^2 - 4x - 8$; (d) $x^4 - 5x^3 + 2x^2 + 20x - 24$;
(e) $x^4 - 3x^3 + x^2 + 3x - 2$.

2.708. Solve the following equations:

(a) (1) $x^3 - 2x^2 + 6x - 27 = 0$; (2) $x^3 - 3x^2 - 4x + 6 = 0$;
(3) $x^3 - 5x^2 - 20x + 64 = 0$; (4) $x^3 - 8x^2 + 13x - 6 = 0$;
(5) $x^3 - 4x^2 + x + 6 = 0$; (6) $x^3 - 6x^2 + 11x - 6 = 0$;
(b) (1) $x^3 - 7x^2 + 16x - 12 = 0$; (2) $x^3 + 2x^2 - 4x - 8 = 0$;
(3) $x^4 - 3x^3 + x^2 + 3x - 2 = 0$; (4) $x^4 - 3x^3 - 8x^2 + 12x + 16 = 0$;
(5) $x^5 + 2x^4 + 2x^2 + 2x^3 + x = 0$; (6) $x^4 - 2x^3 + 5x^2 - 8x + 4 = 0$.

2.709. Solve the given equations knowing that they have one root in common:

(1) $2x^3 - 5x^2 + 6x - 2 = 0$ and $6x^3 - 3x^2 - 2x + 1 = 0$;
(2) $x^4 - x^3 - 22x^2 + 16x + 96 = 0$ and $x^3 - 2x^2 - 3x + 10 = 0$.

2.710. Find the given equations knowing that one of the roots of the first equation is twice one of the roots of the second equation:

(1) $x^3 - 10x^2 - 2x + 20 = 0$ and $x^3 - 7x^2 + 12x - 10 = 0$;
(2) $x^3 - 6x^2 - 39x - 10 = 0$ and $x^3 + x^2 - 20x - 50 = 0$.

2.711. Set up an equation given its roots:

(1) 1; 2; -2; (2) 2; $i\sqrt{3}$; $-i\sqrt{3}$; (3) $1 + i$; $1 - i$; $1 - 2i$; $1 + 2i$; (4) 2; $-i$; (5) 2; -2; $1 - i$; (6) $1 - i$; $3 + 2i$.

2.712. Solve the following equations:

(1) $x^4 - 6x^3 + 15x^2 - 18x + 10 = 0$ if $x_1 = 1 - i$;

(2) $x^4 - 2x^3 + 3x^2 - 2x + 2 = 0$ if $x_1 = i$;

(3) $x^4 - 5x^3 + 3x^2 + 4x - 2 = 0$ if $x_1 = 1 + i$;

(4) $6x^4 - 17x^3 + 23x^2 - 12x + 2 = 0$ if $x_1 = 1 - i$;

(5) $x^3 - 6x^2 + ax - 6 = 0$ if $x_1 = 3$;

(6) $x^3 - 3\sqrt{3x^2} + 7x - \sqrt{3} = 0$.

§ 81. Elements of Vector Algebra. Scalars and Vectors. Equal and Opposite Vectors. Null Vector. Addition and Subtraction of Vectors. Resolving a Vector into Two Noncollinear Vectors

2.713. (Orally). Which of the given quantities are scalars and which are vectors: volume, velocity, temperature, mass, length of a line segment, acceleration, strength of current, voltage of an electric field?

2.714. The vectors $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$, $\overrightarrow{DA}$ coincide with the sides of the square $ABCD$ (Fig. 3). Which of the given vectors are: (a) coplanar; (b) equal; (c) opposite?

2.715. Given three arbitrary vectors **a**, **b**, **c**. Construct the vector of their sum.

2.716. On the sides of the triangle ABC, construct the vectors $\overrightarrow{AB} = \mathbf{a}$, $\overrightarrow{BC} = \mathbf{b}$, and $\overrightarrow{CA} = \mathbf{c}$.

(a) What is the sum $\mathbf{a} + \mathbf{b} + \mathbf{c}$ equal to?

(b) The points M, N, and K are the midpoints of the sides of the triangle. Find the vectors $\overrightarrow{MA}$, $\overrightarrow{CN}$, and $\overrightarrow{BK}$ (Fig. 4).

(c) Three vectors whose moduli are equal have a common origin and form angles of 120°. What is their sum equal to?

2.717. (a) In the rhombus $ABCD$, M is the midpoint of the side CD. Express $\overrightarrow{AM}$ in terms of $\overrightarrow{AB}$ and $\overrightarrow{AD}$.

(b) In the square $ABCD$, O is the midpoint of the side BC. Express $\overrightarrow{DO}$ in terms of $\overrightarrow{DA}$ and $\overrightarrow{DC}$.

2.718. In the parallelogram $MKED$, A is the midpoint of the side MK. Express $\overrightarrow{DA}$ in terms of $\overrightarrow{MD}$ and $\overrightarrow{DE}$.

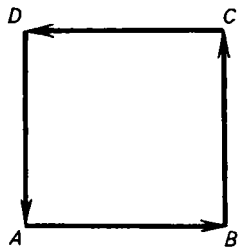

FIG. 3

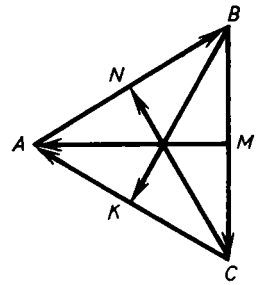

FIG. 4

2.719. Prove that the three medians of a triangle intersect at one point, and this point divides each median in the ratio 1 : 2.

2.720. Given two arbitrary vectors **a** and **b**. Construct the vector of their difference.

2.721. In the parallelogram $ABCD$, the vector $\overrightarrow{AB} = \mathbf{a}$, and the vector $\overrightarrow{AD} = \mathbf{b}$. Express the vectors $\overrightarrow{MA}$, $\overrightarrow{MB}$, $\overrightarrow{MC}$, and $\overrightarrow{MD}$ in terms of **a** and **b**, where M is the point of intersection of the diagonals of the parallelogram.

2.722. (a) $ABCD$ is a parallelogram, O is the point of intersection of the diagonals. Name the vectors: (1) $\overrightarrow{AO} + \overrightarrow{DO}$; (2) $\overrightarrow{OB} - \overrightarrow{AO}$; (3) $\overrightarrow{OC} + \overrightarrow{DA} + \overrightarrow{BA}$; (4) $\overrightarrow{AB} + \overrightarrow{CD}$.

(b) Find x if: (1) $\overrightarrow{AB} = x \cdot \overrightarrow{OD}$; (2) $\overrightarrow{AC} = x \cdot \overrightarrow{AO}$; (3) $\overrightarrow{OC} = x \cdot \overrightarrow{CA}$; (4) $\overrightarrow{BD} = x \cdot \overrightarrow{OB}$.

2.723. In the parallelogram $ABCD$, $\overrightarrow{AB} = \mathbf{a}$; $\overrightarrow{AD} = \mathbf{b}$ (Fig. 5). Verify analytically and geometrically the following equalities:

(a) $(\mathbf{a} + \mathbf{b}) - (\mathbf{a} - \mathbf{b}) = 2\mathbf{b}$;

(b) $(\mathbf{a} + \mathbf{b}) - (\mathbf{b} - \mathbf{a}) = 2\mathbf{a}$;

(c) $(\mathbf{a} + \mathbf{b}) - \mathbf{a} = \mathbf{b}$;

(d) $\mathbf{a}/2 + \mathbf{b}/2 = (\mathbf{a} + \mathbf{b})/2$;

(e) $(\mathbf{a} - \mathbf{b})/2 + \mathbf{b} = (\mathbf{a} + \mathbf{b})/2$;

(f) $\left(\mathbf{a} + \frac{\mathbf{b}}{2}\right) + \left(\mathbf{b} + \frac{\mathbf{a}}{2}\right) = 3/2\left(\frac{\mathbf{a}-\mathbf{b}}{2}\right)$;

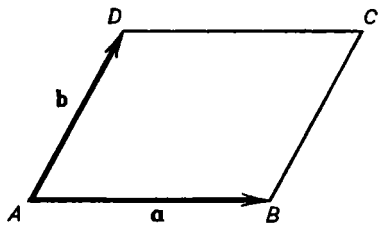

FIG. 5

(g) $(\mathbf{a} + \mathbf{b}/2) + (\mathbf{b} + \mathbf{a}/2) = (3/2)(\mathbf{a} + \mathbf{b})$.

2.724. Prove that the bisector of an angle in a triangle divides the opposite side into line segments which are proportional to the adjacent sides.

2.725. (a) Prove that the midline of a triangle is parallel to the base and its length is half the length of the base; (b) prove that the midline of a trapezoid is parallel to its bases, and its length is equal to half-sum of the lengths of its bases.

2.726. Given a triangle ABC in which $\overrightarrow{CA} = \mathbf{a}$ and $\overrightarrow{CB} = \mathbf{b}$. Resolve $\overrightarrow{CM}$ into components in the directions of $\mathbf{a}$ and $\mathbf{b}$, where M is the midpoint of $[AB]$ and: (1) $|\overrightarrow{AM}| : |\overrightarrow{MB}| = 3 : 10$; (2) $|\overrightarrow{AK}| : |\overrightarrow{KB}| = m : n$.

2.727. Prove that if M is the midpoint of the line segment $[AB]$ and O is an arbitrary point in space, then the equality $\overrightarrow{OM} = (1/2)(\overrightarrow{OA} + \overrightarrow{OB})$ is fulfilled.

2.728. Prove that if M is the point of intersection of the medians of a triangle ABC, and O is an arbitrary point in space, then the equality $\overrightarrow{OM} = (1/3)(\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC})$ is satisfied.

2.729. Prove that the midpoints of the bases of a trapezoid and the point of intersection of the extensions of its lateral sides lie on the same straight line (that is, they are collinear).

2.730. Given a parallelogram $MNPK$, $[MN] \cap [MK] = O$, $A \in [NP]$, $|AN| = |AP|$, $\overrightarrow{MN} = \mathbf{m}$, $\overrightarrow{MO} = \mathbf{n}$. Re-

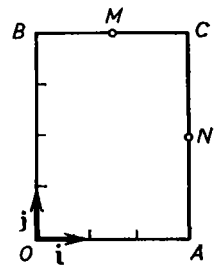

FIG. 6

solve the vector $\overrightarrow{AM}$ along the vectors **m** and **n**.

2.731. Given two points: A (3, 1) and B (−1, 1). Find the length of $\overrightarrow{OC}$ if O is the origin, and the point C lies both on AB and the x-axis.

2.732. The base of the pyramid $SABC$ is an isosceles right triangle ABC, the length of the hypotenuse equals $4\sqrt{2}$, $SC \perp ABC$, $|SC| = 2$. Find the size of the angle between two straight lines: one line passes through the vertex S and the midpoint of AC, the other through the vertex C and the midpoint of AB.

2.733. Given a cube $ABCDA_1B_1C_1D_1$. Find the size of the angle between the indicated vectors and the angle between the straight lines defined by the same pairs of points as the vectors: (1) $\overrightarrow{D_1A_1}$ and $\overrightarrow{CC_1}$; (2) $\overrightarrow{C_1B}$ and $\overrightarrow{DD_1}$; (3) $\overrightarrow{DC_1}$ and $\overrightarrow{A_1B}$; (4) $\overrightarrow{AC}$ and $\overrightarrow{DC_1}$: (5) $\overrightarrow{DA_1}$ and $\overrightarrow{B_1B}$.

2.734. Given: $|\mathbf{a}| = 2$, $|\mathbf{b}| = 1$, $(\widehat{\mathbf{a}, \mathbf{b}}) = 60°$. Find the cosine of the angle between the vectors: (1) **a** and **a** + **b**; (2) **b** and **a** − **b**, (3) **a** + **b** and **a** − **b**.

2.735. Given: $|\mathbf{a}| = |\mathbf{b}|$. Prove that the vectors **a** + **b** and **a** − **b** are perpendicular to each other.

2.736. Prove that the length of the median of a triangle is expressed in terms of the lengths of its sides by the formula $m_a = (1/2)\sqrt{2b^2 + 2c^2 - a^2}$.

§ 82. Multiplying a Vector by a Number. Coordinates of a Vector. Scalar Product of Vectors

2.737. Given an arbitrary vector **a**. Construct the following vectors: (a) 3**a**; (b) (3/2) **a**; (c) −2**a**; (d) −(2/3) **a**.

2.738. (1) Two unit vectors **i** and **j** are laid off on the sides $\overrightarrow{OA}$ and $\overrightarrow{OB}$ (respectively) of the rectangle $OACB$ (Fig. 6). Express the vectors $\overrightarrow{OA}$, $\overrightarrow{AC}$, $\overrightarrow{CB}$, $\overrightarrow{BO}$, $\overrightarrow{OC}$, and $\overrightarrow{BA}$ in terms of **i** and **j** if $|OA| = 3$ and $|OB| = 4$.

(2) M is the midpoint of BC and N the midpoint of AC. Determine the vectors $\overrightarrow{OM}$, $\overrightarrow{ON}$, and $\overrightarrow{MN}$.

2.739. Given two vectors $\mathbf{a}$ and $\mathbf{b}$. (a) Construct the vector $\mathbf{c}$ if $\mathbf{c} = 2\mathbf{a} - \mathbf{b}$. (b) Construct the vector $\mathbf{m} = \mathbf{a} + 3\mathbf{b}/2$.

(c) Construct the vector $\mathbf{n} = 3\mathbf{a}/2 - 2\mathbf{b}/3$. (d) Given two vectors $\mathbf{a}$ and $\mathbf{b}$. Show that $|\mathbf{a}| + |\mathbf{b}| \geqslant |\mathbf{a} + \mathbf{b}|$. How do we read this relation? When does the equality sign appear?

2.740. (a) What condition must be met by the vectors $\mathbf{a}$ and $\mathbf{b}$ so that the following relations occur: (1) $|\mathbf{a} + \mathbf{b}| = |\mathbf{a} - \mathbf{b}|$; (2) $|\mathbf{a} + \mathbf{b}| = |\mathbf{a}| - |\mathbf{b}|$; (3) $|\mathbf{a} - \mathbf{b}| = |\mathbf{a}| - |\mathbf{b}|$?

(b) Three points: A_1 (−1, 1), A_2 (−0.5, −2), and A_3 (2.5, 2) are given on the coordinate plane. Construct the sum $\overrightarrow{A_1A_2} + \overrightarrow{A_2A_3}$.

(c) Construct the sum $\overrightarrow{A_1A_2} + \overrightarrow{A_2A_3} + \overrightarrow{A_3A_4} + \overrightarrow{A_4A_5}$ if A_1 (−4, 4.2); A_2 (6, 4.2); A_3 (−2, −1); A_4 (1, 8); A_5 (4, −1).

(d) Five points are given on the coordinate plane: A (− 4, − 1); B (1, 1.5); C (3.5, 5); D (10, 6), and E (5, −4). Construct: $\overrightarrow{AB} - \overrightarrow{CE}$; $\overrightarrow{AB} - \overrightarrow{DC}$; $\overrightarrow{BE} - \overrightarrow{CD}$.

2.741. (a) When does the projection of a vector on an axis equal: zero; the length of the given vector (in absolute value)?

(b) The vectors $\mathbf{a}$ and $\mathbf{b}$ are symmetric with respect to the straight line m. What are the projections of these vectors on the axis: parallel to the straight line m; perpendicular to this line?

(c) Find the projection of the vector $\mathbf{a}$ on the axis which forms an angle of 120° with the given vector if $|\mathbf{a}| = 8$.

(d) Find the moduli of the sum and difference of the vectors $\mathbf{a} = \{3, -5\}$ and $\mathbf{b} = \{-1, 1\}$. Then find the scalar product of the vectors.

(e) A vector $\overrightarrow{OA}$, whose length is 10 cm, is directed at an angle of 38° to the x-axis. Compute the projections of the vector $\overrightarrow{OA}$ on the coordinate axes.

2.742. Prove that the triangles are isosceles and right-angled if their vertices are represented by the points: (a) A (−3, 4); B (4, 3), and C (0, 0); (b) A (−4, −2); B (−3, 5), and C (0, 1).

2.743. The vectors $\mathbf{a}$ and $\mathbf{b}$ make an angle $\varphi = (2/3)\pi$; $|\mathbf{a}| = 3$; $|\mathbf{b}| = 4$. Compute: (a) $\mathbf{a}\cdot\mathbf{b}$; (b) $\mathbf{a}^2$, $\mathbf{b}^2$; (c) $(\mathbf{a}+\mathbf{b})^2$; (d) $(3\mathbf{a}-2\mathbf{b})(\mathbf{a}+2\mathbf{b})$; (e) $(\mathbf{a}-\mathbf{b})^2$; (f) $(3\mathbf{a}+2\mathbf{b})^2$.

2.744. Given three unit vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ satisfying the condition: $\mathbf{a}+\mathbf{b}+\mathbf{c}=0$. Compute: $\mathbf{a}\cdot\mathbf{b}+\mathbf{b}\cdot\mathbf{c}+\mathbf{a}\cdot\mathbf{c}$.

2.745. Given three vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ satisfying the following conditions: $\mathbf{a}+\mathbf{b}+\mathbf{c}=0$, $|\mathbf{a}|=3$, $|\mathbf{b}|=1$, and $\mathbf{c}=4$. Compute $\mathbf{a}\cdot\mathbf{b}+\mathbf{b}\cdot\mathbf{c}+\mathbf{a}\cdot\mathbf{c}$.

2.746. Two vectors $\mathbf{a}$ and $\mathbf{b}$ form an angle $\varphi = \pi/6$; $|\mathbf{a}|=3$, $|\mathbf{b}|=1$. Compute the size of the angle α between the vectors $\mathbf{p}=\mathbf{a}+\mathbf{b}$, and $\mathbf{q}=\mathbf{a}-\mathbf{b}$.

2.747. Given two vectors: $\mathbf{a}\{4, -2\}$ and $\mathbf{b}\{6, -3\}$. Compute: (a) $\mathbf{a}\cdot\mathbf{b}$; (b) $\mathbf{a}^2$; (c) $\mathbf{b}^2$; (d) $(\mathbf{a}+\mathbf{b})^2$; (e) $(\mathbf{a}-\mathbf{b})^2$.

2.748. Given two vectors: $\mathbf{a}=2\mathbf{i}+3\mathbf{j}$; $\mathbf{b}=2\mathbf{i}-3\mathbf{j}$. Find the scalar product $\mathbf{a}\cdot\mathbf{b}$.

2.749. Given two vectors: $\mathbf{a}=4\mathbf{i}+\mathbf{j}$ and $\mathbf{b}=2\mathbf{i}-3\mathbf{j}$. Find the angle between these vectors.

§ 83. The Distance Between Two Points

2.750. Compute the distance of the given point from the origin (a) M (5, 7); (b) M (—3, 5); (c) A (12, —3); (d) B (x, y, z, t), where x, y, z, t are any real numbers.

2.751. Compute (to within 0.1) the distance between two points: (a) M_1 (2, 5) and N_1 (3, 8); (b) M_2 (—7, —10) and N_2 (—3, 0); (c) A (—2, 1, 3) and B (0, —1, 2); (d) A (—6, 8, 0) and D (2, 5, —2).

2.752. Find the coordinates of the point M' which is symmetric to the point M (—3, 4) about the origin and compute the distance $|M'M|$.

2.753. Compute the area of the triangle with vertices at the points: (a) A (— 2,—3), B (—2, 4), C (3, 4); (b) A (1, 0, —1), B (0, 2, —3), and C (4, 4, 1).

2.754. (a) Find the point M equidistant from the points O (0, 0), A (7, —7), and B (8, 0); (b) given three successive vertices of a parallelogram: A (1, —2, 3), B (3, 2, 1), and C (6, 4, 4). Find its fourth vertex.

2.755. Find the point M on the axis of ordinates whose distance from the point A (—3, 1) is equal to 5.

2.756. (a) On the axis of abscissas, find the point M equidistant from the points A (—1, 0) and B (7, —4); (b) on the axis of ordinates, find the point M equidistant from the point N (3, 5) and the origin.

CHAPTER 3

Trigonometrical Functions

§ 84. The Definition of Trigonometrical Functions. Signs of Trigonometrical Functions. The Values of Trigonometrical Functions of Some Angles (Arcs)

When solving exercises on transformations involving trigonometrical functions and proving trigonometrical identities, the student should bear in mind the following: (1) the denominator of a fraction must never be equal to zero; (2) for the tangent, cotangent, secant, and cosecant functions it is necessary to determine the permissible values of the arguments; (3) for the trigonometrical formulas containing parameters it is necessary to determine the permissible values of these parameters; (4) the results obtained should be investigated.

3.1. Find the radian measure of the following angles:

(a) 1°; 2°; 6°; 9°; 18°; 24°; 30°; 36°; 45°; 54°; 60°; 75°; 90°; 105°; 120°; 135°; 150°; 165°; 180°; 195°; 210°; 225°; 240°; 255°; 270°; 285°; 300°; 315°; 330°; 345°; 360°; 375°; 450°; 540°; 720°; 900°; 1080°;

(b) $9^\circ 15'$; $18^\circ 24'$; $24^\circ 36'$; $27^\circ 45'$; $36^\circ 24'$; $45^\circ 15'$; $54^\circ 42'$; $65^\circ 18'$; $75^\circ 45'$; $105^\circ 12'$; $115^\circ 6'$; $185^\circ 32'$; $325^\circ 35'$; $386^\circ 15'$.

3.2. Express in degrees the arcs whose radian measure is given by the following numbers: 0.5; 0.75; 1; 1.5; 1.75; 2; 2.25; 3; 3.36; $\pi/36$; $5\pi/18$; $3/2\pi$; $4/15\pi$; $\pi/10$; $3/10\pi$; $\pi/4$; $\pi/12$; 0.75π; 2.25π; 2.4π; 5.4π; 7.5π; $3/4\pi$; $\pi/6$; $\pi/3$; $2/3\pi$; $5/6\pi$; $11/6\pi$; $7/4\pi$.

3.3. Find the radian and degree measures of the angles of a triangle if their sizes are in the following ratios: (1) 3 : 4 : 5; (2) 2 : 3 : 7; (3) 1 : 4 : 7; (4) 1 : 3 : 8; (5) 2 : 3 : 4; (6) 5 : 6 : 7.

3.4. Find the radian and degree measures of the interior angles of a convex quadrilateral if their sizes are in the following ratios: (1) 1 : 2 : 4 : 5; (2) 4 : 5 : 7 : 8; (3) 5 : 7 : 9 : 15; (4) 4 : 7 : 11 : 14.

3.5. Given the following angles: $(2\pi/5)$; $(11\pi/20)$; $(7\pi/9)$; $(5\pi/6)$; $(7\pi/18)$; $(14\pi/15)$; 35°; 112°; $(7\pi/24)$; $(5\pi/36)$;

20°. Find their adjacent angles in the radian and degree measures.

3.6. Find the degree and radian measures of circular arcs:

(a) if the arc constitutes the following part of the circumference: 1/5; 1/12;, 5/6; 2/5; 1; 1.75; 0.375;

(b) if they are in the following ratio: (1) 1 : 5 : 7 : 11 : 12; (2) 1/3 : 5/9 : 5/12 : 25/36.

3.7. The radius of a circle is 20 cm long. Determine the length of the given arc of the circle if its degree measure is equal to: (1) 30°; (2) 11°25′; (3) 40°20′; (4) 73°; (5) 7°30′; (6) 15°; (7) 22°30′; (8) 36°; (9) 150°; (10) 210°.

3.8. Construct the acute angle α if:

(1) its sine equals: 1/2; 3/5; 5/7; (−2/3); (−3/4);
(2) its cosine equals: 2/3; 4/5; 7/8; 3/4; (−1/3);
(3) its tangent equals: 4/7; 3; (− 3.5); (−4.5); 22.5;
(4) its cotangent equals: 2; (−1/2); 5; (−2.5); 24.3.

3.9. Which of the angles $0 < \alpha < \pi/2$ and $0 < \beta < \pi/2$ is greater if:

(a) (1) $\cos \alpha = 3/5$ and $\cos \beta = 2/5$; (2) $\cos \alpha = 3/5$ and $\cos \beta = 3/7$;

(b) (1) $\cos \alpha = 3/5$ and $\cos \beta = 2/3$; (2) $\sin \alpha = 3/7$ and $\sin \beta = 4/7$;

(c) (1) $\sin \alpha = 3/5$ and $\sin \beta = 3/7$; (2) $\sin \alpha = 2/5$ and $\sin \beta = 3/7$;

(d) (1) $\tan \alpha = 2$ and $\tan \beta = 3$; (2) $\tan \alpha = 3/5$ and $\tan \beta = 4/7$;

(e) (1) $\cot \alpha = 2$ and $\cot \beta = 3$; (2) $\tan \alpha = -0.5$ and $\tan \beta = -1$.

3.10. In the unit circle, construct an angle α; in which quadrant is the terminal side of the angle α situated if:

(a) (1) $\sin \alpha = 2/5$; (2) $\sin \alpha = -1/3$;
(3) $\sin \alpha = -0.5$; (4) $\cos \alpha = 1/2$;

(b) (1) $\cos \alpha = -0.6$; (2) $\tan \alpha = 2$;
(3) $\sin \alpha = 0.7$; (4) $\cot \alpha = -2.5$;

(c) (1) $\cot \alpha = -1.4$; (2) $\cot \alpha = 3$;
(3) $\tan \alpha = -4.5$; (4) $\cos \alpha = -0.5$?

3.11. (a) Can the sine of any angle be equal to: 2/3; 0.5; 5/3; $-8/11$; $\sqrt{2}/2$; $2/\sqrt{3}$; $\sqrt{3}$; 3/4; π; $m+1/m$; $\frac{m+2}{m+1}$; $\frac{\sqrt{mn}}{m+n}$; $\frac{\sqrt{m(m+1)}}{2m+1}$ $(m \neq 0)$?

(b) Can the cosine of an angle be equal to: 0.7; 2; 5/7; $3/\sqrt{3}$; 25/31; $\sqrt{5}$; $-15/17$; 17/15; 5; π; n^2+1/n^2 $(n \neq 0)$?

(c) Can the tangent of an angle be equal to: 2; -1.5; 3.3; -0.5; 3/5; $\pi/2$; $\frac{1}{\pi}$; -2π?

(d) Can the cotangent of an angle be equal to: 0; -20; π^2; $-\pi$; $\frac{\pi}{4}$; $\frac{1}{3}$; 1.5; $\frac{\pi+1}{2} \cdot 2.5\pi$; $\frac{2\sqrt{2}+1}{4}$; $\frac{\sqrt{17}}{\sqrt{3}}$; $\frac{20}{\sqrt{2}+\sqrt{3}}$?

3.12. Which is greater:
(1) $\sin 25^\circ$ or $\tan 25^\circ$; (2) $\sin 320^\circ$ or $\tan 320^\circ$; (3) $\cos 75^\circ$ or $\cot 75^\circ$; (4) $\cos 160^\circ$ or $\cot 160^\circ$; (5) $\tan 1$ or $\sin 1$; (6) $\cot 1$ or $\cos 1$; (7) $\tan(\pi/4)$ or $\tan(\pi/3)$; (8) $\cos(3\pi/4)$ or $\cos(5\pi/6)$; (9) $\sin(5\pi/4)$ or $\sin(7\pi/6)$; (10) $\cot(11\pi/6)$ or $\cot(7\pi/4)$?

3.13. Prove the given inequalities and determine for what values of α the equality sign occurs:
(1) $|\cot\alpha| \geqslant |\cos\alpha|$; (2) $\tan\alpha + 1/\tan\alpha \geqslant 2$ $(\tan\alpha > 0)$; (3) $(\tan\alpha + 1/\tan\alpha)^2 \geqslant 4$; (4) $\sin^2\alpha - \sin\alpha\cos\alpha + \cos^2\alpha \geqslant \sin\alpha\cos\alpha$.

3.14. Given: $\tan\alpha = \cot\alpha$. Write a general formula for the angles satisfying this equality.

3.15. What trigonometrical functions of an obtuse angle of a triangle can take on negative values?

3.16. Determine the signs of the following expressions:

(a) $\sin 110^\circ$; $\cot 220^\circ$; $\tan(-95^\circ)$; $\cos 600^\circ$; $\cos 2$; $\cot 1.7$;

(b) $\cos 200^\circ$; $\sin 280^\circ$; $\cot(-230^\circ)$; $\cot 500^\circ$; $\sin(-3)$;

(c) $\tan 160^\circ$; $\cos 340^\circ$; $\sin(-130^\circ)$; $\tan 670^\circ$; $\tan 10$;

(d) $\sin(3\pi/4)$; $\tan 2$; $\cot(4\pi/5)$; $\cos(2\pi/3)$; $\sin 4$; $\cos(-1.5)$; $\tan 0.3$;

(e) $\sin 1.2 - \tan 1.2$; $\cos 0.5 - \cot 0.5$; $\sin 2 - \cos 3$; $\tan 2 - \sin 3$; $\tan 1 - \cot 1$;

(f) $\sin 110^\circ \cdot \sin 132^\circ$; $\tan 112^\circ \cdot \cos 165^\circ$; $\sin 1 \cdot \cos 2$;

(g) $\cos 318^\circ \cdot \tan(-240^\circ)$; $\sin 5 \cdot \tan 5$; $\sin 3 \cdot \cos 4 \cdot \cot 5$; $\sin(-118^\circ) \cdot \cos 118^\circ \cdot \tan 118^\circ$; $\cos 123^\circ \cdot \tan 231^\circ \cdot \sin 312^\circ$;

(h) $\tan\alpha \cdot \cot\alpha \cdot \sin^2\alpha \cdot \cos^2\alpha$; $\tan\alpha(1 + \cot\alpha) - \tan\alpha$; $\sin^2\alpha \cdot \cot^2\alpha + \cos^2\alpha \cdot \tan^2\alpha$; $\sin^2\alpha - 2\sin\alpha\cos\alpha + \cos^2\alpha + 1$;

$\tan^2\alpha - \sin^2\alpha$; $\cot^2\alpha - \cos^2\alpha$.

3.17. For what values of a are the given equalities possible to occur:

(1) $\sin\alpha = \frac{2}{a}$, $a \neq 0$; (2) $\cos\alpha = \frac{7}{a+1}$, $a \neq -1$;

(3) $\sin\alpha = \frac{a}{a^2+1}$; (4) $\sin\alpha = \frac{2a-3}{2-3a}$, $a \neq \frac{2}{3}$;

(5) $\cos\alpha = \frac{5a-2}{2-3a}$, $a \neq 2/3$; (6) $\tan\alpha = \frac{5a^2}{a^{-2}}$, $a \neq 2$;

(7) $\cot\alpha = \frac{8}{4-a^2}$, $a \neq \pm 2$; (8) $\tan\alpha = \frac{7a+11}{a^2-49}$, $a \neq \pm 7$?

3.18. In what part of the interval $(0, 2\pi)$ is the expression:

(1) $\sin x \cos x$ positive? (2) $\tan x \sin x$ negative?

(3) $\frac{\cos x}{\cot x}$ positive? (4) $\tan x \cdot \cos x$ and $\frac{\cos x}{\cot x}$ negative?

3.19. If α, β, and γ are the angles of a triangle, then:

(1) what sign has the sum $\sin\alpha + \sin\beta + \sin\gamma$?

(2) determine the sign of the given expression:

(a) $\cos(\alpha/2) + \cos(\beta/2) + \cos(\gamma/2)$;

(b) $\tan(\alpha/2) + \tan(\beta/2) + \tan(\gamma/2)$.

3.20. Using the definition of trigonometrical functions, compute the values of sine, cosine, tangent, and cotangent of the following angles (arcs): 0°, 135°, 2π, 180°, $7\pi/6$, 225°, 240°, 270°, $5\pi/3$, 315°, 330°, 2.5π.

3.21. (a) Prove that the length of any chord of the unit circle is equal to twice the sine of half the central angle corresponding to the chord;

(b) using the proved assertion, find the values of trigonometrical functions of the following angles (arcs): 30°, 45°, 60°, 120°, $3\pi/4$, $5\pi/6$.

3.22. Compute:

(a) (1) $\tan 45^\circ \cdot \sin 60^\circ \cdot \cot 30^\circ$;

(2) $3\cos 180^\circ + 5\cot 270^\circ - 2\sin 360^\circ - \tan 60^\circ$;

(3) $\cos(\pi/3) + 2\sin(\pi/6) + (1/2)\tan^2(\pi/3) - \cot(\pi/4)$;
(4) $\sin 150^\circ \cdot \sin 240^\circ - \tan 360^\circ \cdot \cos 315^\circ - \cot(-30^\circ) \times \sin^2 330^\circ + 3\tan^2 30^\circ$;
(5) $\cot 225^\circ - \cos 240^\circ - \sin^2 120^\circ + 0.75\tan^2 210^\circ$;
(6) $(3/2)\cot(5\pi/3) + \tan(7\pi/4) + (3/4)\cot^2(4\pi/3) + \cos^2(11\pi/6) + \sin(2\pi/3)$;

(b) (1) $\dfrac{4 - 2\tan^2 45^\circ + \cot^4 60^\circ}{3\sin^3 90^\circ - 4\cos^2 60^\circ + 4\cot 45^\circ}$;
(2) $3\sin^2(\pi/2) - (2\cot(\pi/4)^3 - 4\cos^2(\pi/6) + 3\cot^3(\pi/2)$;
(3) $m^2\sin^2(\pi/2) - n^2\sin(3\pi/2) + 2mn\cos 2\pi$;
(4) $\dfrac{a^2\cos 0 - b^2\sin(\pi/2)}{a\tan^2(\pi/4) + b\cos\pi - 2ab\cot(3\pi/2)}$.

3.23. (Orally). Evaluate the following expressions:
(1) $5\sin 90^\circ + 2\cos 0^\circ + 2\sin 270^\circ + 10\cos 180^\circ$;
(2) $3\tan 0^\circ + 2\cos 90^\circ + 3\sin 270^\circ - 3\cos 180^\circ$;
(3) $4\sin\pi - 2\cos(3\pi/2) + 3\sin 2\pi - \tan\pi$;
(4) $6 - \sin 2\pi - 3\cos\pi + 2\sin(\pi/2)\cos 2\pi$;
(5) $2\sin(\pi/4) + 3\cos 2\pi - 5\tan 2\pi$;
(6) $4\tan 2\pi - 2\sin(\pi/2) + 3\cos(3\pi/2) - 4\tan\pi$.

3.24. Prove the given inequalities:
(1) $\cos 30^\circ + \cos 45^\circ > 1$; (2) $\sin(\pi/6) + \sin(\pi/3) > 1$;
(3) $\tan(\pi/4) + \cot(\pi/6) > 2$; (4) $\sin(\pi/3) + \cos(\pi/6) > 1$.

3.25. Simplify:
(1) $m^2\tan 225^\circ - 4mn\sin 210^\circ + 3n^2\tan^2 30^\circ$;
(2) $a^3\tan(\pi/4) + a^2b\tan^2(\pi/3) + 9ab^2\cot^2(\pi/3) + 2b^3\cos(\pi/3)$;
(3) $2\cos^2 45^\circ - 3q\cot 45^\circ + q^2\tan^2 60^\circ - 2q^3\sin^2 45^\circ$;
(4) $a\cot(\pi/4) + 2b\sin(\pi/4) - 3a\cot(\pi/2) - 2a\cos 0$.

3.26. Evaluate the following expressions:
(a) $\sin\alpha + \cos\alpha$ for: (1) $\alpha = 0$; (2) $\alpha = \pi/2$; (3) $\alpha = 180'$; (4) $\alpha = 3\pi/2$;
(b) $\sin(\alpha + \pi/4) + 2\sin(\alpha - \pi/4) + 4\cos 2\alpha + 2\cos(\alpha + 3\pi/4)$ for: (1) $\alpha = \pi/4$; (2) $\alpha = 3\pi/4$;

(c) $\sin\alpha - \cos 2\alpha + \cos 3\alpha + \sin 2\alpha$ for: (1) $\alpha = 30°$; (2) $\alpha = \pi/4$; (3) $\alpha = \pi/3$; (4) $\alpha = 90°$;

(d) $\sin^2 t + 7\cos^2 t$; $\sin t - 2\cos 2t$ for: $t = 0$; $\pi/6$; $\pi/4$; $\pi/3$;

(e) $\sin^2 t - \cos^2 t$; $\sin^3 t + \cos^3 t$; $\sin^4 t + \cos^4 t$; $\sin^4 t - \cos^4 t$ for: $t = 0$; $\pi/6$; $\pi/4$; $\pi/3$.

3.27. Simplify:

(1) $a^2 \sin(\pi/2) + 2ab \sec\pi - b^2 \sin(3\pi/2)$;

(2) $a^2 \operatorname{cosec} 90° - 2ab \sin 180° + b^2 \operatorname{cosec} 270°$;

(3) $a^2 \sin 2\pi + 2ab \cos(3\pi/2) + b^2 \tan 2\pi$;

(4) $a^3 \cot 270° + b^3 \tan 180°$;

(5) $4a^2 \sin^4(\pi/4) - 6ab \tan^2(\pi/6) + (b \cot(\pi/4))^2$;

(6) $\dfrac{(2a\cos(\pi/3))^3 - (b\cos 0)^3 + (3ab\sin 0)^2}{(5a\cos(\pi/2))^3 + 2a\sin(\pi/6) - 2b\cos^2(\pi/4)}$;

(7) $\dfrac{\left(a\sin\left(\frac{\pi}{2}\right)\right)^2 - \left(b\tan\left(\frac{\pi}{4}\right)\right)^2}{2a^2\sin(\pi/6) - 2ab\cos 0 + (b\cot(\pi/4))^2}$.

§ 85. Relationships Between Trigonometrical Functions of the Same Argument

3.28. Express the trigonometrical functions of the argument α in terms of: (1) $\sin\alpha$; (2) $\cos\alpha$; (3) $\tan\alpha$; (4) $\cot\alpha$ and tabulate the obtained relationships.

3.29. Is it possible for the following equalities to occur for one and the same argument α:

(a) $\sin\alpha = -12/13$, $\cos\alpha = 5/13$, $\tan\alpha = -12/5$;

(b) $\sin\alpha = -3/5$, $\cos\alpha = -4/5$, $\tan\alpha = 3/4$;

(c) $\sin\alpha = 3/2$, $\cos\alpha = -1/2$, $\cot\alpha = -\sqrt{3}$;

(d) $\sin\alpha = 8/17$, $\cos\alpha = 15/17$, $\cot\alpha = 15/8$;

(e) $\cos\alpha = -1/7$, $\sin\alpha = 4\sqrt{3}/7$, $\tan\alpha = -4\sqrt{3}$.

3.30.

(a) Given: $\sin\alpha = -12/13$ and $\pi < \alpha < 3\pi/2$. Find: $\cos\alpha$; $\tan\alpha$; $\cot\alpha$.

(b) Given: $\tan\alpha = -3\frac{15}{16}$ and $\frac{\pi}{2} < \alpha < \pi$. Find: $\sin\alpha$; $\cos\alpha$; $\cot\alpha$; $\sec\alpha$; $\operatorname{cosec}\alpha$.

(c) Given: $\sin \alpha = -\sqrt{1-a^2}$; $\pi < \alpha < 3\pi/2$. Find: $\cos \alpha$, $\sec \alpha$. What must the number a be?

(d) Given: $\cos \alpha = 1 + a$; $0 < \alpha < \pi/2$. Find: $\sin \alpha$, $\operatorname{cosec} \alpha$. What must the number a be?

(e) For what values of α is the given equality satisfied: (1) $\sqrt{1-\sin^2\alpha} = -\cos \alpha$; (2) $\tan \alpha \cot \alpha = 1$?

3.31.

(a) Compute: $\dfrac{\tan \alpha + \cot \alpha}{\tan \alpha - \cot \alpha}$ if $\tan \alpha = \dfrac{\sqrt{6}}{2}$.

(b) $\dfrac{1+\tan \alpha}{1-\tan \alpha}$ if $\sin \alpha = \dfrac{3}{5}$, $0 < \alpha < \dfrac{\pi}{2}$.

(c) Compute: $\dfrac{\sin \alpha - \cos \alpha}{\sin \alpha + \cos \alpha}$ if: (1) $\tan \alpha = \dfrac{1}{3}$; (2) $\cot \alpha = \dfrac{20}{9}$.

(d) Compute: $\dfrac{\sin \alpha}{\sin^3 \alpha + 3\cos^3 \alpha}$ if: (1) $\tan \alpha = 2$; (2) $\cot \alpha = \dfrac{3}{4}$.

3.32. Given: $\sin \alpha + \cos \alpha = p$. Find:
(1) $\sin \alpha \cos \alpha$; (2) $\sin^2 \alpha - \cos^2 \alpha$; (3) $\sin^3 \alpha \pm \cos^3 \alpha$; (4) $\sin^3 \alpha \pm \cos^4 \alpha$; (5) $\sin^6 \alpha \pm \cos^6 \alpha$.

3.33. Given: $\tan \varphi + \cot \varphi = m$. Find:
(1) $\tan^2 \varphi + \cot^2 \varphi$; (2) $\tan^2 \varphi - \cot^2 \varphi$; (3) $\tan^3 \varphi + \cot^3 \varphi$;
(4) $\tan^6 \varphi + \cot^6 \varphi$; (5) $\tan^3 \varphi - \cot^3 \varphi$; (6) $\tan^6 \varphi - \cot^6 \varphi$.

3.34. Given: $\sin x \cos x = 0.4$. Compute:
(1) $\dfrac{\sin x + \cos x}{\sin x - \cos x}$; (2) $\dfrac{\sin x + 5\cos x}{2\cos x - 5\sin x}$ if $\tan x = 3$.

3.35. Prove the given inequalities for $0 < \alpha < \pi/2$: (1) $\tan \alpha + \cot \alpha \geqslant 2$; (2) $\tan^2 \alpha + \cot^2 \alpha \geqslant 2$; (3) $(\tan \alpha + \cot \alpha)^2 \geqslant 4$. Find out in each case for what α the equality sign occurs.

3.36. Simplify the following expressions:

(a) (1) $\sin^2\alpha + \cos^2\alpha + \cot^2\alpha$; (2) $1 - \sin\alpha \cot\alpha \cos\alpha$;

(3) $(1+\cos\alpha)\cdot\cot^2\alpha(1-\cos\alpha)$; (4) $\frac{1-\sin^2\alpha}{1-\cos^2\alpha}+\tan\alpha\cot\alpha$;

(5) $\frac{\sin^2\alpha-\tan^2\alpha}{\tan^2\alpha\cos^2\alpha-1}$;

(b) (1) $1+\tan^2\alpha-\tan\alpha(\cos^2\alpha+1)$; (2) $\frac{\sin^2\alpha}{1+\cos\alpha}$:

(3) $\tan^2\alpha-\sin^2\alpha$;

(4) $\frac{\sin^2\alpha}{\sec^2\alpha-1}+\frac{\cos^2\alpha}{\operatorname{cosec}^2\alpha-1}$; (5) $\frac{\sin^2 t}{\tan^2 t}+\frac{\cos^2 t}{\cot^2 t}+\frac{\tan t}{\cot t}$;

(c) (1) $(1+\sec 2\alpha+\tan 2\alpha)(1-\sec 2\alpha+\tan 2\alpha)$;

(2) $(1-\sin^2\alpha)\cot^2\alpha+1-\cot^2\alpha$;

(3) $\cos\alpha(1+\sec\alpha+\tan\alpha)(1-\sec\alpha+\tan\alpha)$;

(4) $\sin^2\alpha(1+\operatorname{cosec}\alpha+\cot\alpha)(1-\operatorname{cosec}\alpha+\cot\alpha)$;

(5) $\frac{2\sin^2\alpha+\cos^2\alpha-1}{1-\cot^2\alpha\sin^2\alpha}$; (6) $\frac{\tan 2\alpha+\cot 3\beta}{\cot 2\alpha+\tan 3\beta}$;

(d) (1) $\sqrt{\sin^2\alpha(1+\cot\alpha)+\cos^2\alpha(1+\tan\alpha)}$;

(2) $\frac{\cos\alpha\tan\alpha}{\sin^2\alpha}-\cot\alpha\cos\alpha$; (3) $\cos^4\alpha+\sin^2\alpha\cdot\cos^2\alpha+\sin^2\alpha$;

(4) $3(\sin^4\alpha+\cos^4\alpha)-2(\sin^6\alpha+\cos^6\alpha)$.

3.37. Evaluate the given expressions:

(1) $\frac{\cot\alpha+\cos\alpha}{1+\sin\alpha}$ for $\alpha=\frac{\pi}{4}$;

(2) $\frac{\cot\beta-\cot\alpha}{\tan\alpha-\tan\beta}$ for $\alpha=0.25\pi$; $\beta=\frac{\pi}{6}$:

(3) $\frac{\tan\alpha-\sin\alpha}{1-\cos\alpha}$ for $\alpha=0.9643$.

3.38. Prove the following identities:

(1) $\frac{1+\tan^4\alpha}{\tan^2\alpha+\cot^2\alpha}=\tan^2\alpha$;

(2) $\frac{1-2\sin^2\alpha\cos^2\alpha}{\sin^2\alpha\cos^2\alpha}-\frac{\sec^2\alpha}{\operatorname{cosec}^2\alpha}=\cot^2\alpha$;

(3) $\operatorname{cosec}^2\alpha-\cot^2\alpha-\frac{\cos^2\alpha-1}{1-\sin^2\alpha}=\sec^2\alpha$;

(4) $\operatorname{cosec}\alpha - \cot\alpha - \dfrac{\sin\alpha+\cos\alpha}{\cos\alpha} + \dfrac{\sec\alpha-1}{\sin\alpha} = -1;$

(5) $\dfrac{\sin^2\alpha-1}{1-\cos^2\alpha} - \sec^2\alpha + \tan^2\alpha = -\operatorname{cosec}^2\alpha;$

(6) $\dfrac{\tan\alpha}{\tan\alpha+\cot\alpha} = \sin^2\alpha;$ (7) $\dfrac{\cos^2\alpha-\sin^2\alpha}{\cot^2\alpha-\tan^2\alpha} = \dfrac{\sin^2\alpha}{\sec^2\alpha};$

(8) $\dfrac{\sin^2\alpha}{\sec^2\alpha-1} + \dfrac{\cos^2\alpha}{\operatorname{cosec}^2\alpha-1} = 1.$

3.39. Prove the listed identities:

(1) $\dfrac{1-\sin^4\alpha-\cos^4\alpha}{\cos^4\alpha} = 2\tan^2\alpha;$

(2) $\dfrac{(\sin\alpha-\cos\alpha)^2-1}{\cot\alpha-\sin\alpha\cos\alpha} - 2\sec^2\alpha + 2\tan\alpha\cdot\cot\alpha = 0;$

(3) $\dfrac{\sin^3\alpha-\cos^3\alpha}{\sin\alpha-\cos\alpha} - \dfrac{\cos\alpha}{\sqrt{1+\cot^2\alpha}} - 2\tan\alpha\cdot\cot\alpha = -1$ if $\pi/2 < \alpha < \pi;$

(4) $1-(\sin^6\alpha+\cos^6\alpha) = 3\sin^2\alpha\cdot\cos^2\alpha;$

(5) $\sin^3\alpha(1+\cot\alpha)+\cos^3\alpha(1+\tan\alpha) = \sin\alpha+\cos\alpha;$

(6) $(\sin\alpha+\cos\alpha)^2-1 = \dfrac{2}{\tan\alpha+\cot\alpha};$

(7) $\tan^2\alpha-\sin^2\alpha = \tan^2\alpha\cdot\sin^2\alpha;$

(8) $\cot^2\alpha-\cos^2\alpha = \cot^2\alpha\cdot\cos^2\alpha;$

(9) $\sin^6\alpha+\cos^6\alpha-\dfrac{3}{4}\left(\dfrac{1}{\sec^2\alpha}-\dfrac{1}{\operatorname{cosec}^2\alpha}\right)^2 = \dfrac{1}{4};$

(10) $\sqrt{1+2\sin\alpha\cos\alpha}-\sqrt{1-2\sin\alpha\cos\alpha} = 2\sin\alpha$ if $0 < \alpha < \pi/4.$

3.40. Find the greatest and least values of the given functions:

(1) $y = 3\sin^2 x + 2\cos^2 x;$ (2) $y = \cos^2 x - 2\sin^2 x;$
(3) $y = a\sin^2 x + b\cos^2 x$ $(a > 0,\ b > 0);$
(4) $y = 12\sin^2 x + 5\cos^2 x;$
(5) $y = 5 - 3\sin^2 x;$ (6) $y = 3\cos^2 x - 7\sin^2 x;$
(7) $y = 3/8 + (5/8)\cos^2 x - \sin^2 x;$ (8) $y = 0.28 - 0.28\cos^2 x - \sin^2 x.$

§ 86. The Basic Properties of Trigonometrical Functions

Evenness and Oddness of Trigonometrical Functions

3.41. Evaluate: $\cos(-45°)$; $\cot(-1.5\pi)$; $\tan(-\pi/6)$; $\sin(-\pi/3)$; $\cos(-\pi/3)$; $\cot(-\pi/4)$; $\cos(-\pi)$; $\sin(-3\pi/2)$; $\sec(-\pi/3)$; $\operatorname{cosec}(-\pi/4)$; $\sin(-135°)$; $\cos(-240°)$; $\tan(-210°)$.

3.42. Find the value of the given expressions:

(a) (1) $\sin(-45°) + 10\cot(-270°)\cdot\tan(-30°) + 1$;

(2) $\sin(-\pi/3) + 4\cos(-\pi/3)\cos(-\pi/4) - 5$;

(3) $-2\sin(-\pi/4) + 2\sqrt{2}\cos(-\pi/3) - 24\sin(-\pi/6)\cos(-\pi/3)$;

(4) $5\tan(-\pi/4) + 3\sin(-\pi/3) + 2\cos(\pi/3)$;

(b) (1) $\sin(-\pi/6) - \cos(-\pi/3) + \tan^2(-\pi/4) + |\cos\pi| + \cot(\pi/2)$;

(2) $|\sin(-\pi/4)| + \cos(-\pi/4) + 1.5\tan^2(-\pi/6) + 2\sin 2\pi + 1$;

(3) $\cos(-\pi/4) - 3|\cot(-\pi/3)| + \sin(-\pi/4) - 4\sin^2(-\pi/6)$;

(4) $(2a\cos(-\pi/3))^3 - 4(a\cot(-\pi/6))^3 + 6\tan 0$;

(5) $5\tan 0 + 2\sin(-\pi/6) - 3\cot(-\pi/4) + 4\cos(-\pi/2)$;

(6) $\operatorname{cosec}(-3\pi/2) + \cos(-\pi) + 2\tan(-\pi) + \sec(-2\pi)$.

3.43. Show that the following functions are even:

(1) $y = \sin^4 x$; (2) $y = x^2 + \tan^4 x$;

(3) $y = \dfrac{1+2\cos x}{x^6}$; (4) $y = \dfrac{\cos x \sin x}{\tan x + \cot x}$;

(5) $y = \dfrac{\sin x - \tan x}{\sin x + \cot x}$; (6) $y = \dfrac{3x + 2\sin x}{\cot^3 x}$;

(7) $y = \sin|x|$; (8) $y = |\sin x|$;

(9) $y = \dfrac{x\sin x}{x^2 + \cot^2 x}$; (10) $y = \dfrac{\tan^2 x + 1}{\sec x}$.

344. Show that the following functions are odd:

(1) $y=\cot^5 x$; (2) $y=x+\tan x$; (3) $y=\dfrac{\cos^4 x+1}{\sin^3 x}$;

(4) $y=-\tan^3 x$; (5) $y=\dfrac{x^2\cdot\tan x}{1+\sec x}$; (6) $y=\dfrac{x+\sin x}{\cot^2 x+\tan^2 x}$.

3.45. Decide whether the given function is even, odd, or neither:

(a) (1) $y=\sin x+\cot x$; (2) $y=\sin x+\cos x$;
(3) $y=\dfrac{1+\sin x}{1-\sin x}$;
(4) $y=x^4+\sin^2 x+1$; (5) $y=x^3+\sin^3 x+1$;
(6) $y=\log|\sin x|$;
(7) $y=x\cdot\sin x$; (8) $y=x\cdot\cos x$; (9) $y=3^{\sin x}$;
(10) $y=2^{\cos x}$;
(11) $y=\sin x\cdot\cos x$; (12) $y=x^3\cdot\cot x$;

(b) (1) $y=x^2\cdot\tan x$; (2) $y=\sin 3x\cdot\tan x$;
(3) $y=\sin 2x\cdot\sec x$;
(4) $y=\cos^2 x\cdot\operatorname{cosec} x$; (5) $y=(\sin x)/x$; (6) $y=(\cos x)/x$;
(7) $y=x^3/\cos x$; (8) $y=x^3/\sin x$; (9) $y=(x^2-1)/\cos^2 x$;
(10) $y=(x^2+1)/\sin^2 x$; (11) $y=(\tan^{41} x)/x^2$;
(12) $y=(1+\cos x)/x^2$.

Periodicity of Trigonometrical Functions

3.46. Evaluate:

(a) (1) $\sin(-1125°)$; (2) $\tan(7.25\pi)$; (3) $\cos(-855°)$;
(4) $\cot\left(-9\frac{1}{3}\pi\right)$; (5) $\sin 2023°+2\cos 485°+\sin 2383°-\cos 125°$;

(b) (1) $\cos(4.5\pi)$; (2) $\tan(10\pi/3)$;
(3) $\cot(10\pi/3)\cdot\sin(13\pi/3)$; (4) $\dfrac{2\sin 3281°-\sin 1481°}{2\tan 585°}$;
(5) $\sin^2(7/2\pi)+\cos^2(7\pi/2)$;

(c) (1) $4\sin 810°+3\cos 720°-3\sin 630°+5\cos 900°$;
(2) $5\tan 540°+2\cos 1170°+4\sin 990°-3\cos 540°$;
(3) $100\cot^2 990°+25\tan 540°-3\cos^2 990°$;

(4) $2 \tan 1095° + \tan 915° - \tan (-195°)$ if $\tan 15° = 2 - \sqrt{3}$;
(5) $\tan 210° + \cot 570° + 4 \tan 1290° + 3 \cot 390°$.

3.47. Write the general form of the angles if: (1) $\sin \alpha = 0.5$; $\sin \alpha = -\sqrt{2}/2$; (2) $\cos \alpha = \sqrt{3}/2$; $\cos \alpha = -1$; (3) $\tan \alpha = -\sqrt{3}$; $\tan \alpha = 1$; $\tan \alpha$ does not exist.

3.48. Find the least positive period of the following functions:

(a) (1) $y = \sin 2x$; (2) $y = \tan 3x$; (3) $y = \cos (x/2)$;
(4) $y = \cot (x/4)$; (5) $y = \cos (2x/3)$; (6) $y = \tan x + \cot x$;
(7) $y = \sec 3x$; (8) $y = \sin x + \cos x$; (9) $y = \operatorname{cosec} 4x$;
(10) $y = \sin (x/2) + \cos (x/2)$;
(11) $y = \sin^2 x + \tan x$; (12) $y = \cos^2 x + \cot x$;

(b) (1) $y = \sin^2 x + \sin (\pi/2 - x)$; (2) $y = |\sin x|$;
(3) $y = \cos 2x + 2 \cos 3x$; (4) $y = \sin (4\pi x + 2)$;
(5) $y = \tan (\pi x/2)$;
(6) $y = \tan (3x + \pi/4)$; (7) $y = 3 \sin (2x - \pi/8)$;
(8) $y = \sin (x/3 - \pi/4)$;
(9) $y = \cos (x/4 - \pi/4)$; (10) $y = 2^{\sin x}$; (11) $y = 1/(\sin 4x)$.

§ 87. A Change in the Value of a Trigonometrical Function with a Change in the Argument

3.49. In which quadrant do the sine and cosine simultaneously: (a) increase? (b) decrease? (c) $\sin x > 0$, $\cos x > 0$; (d) $\sin x < 0$, $\cos x < 0$?

3.50. Determine whether the given function increases or decreases:

(a) $y = \sin (x + \pi/3)$; $y = \cos (x - \pi/6)$; $y = \tan (x + \pi/2)$ if $\pi/6 \leqslant x \leqslant \pi/3$;

b) $y = -\cos x$; $y = 1 - \cos x$; $y = 1/\cos x$; $y = -1/\cos x$ if $\pi/4 \leqslant x \leqslant \pi/3$.

For $x \in (0°, 360°)$ single out the intervals of constant sign.

3.51. (a) In the given expression, how does the sign change: (1) $1/\sqrt{2} - \sin x$; (2) $1 + \cos x$; (3) $\cos x - \sqrt{3}/2$; (4) $\tan x - 1$ if $0° < x < 360°$.

(b) Which is greater: (1) 0.5 or sin 1; (2) 0.5 or cos 0.91; (3) sin 1 or sin 40°; (4) cot 0.1 or cot 1?

3.52. Determine whether the given function increases or decreases:

(1) $y = \sin 2x$ if $0 \leqslant x \leqslant \pi/4$;
(2) $y = \tan (3x/2)$ if $0 \leqslant x \leqslant \pi/6$;
(3) $y = \cos (x/2)$ if $\pi \leqslant x \leqslant 3\pi/2$;
(4) $y = 1 + \sin (-x)$ if $\pi/2 \leqslant x \leqslant 3\pi/2$;
(5) $y = 3 + \cos x$ if $\pi/2 \leqslant x \leqslant \pi$;
(6) $y = 3/2 - 2 \sin 2x$ if $\pi/4 \leqslant x \leqslant 3\pi/4$;
(7) $y = 2/(3 + \cos x)$ if $0 \leqslant x \leqslant \pi/2$;
(8) $y = 5/2 - 2 \cos 2x$ if $\pi/4 \leqslant x \leqslant 3\pi/4$;
(9) $y = 2 - \tan x$ if $2\pi/3 \leqslant x \leqslant 5\pi/6$;
(10) $y = 3 - \cos x$ if $\pi \leqslant x \leqslant 5\pi/4$.

3.53. Find the range of the given function:

(a) (1) $y = 1 + \sin \alpha$; (2) $y = 1 - \cos 2\alpha$; (3) $y = \sin |\alpha|$; (4) $y = |\cos \alpha|$;
(5) $y = 1/(3 - 2 \cos \alpha)$; (6) $y = 1/2 - \cos \alpha$; (7) $y = (1/2) \sin \alpha$;
(8) $y = 2 \cos (\alpha/2)$;

(b) (1) $y = \sec \alpha$; (2) $y = \operatorname{cosec} \alpha$; (3) $y = -3 + 2 \cos \alpha$;
(4) $y = 2 \cos (\alpha/2) + 1$; (5) $y = 3 \sin x + 5$; (6) $y = (1 - \sin x)/2$;
(7) $y = 1 + \cos^2 x$; (8) $y = -1 \sin^2 x$.

3.54. (a) On the unit circle, mark the angles (arcs) for which: (1) $\sin \alpha \geqslant 1/2$; (2) $\cos \alpha \geqslant \sqrt{3}/2$.

(b) On the unit circle, mark the intervals and write the general formula for the angle α if: (1) $\sin \alpha \geqslant 1/2$; (2) $\cos \alpha \geqslant \sqrt{3}/2$; (3) $\tan \alpha \geqslant \sqrt{3}$; (4) $\cot \alpha \geqslant 1$.

3.55. Find all the values of the angle α if; (1) $0° < \alpha < 90°$ and $\sin \alpha < 40°$; (2) $0° < \alpha < 90°$ and $\sin \alpha > \cos \alpha$; (3) $0° < \alpha < 180°$ and $\cos \alpha < \cos 72°$; (4) $180° < \alpha < 270°$ and $\tan \alpha < \tan 50°$;
(5) $180° < \alpha < 270°$ and $\tan \alpha < \cot \alpha$; (6) $0° < \alpha < 90°$ and $\cos \alpha > \cos 45°$;
(7) $90° < \alpha < 270°$ and $\sin \alpha < \sin 30°$; (8) $180° < \alpha < 270°$ and $\tan \alpha > \tan 60°$.

3.56. In the interval from 0° to 360°, find all the values of the angle α for which the given condition is satisfied:

(1) $\sin \alpha < \sqrt{2}/2$; (3) $\tan \alpha < 1$; (3) $\cos \alpha < -1/2$;
(4) $\cos \alpha < -\sqrt{2}/2$;
(5) $\cot \alpha > -1$; (6) $\tan \alpha > -\sqrt{3}$; (7) $\cot \alpha \geqslant 1/\sqrt{3}$;
(8) $\sin \alpha \leqslant 0.5$;
(9) $\cos \alpha \geqslant \sqrt{3}/2$; (10) $\tan \alpha \leqslant -1/\sqrt{3}$.

3.57. Determine the sign of each of the given differences:
(1) $\sin 2 - \sin 2.1$; (2) $\cos 1 - \cos 0.9$; (3) $\sin 1 - \sin 1.1$;
(4) $\cos 3.4 - \cos 3.5$.

3.58. What values can be assigned to α in the open interval (0°, 360°) for the given inequalities to be satisfied:
(1) $\sin 200° > \sin \alpha$; (2) $\cos \alpha < \cos 150°$; (3) $\tan \alpha < \tan 225°$;
(4) $\cot \alpha \leqslant \tan \alpha$; (5) $\sin \alpha \leqslant \cos \alpha$; (6) $\cos \alpha > \cos 45°$;
(7) $\tan \alpha > \cot 30°$; (8) $\sin \alpha < \sin 60°$; (9) $\cos 1° \cos \alpha$?

3.59. For what values of α does the function $y = \sin \alpha \cdot \cos \alpha$:

(1) Vanish?
(2) Take on positive values?
(3) Take on negative values?

Answer similar questions with respect to the following functions:

(1) $y = \cos \alpha \cdot \tan \alpha$; (2) $y = \sin \alpha \cdot \cot \alpha$; (3) $y = \cos \alpha \cot \alpha$;

(4) $y = \sin \alpha \cdot \tan \alpha$; (5) $y = 1 + \cos \alpha$; (6) $y = \sin \alpha - 1$;
(7) $y = \sqrt{3} + \tan \alpha$; (8) $y = 1 - \cot \alpha$.

§ 88. Reduction Formulas

3.60. Prove the following identities:

(a) (1) $\sin(45^\circ + \alpha) = \cos(45^\circ - \alpha)$;
(2) $\cos(\pi/4 + \alpha) = \sin(\pi/4 - \alpha)$;
(3) $\tan(\pi/4 + \alpha) = \cot(\pi/4 - \alpha)$;
(4) $\frac{1}{2\sin 10^\circ - 2\sin 50^\circ} = (-1/2)\sec 160^\circ$.

(b) (1) $\tan 435^\circ + \tan 375^\circ = 4$; (2) $\tan 255^\circ - \tan 195^\circ = 2\sqrt{3}$;
(3) $(\sin 160^\circ + \sin 40^\circ)\cdot(\sin 140^\circ + \sin 20^\circ) + (\sin 50^\circ - \sin 70^\circ)\cdot(\sin 130^\circ - \sin 110^\circ) = 1$;
(4) $(\cos 70^\circ + \cos 50^\circ)\cdot(\cos 310^\circ + \cos 290^\circ) + (\cos 40^\circ + \cos 160^\circ)\cdot(\cos 320^\circ - \cos 380^\circ) = 1$.

3.61. Simplify:

(a) (1) $\cot(\alpha - \pi/4)\cdot\cos(3\pi/4 - \alpha) - \sin(3\pi/4 - \alpha)$;
(2) $\cos^2(17\pi/8 - \alpha) + \cos(3\pi/8 + \alpha)\sin(\pi/8 - \alpha)$;
(3) $\frac{\sin(\pi/4-\alpha)}{\sec(\pi/4+\alpha)} - \cos^2(\pi/4+\alpha)$;

(b) (1) $2\cos^2(\alpha/2 - 3\pi/2) + \sqrt{3}\cos(5\pi/2 - \alpha) = 1$;
(2) $1 - \cos(2\alpha - \pi) - \cos(4\alpha + \pi) + \cos(6\alpha - 2\pi)$;
(3) $\sin(5\pi/2 - 2\alpha) + 2\sin^2(2\alpha - 3\pi/2) - 1$;

(c) (1) $1 + \cot(3\pi/2 - 4\alpha) + \operatorname{cosec}(5\pi/2 + \alpha)$;
(2) $1 + \cos(2\alpha + 270^\circ) + \sin(2\alpha + 450^\circ)$;
(3) $1 - \cos(2\alpha - 270^\circ) + \sin(2\alpha + 270^\circ)$.

3.62. Compute:

(a) (1) $\tan 41^\circ\cdot\tan 42^\circ\cdot\tan 43^\circ \ldots \tan 49^\circ$;
(2) $\cot 5^\circ\cdot\cot 15^\circ\cdot\cot 25^\circ \ldots \cot 85^\circ$;
(3) $(\sin 10^\circ + \sin 20^\circ + \sin 30^\circ + \sin 40^\circ) - (\cos 50^\circ + \cos 60^\circ + \cos 70^\circ + \cos 80^\circ)$;
(4) $\sin(-660^\circ) + \cos(-225^\circ) + \tan(-150^\circ) + \cot(-420^\circ) + \cot(-420^\circ)$;

(b) (1) $4 \sin 330^\circ \cdot \cos(-240^\circ) \cdot \tan 300^\circ - 2 \cos 150^\circ \times \tan(-315^\circ)$;

(2) $\sin 395^\circ \cdot \sin 505^\circ + \cos 575^\circ \cdot \cos 865^\circ + \tan 606^\circ \times \tan 1104^\circ$;

(3) $\cos 675^\circ \cdot \sin 405^\circ + \tan 562^\circ \cdot \tan 788^\circ + \dfrac{1}{\cos 660^\circ} \cdot \dfrac{1}{\cos 1200^\circ}$;

(4) $\sin 20^\circ \cdot \cos 70^\circ + \sin^2 110^\circ \cdot \cos^2 250^\circ + \sin 290^\circ \times \cos^2 340^\circ$;

(c) (1) $\sin 160^\circ \cdot \cos 110^\circ + \sin 250^\circ \cdot \cos 340^\circ + \tan 110 \times \tan 340^\circ$;

(2) $\dfrac{\sin^2(-225^\circ) - \cos 675^\circ \cdot \cot 240^\circ}{\cos 300^\circ + \sin(-315^\circ) \cdot \tan^2(-210^\circ) \cos 750^\circ}$.

3.63. Simplify:

(a) (1) $\tan^2(11\pi/3) + \sin 5.5\pi - \cos(-26\pi/3)$;
(2) $2 \sin(19\pi/6) + \cot(-7.25\pi) + 4 \cos^2(31\pi/6)$;

(3) $\dfrac{\tan(\pi - \alpha) \cdot \cos(\pi + \alpha)}{\tan(3\pi/2 + \alpha) \cdot \cos(1.5\pi - \alpha)}$;

(b) (1) $\tan(\pi - \alpha) \cdot \tan(3\pi/2 + \alpha) + \sin(2\pi - \alpha) \times \cos(3\pi/2 + \alpha) - \cos^2(\pi - \alpha)$;

(2) $\dfrac{\sin(x - \pi) \cdot \cos(x - 2\pi) \cdot \sin(2\pi - x)}{\sin(\pi/2 - x) \cdot \cot(\pi - x) \cdot \cot(3\pi/2 + x)}$;

(3) $\dfrac{\cos 304^\circ \cdot \tan 416^\circ - \tan 214^\circ \tan(-56^\circ)}{\cot 214^\circ + \cos 326^\circ \cdot \cot(-56^\circ)}$.

3.64. Prove the following identities:

(a) 1) $\sin(3\pi/2 + \alpha) + \cot(\pi/2 + \alpha) - \sin(\pi - \alpha) \cot(\alpha - 3\pi/2) = -(\sin\alpha + \cos\alpha)$;

(2) $\dfrac{\sin^2(3\pi/2 + \alpha)}{\cot^2(\alpha - 2\pi)} + \dfrac{\sin^2(-\alpha)}{\cot^2(3\pi/2 + \alpha)} = 1$;

(3) $\sin(2\pi - \varphi) \cdot \tan(3\pi/2 - \varphi) - \cos(\varphi - \pi) - \sin(\varphi - \pi) = \sin\varphi$;

(4) $\sin(\pi/3 - \beta) \tan(2\pi/3 + \beta) \cdot \cos(5\pi/3 + \beta) + \tan(\pi + \beta) \cdot \tan(3\pi/2 - \beta) = \cos^2(\pi/3 - \beta)$;

(b) (1) $\cos 20^\circ + \cos 40^\circ + \cos 60^\circ + \ldots + \cos 160^\circ + \cos 180^\circ = -1$;

(2) $\cot 15^\circ + \cot 30^\circ + \cot 45^\circ + \ldots + \cot 165^\circ = 0$;

(3) $\dfrac{\sin^2(-212^\circ)\cdot\cos 302^\circ-\cos^3(-148^\circ)}{\sin(-82^\circ)\cdot\cos(-8^\circ)+\sin 368^\circ\cdot\sin(-172^\circ)-\sin 58^\circ\cdot\sin 148^\circ}=\cos 32^\circ-\sin 32^\circ;$

(4) $\dfrac{\cos\alpha\tan(\pi+\alpha)\tan(3\pi/2-\alpha)\cos(13\pi/2+\alpha)}{\sin(\pi/2+\alpha)\cdot\sin(7\pi/2-\alpha)}=\tan\alpha;$

(c) (1) $\dfrac{\cot(270^\circ-\alpha)}{1-\tan^2(\alpha-180^\circ)}\cdot\dfrac{\cot^2(360^\circ-\alpha)-1}{\cot(180^\circ+\alpha)}=1;$

(2) $\dfrac{\cos^2(\alpha-270^\circ)}{\operatorname{cosec}^2(\alpha+90^\circ)-1}+\dfrac{\sin^2(\alpha+270^\circ)}{\sec^2(\alpha-90^\circ)-1}=1;$

(3) $\dfrac{\sin^2 450^\circ+\sin(-270^\circ)\cos 240^\circ\cdot\tan(-405^\circ)}{\cot^2 510^\circ-2\sin 810^\circ\cos^2(-150^\circ)}=1.$

3.65. Find the range of the given function:

(a) $y = 1/2 + \cos(\pi - x)$ if $3\pi/2 < x < 2\pi$;

(b) $y=\dfrac{\sin^2(\pi-x)}{+\sin(\pi/2-x)}$ if $\pi/2<x<\pi$;

(c) $y = \cot(\pi - x) + 1$ if $5\pi/4 < x < 3\pi/2$.

3.66. Find:

(1) $\tan(270^\circ + \alpha)$ if $\sin\alpha = 7/25$ and $90^\circ < \alpha < 180^\circ$;

(2) $\cos(\pi - \alpha)$ if $\sin\alpha = -0.8$ and $\pi < \alpha < 3\pi/2$;

(3) $\cot(360^\circ - \alpha)$ if $\cos\alpha = -0.6$ and $\pi/2 < \alpha < \pi$;

(4) $8\sin(450^\circ - \alpha) + 5\sin(\alpha - 90^\circ)$ if $\alpha = 2.4458$;

(5) $3.\tan(3\pi/2 - \alpha) + 2\cot(\pi + \alpha)$ if $\alpha = 75^\circ$;

(6) prove that $\dfrac{1+\cos 2x}{\cos(270^\circ+x)+\sin 3x}<0$ for $x=-120^\circ$.

§ 89. Trigonometrical Functions of a Numerical Argument. Graphs and Properties of Trigonometrical Functions

3.67. (a) What is the arrangement on the number line and on the number circle of the points corresponding to the numbers: (1) a and $-a$; (2) a and $a \pm \pi$; (3) $a + \pi$ and $a - \pi$; (4) a and $a + 2k\pi$, where $k = 0, \pm 1, \pm 2, \ldots$

(b) Write the set of numbers corresponding to: (1) the given point A (a) on the number circle; (2) the point symmetric to the point A (a) about: the axis of ordinates, the axis of abscissas, the origin.

3.68. Determine the sign of the given difference:

(1) $\cos\left(3\tan^2\frac{\pi}{6}\right)-\cot 1$;

(2) $\tan\left(\sin\frac{5}{6}\pi\right)-\sin\left(\frac{1}{2}\tan\frac{5}{4}\pi\right)$.

3.69. Prove the following inequalities:

(a) (1) $\sin(\pi/2-\cos\alpha)>0$; (2) $|\cos(3\pi/2+\sin\alpha)|\leqslant 1$; (3) $|\cos x+3\sin x|<4$;

(b) (1) $5\sin x+2\cos x+1<8$; (2) $5\cos x+2\sin x-1<6$; (3) $|3\cos x-2\sin x|<5$.

3.70. Find the domain of definition of the given function:

(a) (1) $y=\sqrt{1-2\cos x}$; (2) $y=\cos(\log x)$; (3) $y=\sqrt{\sin x}$;

(b) (1) $y=1/(1+\tan x)$; (2) $y=\sqrt{\cos(\sin x)}$; (3) $y=\sqrt{x}/(\sin\pi x)$;

(c) (1) $y=\sin(1/x)$; (2) $y=\tan\sqrt{x}+\operatorname{cosec} x$; (3) $y=1/(1+\sin x)$.

3.71. Find the range of the given function:

(a) (1) $y=1+\sin^2 x$; (2) $y=|\cos x|$; (3) $y=1/(2-\cos 2x)$;

(b) (1) $y=\tan(\sin\pi x)$; (2) $y=1-2|\sin 3x|$; (3) $y=x\cdot\cot x$;

(c) (1) $y=2^{\sin x}+1$; (2) $y=\sec(\pi\cos x)$; (3) $y=1-|\sin x|$;

(d) (1) $y=\sin(\sin x)$; (2) $y=1-\cos(\sin x)$.

3.72. Which of the following functions are periodic:

(a) (1) $y=2^{\cos x}$; (2) $y=3^{\sin x}$; (3) $y=\log(\sin x)$; (4) $y=x+\sin x$;

(b) (1) $y=\frac{1}{\tan x+1}$; (2) $y=x\cdot\cot x$; (3) $y=\frac{x}{\cos x}$; (4) $y=\tan x+\cos x$.

3.73. Determine the period of the given function: (1) $y = \tan(x - \pi/4)$; (2) $y = \cot(x/2 + 1)$; (3) $y = 3\sin(3x + \pi/6)$; (4) $y = 4\cos 2x$; (5) $y = \cos(x/2)$; (6) $y = \sin 4x$.

3.74. Find the roots of the given function:

(a) (1) $y = \cos 2x$; (2) $y = \sin 3x$; (3) $y = \tan(x/2)$; (4) $y = \cot(\pi + x)$;

(b) (1) $y = \cos(\sin x)$; (2) $y = \sin(\tan x)$; (3) $y = \sec x$; (4) $y = \operatorname{cosec}(x + 1)$;

(c) (1) $y = 1 + \tan^2 x$; (2) $y = \cot(\cos x)$; (3) $y = \sin(\sec x)$; (4) $y = \cot^2 x - 1$.

Find the greatest values of the given functions:

(1) $y = \frac{1}{2}\cos\frac{x}{2}$; (2) $y = 2\sin\left(x - \frac{\pi}{4}\right)$; (3) $y = \sin^2 x$; (4) $y = \cos(-x)$.

Find the least values of the given functions:

(a) (1) $y = \cos(2x + \pi/2)$; (2) $y = \sin(x + \pi/4)$; (3) $y = \sin(x + 1)$;

(b) (1) $y = -\cos x = y$; (2) $|\tan(x + \pi/3)|$; (3) $y = |\cot x/2|$.

For what values of x are the following inequalities valid: (a) $\cos x > 0$; (b) $\sin x > 0$; (c) $\tan x < 0$; (d) $\cot x < 0$; (e) $\sin(3x + 1) > 0$; (f) $\cos 3x < 0$; (g) $\tan(-2x) < 0$?

3.75. Compute: $\sin 0.785$; $\sin 0.4$; $\sin 0.3$; $\cos 0.6$; $\cos 0.5$; $\cos 0.7$; $\cos 2$; $\sin 50$; $\tan\left(\cot\frac{\pi}{4}\right)$; $\cot\left(\tan\frac{\pi}{3}\right)$; $\tan 1.4$; $\tan 1.8$; $\cos(\pi - 1)$; $\tan 0.5231$.

3.76. Evaluate the given function:

(a) (1) $y = \sin(x/2)$ for: $x = 0.2375$; $x = 1.249$; $x = 14.78$; (2) $y = \cos(\cos x)$ for: $x = 0.7632$; $x = 3$; $x = 0.9$; (3) $y = \sin(\sin x)$ for: $x = 1.483$; $x = 0.4529$; $x = 1/2$;

(b) (1) $y = \sin(\cos x)$ for: $x = 0.4529$; $x = 0.5$; (2) $y = \sin x^2$ for: $x = 0.5$; $x = 1$; $x = 5$; (3) $y = \cos x^2$ for: $x = 0.5$; $x = -1$; $x = 3$.

3.77. Determine the sign of the given expression: (a) $\sin 2 \cdot \sin 3 \cdot \sin 5$; (b) $\cos 2 \cdot \cos 3 \cdot \cos 5$; (c) $\cos 2.3 - \cos 2$; (d) $\tan 2.7 - \tan 5$.

3.78. Which is greater:

(1) $\cos 2 \cdot \cos(-2) - 2$ or $\cos 2.7$;

(2) $(\sin \pi - 1)\sin 3$ or $\sin 8$;

(3) $\sin 1$ or $\sin 10°$;

(4) $\cos 2 \cdot \tan 2$ or $\sin 2 \cdot \cos 0.5$.

3.79. Using the graph $y = \sin x$, sketch the graphs of the following functions:

(1) $y = -\sin x$; (2) $y = \sin x - 2$; (3) $y = \sin 2x$;
(4) $y = \sin |x|$; (5) $y = -0.5 \sin x$;
(6) $y = \sqrt{1 - \cos^2 x}$; (7) $y = 1/\sin x$; (8) $y = -\sin |x|$;
(9) $y = \sin (x - \pi/4)$.

3.80. Graph the following functions:

(1) $y = \cos 2x$; (2) $y = \cos (x/2)$; (3) $y = \cos (x - \pi/3)$;
(4) $y = -\cos x$; (5) $y = \tan 2x$; (6) $y = \tan (x/2)$;
(7) $y = |\tan x|$; (8) $y = \tan x \cdot \cot x$.

§ 90. Inverse Trigonometrical Functions

3.81. Find (orally):

(a) arcsin (1/2); arcsin 1; arcsin (—1); arcsin 0;

(b) arctan 1; arctan $(-\sqrt{3})$; arctan 0, arctan $(\sqrt{3}/3)$;

(c) arccos 0; arccos (—1/2); arccos (—1); arccos $(\sqrt{3}/2)$;

(d) arccot (—1); arccot 0; arccot $(-\sqrt{3}/3)$; arccot $\sqrt{3}$;

(e) arcsin $(-\sqrt{3}/2)$; arctan (—1); arccot 1; arccos $(\sqrt{2}/2)$.

3.82. Which of the functions $y = \arcsin m$, $y = \arccos m$, $y = \arctan m$, $y = \operatorname{arccot} m$ and for what values of m can take on: (1) negative values? (2) values greater than $\pi/2$? (3) values equal to zero? (4) values equal to 1?

3.83. Using the tables, find in degrees and radians:

(a) arcsin 0.6820; arcsin (—0.3420);

(b) arctan 3.655; arctan 0.6420;

(c) arccos (—0.4384); arccos (—0.8910);

(d) arccot 0.0840; arccot (—4348).

3.84. In the unit circle, construct the following angles (arcs):

(a) (1) arcsin 2/5; (2) arcsin (—0.6);

(b) (1) arccos 0.75; (2) arccos (—2/3);

(c) (1) arctan (4/3); (2) arctan (—1.4);

(d) (1) arccot (2/3); (2) arccot (—2.5).

3.85. Prove geometrically:

(a) $\arcsin(-m) = -\arcsin m$, $|m| \leqslant 1$;

(b) $\arccos(-m) = \pi - \arccos m$, $|m| \leqslant 1$;

(c) $\arctan(-m) = -\arctan m$, $m \in R$;

(d) $\operatorname{arccot}(-m) = \pi - \operatorname{arccot} m$, $m \in R$.

3.86. Find the function inverse to the given one:

(a) (1) $y = (1/2)\sin x$; (2) $y = \cos 2x$; (3) $y = (1/4)\tan x$;

(b) (1) $y = 3\tan(x/2)$; (2) $y = \cot 3x$; (3) $y = 3\sin 2x$; (4) $y = \arcsin(x/2)$;

(c) (1) $y = 2\arctan x$; (2) $y = (1/2)\arccos x$; (3) $y = 5\arccos 3x$;

(d) $y = 1 + 3\arccos(x/2)$; (2) $y = 3\operatorname{arccot}(2/x)$; (3) $y = 5\cot(x/2)$.

3.87. Find the domain of the given function:

(a) (1) $y = \arccos(1 - x)$; (2) $y = \operatorname{arccot}(1/x)$; (3) $y = \arccos(1/x)$;

(b) (1) $y = \operatorname{arccot}\sqrt{x}$; (2) $y = \arccos 2^x$; (3) $y = \arcsin\frac{2x}{1+x^2}$;

(c) (1) $y = \arccos(\cos x)$; (2) $y = \arcsin(\sin x)$; (3) $y = \arcsin|\tan x|$;

(d) (1) $y = \arccos|x|$; (2) $y = \arctan|x|$; (3) $y = \sqrt{\arcsin x}$;

(e) (1) $y = \arcsin\sqrt{1-x} + \arcsin\sqrt{x}$; (2) $y = \arcsin\frac{x}{x-1}$; (3) $y = \arctan x - \arctan(1/x)$; (4) $y = \arccos x + \arccos(x/2 + (1/2)\sqrt{3-3x^2})$;

(f) (1) $y = \arcsin 3x$; (2) $y = \arctan(x/2)$; (3) $y = \arcsin(x+1)$; (4) $y = \arcsin\sqrt{x}$; (5) $y = \arcsin\frac{1}{x-2}$;

(g) (1) $y = \arcsin(\log x)$; (2) $y = \arcsin(1 + \tan^2 \pi x)$; (3) $y = \arccos|x+1|$; (4) $y = \arccos\sqrt{x^2+2x+1}$.

3.88. Find the domain and range of the given function:
(a) $y = \arcsin x$; (b) $y = \pi + \arccos x$; (c) $y = -(1/3) \times \arctan x$; (d) $y = \arcsin |x|$; (e) $y = \dfrac{5}{\arcsin (x-2)}$; (f) $y = 1 + \arctan (x-1)$; (g) $y = 2\pi - \operatorname{arccot} |x|$; (h) $y = \dfrac{1}{2 + \arccos x}$; (i) $y = \dfrac{1}{\arcsin x}$ if $1/2 \leqslant x \leqslant 1$.

3.89. Find x from the given equation:
(a) (1) $\arccos (x - 1) = \pi/2$; (2) $\arctan x = -64°$; (3) $\arctan (x + 1) = 0$; (4) $\arccos (x/2) = 3\pi/4$;
(b) (1) $\operatorname{arccot} x = \pi/18$; (2) $\operatorname{arccot} (x - 1) = 14°$; (3) $\operatorname{arccot} 0.2x = 2\pi/3$; (4) $\arctan 2x = 72°$;
(c) (1) $\arccos (x + 2) = \pi/3$; (2) $\arctan x = -\pi/4$; (3) $\operatorname{arccot} (x - 3) = \pi/3$; (4) $\arcsin (3x + 2) = 2\pi/5$.

3.90. Evaluate the following expressions:
(a) (1) $2 \arcsin (\sqrt{2}/2)$; (2) $3 \arctan 1$; (3) $\arctan 0 + (1/3) \arctan \sqrt{3}$; (4) $0.2 \arccos (-1) - \arcsin 0.5$;
(b) (1) $\operatorname{arccot} (-1) + \arccos (0.5\sqrt{2}) + (1/2) \arccos (-1) + 2 \arcsin (-0.5\sqrt{3})$;
(2) $\arcsin (\sqrt{3}/2)$; (3) $\arcsin (-1/2)$; (4) $\arccos (-1/2)$; (5) $\arctan (-\sqrt{3})$.

3.91. Compute;
(1) $\cos (\arccos 0.45)$; (2) $\tan (\arctan (-3)) + \arccos (\cos 2\pi)$; (3) $\arctan (\cot (\pi/6))$; (4) $\operatorname{arccot} (\sin (2\pi/3)) \cos (\arcsin (-4/5))$; (5) $\arcsin (\sin (2\pi/3))$; (6) $\arccos (\cot (-\pi/4))$;
(7) $\sin (\arccos (-1/2) + 3 \arcsin (\sqrt{3}/2))$;
(8) $\cos (\arctan (\sqrt{3}/3) - 2 \operatorname{arccot} 1)$; (9) $\operatorname{arccot} (\tan (-37°))$.

3.92. Solve the following equations:
(1) $\arccos x = 2\pi/3$; (2) $\arcsin (x + 2) = -\pi/6$; (3) $\arctan x = -\pi/4$; (4) $(\arctan x)^2 - 4 \arctan x + 3 = 0$;
(5) $6 \arcsin (x^2 - 6x + 8) = \pi$;
(6) $4 \arctan (x^2 - 3x - 3) - \pi = 0$;
(7) $\arccos (\cos -\sqrt{3}/2) = x$;
(8) $\arctan x + \arctan 3x = \pi/2$.

3.93. Prove that:
(1) $\arcsin x + \arccos x = \pi/2$;
(2) $\arccos x = \begin{cases} \arcsin \sqrt{1 - x^2} & \text{if } 0 \leqslant x \leqslant 1; \\ \pi - \arcsin \sqrt{1 - x^2} & \text{if } -1 \leqslant x < 0; \end{cases}$

(3) $\arcsin(-x) = -\arcsin x$;

(4) $\arccos(-x) = \pi - \arccos x$.

3.94. Sketch the graphs of the following functions: (1) $y = \arcsin(x/2)$; (2) $y = 2 + \arcsin x$; (3) $y = \arcsin(1/x)$; (4) $y = |\arccos x|$; (4) $y = \pi - \arccos x$; (5) $y = \arccos(1/2)$; (6) $y = \arctan 2|x|$.

3.95. Prove that:

(1) $\tan(\arccos x) = \sqrt{1-x^2}/x$; $-1 \leqslant x \leqslant 1$; $x \neq 0$;

(2) $\tan(\arcsin x) = x/\sqrt{1-x^2}$; $-1 < x < 1$;

(3) $\sin(\arctan x) = x/\sqrt{1+x^2}$ if $-\infty < x < +\infty$;

(4) $\sin(\operatorname{arccot} x) = 1/\sqrt{1+x^2}$ if $-\infty < x < +\infty$;

(5) $\cos(\operatorname{arccot} x) = 1/\sqrt{1+x^2}$ if $-\infty < x < +\infty$;

(6) $\cos(\operatorname{arccot} x) = x/\sqrt{1+x^2}$ if $|x| < 1$;

(7) $\cot(\arcsin x) = \sqrt{1-x^2}$ if $|x| < 1$;

(8) $\cot(\arccos x) = x/\sqrt{1-x^2}$ if $|x| < 1$.

§ 91. Addition Formulas

3.96. Compute:

(a) $\cos 15°$; $\sin 75°$; $\tan 105°$; $\cot 225°$; $\cos 105°$; $\sin(13\pi/12)$; $\tan 15°$; $\tan(5\pi/12)$;

(b) (1) $\sin 10° \cos 20° + \cos 10° \sin 20°$;

(2) $\sin 56° \cos 15° - \cos 56° \sin 15°$;

(3) $\cos 13° \cos 17° - \sin 17° \sin 13°$;

(4) $\sin(\pi/8) \sin(\pi/3) - \cos(\pi/8) \cos(\pi/3)$;

(c) (1) $\cos\alpha + \cos(120° - \alpha) + \cos(120° + \alpha)$;

(2) $\cos^2\alpha + \cos^2(60° + \alpha) + \cos^2(60° - \alpha)$;

(d) (1) $\cos(60° - \alpha) + \cos(60° + \alpha)$;

(2) $\sin(\alpha + \pi/6) - \sin(\alpha - \pi/6)$;

(e) $(1 + \tan\alpha)(1 + \tan\beta)$ if $\alpha + \beta = \pi/4$.

3.97. (a) Find the value of $\cos\beta$ if $\cos\alpha = 1/7$; $\cos(\alpha + \beta) = -11/14$, if α and β are acute positive angles.

(b) Find the value of $\sin\alpha$ if $\sin(45° - \alpha) = -2/3$, $\pi/2 > \alpha > \pi/4$.

(c) Prove that $\alpha + \beta = 90°$ if $\sin\alpha = 8/17$; $\sin\beta = 15/17$, $0° < \alpha < 90°$; $0° < \beta < 90°$.

(d) Prove that $\alpha + \beta = \pi/4$ if $\tan \alpha = 1/\sqrt{5}$; $\sin \beta = 1/\sqrt{10}$; $0° < \alpha < \pi/2$; $0 < \beta < \pi/2$.

(e) Find the value of $\sin 35°$ if $\sin 10° = P$.

3.98. α and β are acute positive angles: $\sin \alpha = 5/13$ and $\cos \beta = 3/5$. Compute:

(a) $\cos(\alpha + \beta)$; (b) $\sin(\alpha + \beta)$; (c) $\tan(\alpha + \beta)$; (d) $\sec(\alpha + \beta)$.

3.99. Given: $\cot \alpha = 8/15$; $\cot \beta = -7/24$; $0 < \alpha < \pi/2$; $\pi/2 < \beta < \pi$. Compute:

(1) $\sin(\alpha + \beta)$; (2) $\tan(\alpha + \beta)$; (3) $\cos(\alpha + \beta)$;
(4) $\cot(\alpha - \beta)$; (5) $\sec(\alpha - \beta)$; (6) $\operatorname{cosec}(\alpha + \beta)$.

3.100. Simplify:

(a) (1) $\sin 12° \cos 18° + \sin 18° \cos 12°$;

(2) $\cos 5 \cos 2 - \sin 5 \sin 2$;

(3) $\dfrac{\cot 2\alpha + \tan 2\alpha}{1 + \tan 2\alpha \tan 4\alpha}$;

(b) (1) $\sin(x + 45°) \sin(x - 45°) + \cos(x + 45°) \cos(x - 45°)$;

(2) $\sin nx \cos x + \cos nx \sin x$;

(3) $\sin 20° + \sin 13° \sin 57° - \sin 33° \sin 77°$;

(4) $\dfrac{\sin 56° \sin 124° - \sin 34° \cos 236°}{\cos 28° \cos 88° + \cos 178° \sin 208°}$;

(5) $\cos 11° \cos 21° + \cos 69° \cos 79°$;

(6) $\dfrac{\cos 18° \cos 28° + \cos 108° \sin 208°}{\sin 34° \cos 416° + \sin 236° \sin 304°}$;

(c) (1) $\sin 6 \cot 3 - \cos 6$; (2) $\cos 2\alpha + \sin 2\alpha \tan \alpha$;

(d) (1) $\sin^2 \alpha + \sin^2 \beta + 2 \sin\alpha \sin \beta \cos(\alpha + \beta)$;

(2) $\dfrac{\cos \alpha + \sin \alpha}{\cos \alpha - \sin \alpha} \tan(\pi/4 + \alpha)$;

(3) $\dfrac{\cot \alpha + \cot \beta}{\cot \alpha - \cot \beta} - \tan(\alpha + \beta) \cot \beta$.

3.101. (a) Given: $\sin(\pi/4 - \alpha) = -0.1\sqrt{2}$; $\pi < \alpha < 3\pi/2$. Find $\sin \alpha$.

(b) Prove that:

(1) $(\sqrt{2}/2)(\cos \alpha + \sin \alpha) = \cos(\pi/4 - \alpha)$;

(2) $(1/2)(\cos \alpha + \sqrt{3} \sin \alpha) = \cos(\pi/3 - \alpha)$.

3.102. Prove that: (a) $\arctan(3+2\sqrt{2})-\arctan(\sqrt{2}/2)=\pi/4$;

(b) $\arcsin(5/13)+\arcsin(12/13)=\pi/2$;

(c) $\arcsin m+\arccos m=\pi/2$; $-1\leqslant m\leqslant 1$.

3.103. (a) Given: $\tan(\alpha+45^\circ)=3$; $\pi<\alpha<3\pi/2$. Find $\tan\alpha$.

(b) The sum of the tangents of two numbers is equal to 2, and the tangent of the sum of these numbers is equal to 4. Find the tangent of each number.

3.104. (a) Given: $\tan\alpha=-0.5$; $\tan\beta=3$; $\pi/2<\alpha<\pi$; $0<\beta<\pi/2$. Find $\tan(\alpha+\beta)$.

(b) The ratio of the sine of the sum of two numbers to the sine of their difference is equal to 0.5. The difference of the tangents of these numbers is equal to 2. Find the sum of tangents of these numbers.

3.105. (1) Express $\sin(\alpha+\beta+\gamma)$; $\cos(\alpha+\beta+\gamma)$; $\tan(\alpha+\beta+\gamma)$; $\sin(\alpha-\beta-\gamma)$ in terms of the trigonometrical functions of the arguments α, β, γ.

(2) Find $\sin(\alpha+\beta+\gamma)$; $\cos(\alpha+\beta+\gamma)$; $\tan(\alpha+\beta+\gamma)$ if $\sin\alpha=0.6$; $\sin\beta=12/13$; $\sin\gamma=7/25$ and α, β, γ are acute positive angles.

3.106. Given: $\tan\alpha=1/12$; $\tan\beta=2/5$; $\tan\gamma=1/3$; α, β, γ are acute positive angles. Prove that $\alpha+\beta+\gamma=\pi/4$.

3.107. Compute:

(a) (1) $\sin(\arcsin(15/17)+\arccos(-12/18))$;

(2) $\sin(\arccos(7/25)-\arctan(4/3))$;

(3) $\sin(\operatorname{arccot}(-1.05)-\arctan(2.4))$;

(4) $\arctan\left(\tan\left(\arctan\frac{2a-b}{b\sqrt{3}}+\arctan\frac{2b-a}{a\sqrt{3}}\right)\right)$, where $a\neq 0$ and $b\neq 0$;

(b) (1) $\arctan(\operatorname{arccot}(1/9)+\arccos(4/5))$;

(2) $\arcsin\left(\cos 2\left(\arcsin(1/3)+\arcsin\left(\frac{1}{3\sqrt{11}}\right)+\arcsin\left(\frac{3}{\sqrt{11}}\right)\right)\right)$;

(3) $\operatorname{arccot}\left(\cot\left(\arcsin\sqrt{\frac{2-\sqrt{3}}{4}}+\arccos\frac{\sqrt{3}}{2}\right)+\arccos\frac{1}{\sqrt{2}}\right)$;

(4) $\arccos(\cos(2\operatorname{arccot}(\sqrt{2}-1)))$.

3.108. Prove the following identities:

(a) (1) $\cos(a+b)\cdot\cos(a-b)=\cos^2 a-\sin^2 b$;

(2) $\sin(a+b)\cdot\sin(a-b)=\sin^2 a-\sin^2 b$;

(3) $\cos^2 a-\cos(\pi/3+a)\cdot\cos(\pi/3-a)=3/4$;

(4) $\sin 2a+\cos 2a\cdot\cot a=\cot a$;

(b) (1) $\sin 2a-\cos 2a\cdot\tan a=\tan a$;

(2) $\dfrac{\sin a+2\sin(\pi/3-a)}{2\cos(\pi/6-a)-\sqrt{3}\cos a}=\sqrt{3}\cot a$;

(3) $\cos(\pi/12)+\sin(\pi/12)=\dfrac{\sin(\pi/3)}{\cos(\pi/4)}$;

(4) $\dfrac{\tan^2 a-\tan^2 b}{1-\tan^2 a\cdot\tan^2 b}=\tan(a+b)\cdot\tan(a-b)$;

(5) $\cos^2 a+\cos^2(a+b)-2\cos a\cdot\cos b\cdot\cos(a+b)=\sin^2 b$;

(c) (1) $0.5\sin^2 a+\sin(\pi/4+a)\cdot\sin(\pi/4-a)=1/2\cos^2 a$; (2) $\tan(\pi/4+a)-\tan a=1+\tan(\pi/4+a)\cdot\tan a$;

(3) $\dfrac{1+\tan a\cdot\tan(\pi/3)}{1-\tan a\cdot\tan(\pi/3)}=\dfrac{\cos(a-\pi/3)}{\cos(a+\pi/3)}$;

(4) $\dfrac{\sqrt{2}\cos a-2\cos(\pi/4+a)}{2\sin(\pi/4+a)-\sqrt{2}\sin a}=\tan a$;

(5) $\cos(\pi/12)-\sin(\pi/12)=\cos(\pi/4)$;

(6) $\dfrac{\tan a+\tan b}{\tan(a+b)}+\dfrac{\tan a-\tan b}{\tan(a-b)}=2$.

§ 92. The Double-angle and Half-angle Formulas

The Double-angle Formulas

3.109. (a) Express $\sin 2a$ and $\cos 2a$: (1) only in terms of $\sin a$; (2) only in terms of $\cos a$.

(b) Express $\sin a$ and $\cos a$: (1) in terms of $\sin(a/2)$ and $\cos(a/2)$; (2) in terms of $\cot(a/2)$; (3) only in terms of $\sin(a/2)$; (4) only in terms of $\cos(a/2)$; (5) only in terms of $\tan(a/2)$.

3.110. Compute: (a) $\sin 2a$; $\cos 2a$; $\tan 2a$ if: $\sin a=0.6$, $0<a<\pi/2$;

(b) $\sin a$; $\cos a$; $\tan a$ if $\cos(a/2)=-8/17$; $\alpha\in(90°, 180°)$;

(c) $\sin 2\alpha$ if $\sin\alpha+\cos\alpha=b$.

3.111. Express $\sin 3a$; $\cos 3a$, and $\tan 3a$ in terms of $\sin a$; $\cos a$; and $\tan a$, respectively.

3.112. Compute (without using tables):

(1) $\sin 18^\circ \cdot \cos 36^\circ$; (2) $\sin(\pi/24) \cdot \cos(\pi/24) \cdot \cos(\pi/12)$;
(3) $1 - 2\sin^2(\pi/8)$; (4) $1 - 8\sin^2(\pi/16) \cdot \cos^2(\pi/16)$;
(5) $\cos^4 15^\circ - \sin^4 15^\circ$; (6) $\sin(2 \arcsin 0.5)$;
(7) $\cos(2 \arccos(-1))$; (8) $\sin(2 \arcsin 1)$;
(9) $\tan(2 \arctan(-1))$.

3.113. Simplify:

(a) (1) $2\sin 15^\circ \cdot \cos 15^\circ$; (2) $\cos^2(\pi/8) - \sin^2(\pi/8)$;
(3) $\dfrac{2\tan 40^\circ}{1-\tan^2 40^\circ}$;

(b) (1) $\sin 13^\circ \cdot \sin 77^\circ$; (2) $\sin^2 26^\circ - \sin^2 64^\circ$;
(3) $1 - 2\sin^2(\pi/4)$;

(c) (1) $\sin(\pi/8) \cdot \cos(\pi/8)$; (2) $\cos^2(\pi/12 - 1)$;
(3) $\dfrac{\tan 15^\circ}{1-\tan^2 15^\circ}$;

(d) (1) $\dfrac{1-\tan^2(\pi/8)}{\tan(\pi/8)}$; (2) $2\sin 40^\circ \cdot \sin 50^\circ$;
(3) $\cos(\pi/4 - a/2) \cdot \cos(\pi/4 + a/2)$;

(e) (1) $(\sin 10^\circ + \sin 80^\circ)(\cos 80^\circ - \cos 10^\circ)$;
(2) $(\sin a - \cos a)^2 + \sin 2a$;
(3) $(\cos(\pi/4 + 2a) + \cos(\pi/4 - 2a))^2 - \cos 4a$;
(4) $\dfrac{2\cos^2\alpha - 1}{2\tan(\pi/4-\alpha)\sin^2(\pi/4+\alpha)}$;

(f) (1) $\dfrac{1+\sin 2x}{(\sin x+\cos x)^2}$;
(2) $\cos^2 a - 4\sin^2(a/2) \cdot \cos^2(a/2)$;
(3) $1 - 8\sin^2 a \cdot \cos^2 a$; (4) $2 \sin x \cos x(\cos^2 x - \sin^2 x)$;
(5) $\cos 4\alpha + \sin 4\alpha \cdot \tan 2\alpha$;

(g) (1) $\cos^4 x - 6\cos^2 x \cdot \sin^2 x + \sin^4 x$;
(2) $\dfrac{\cot a - \tan a}{\cos 2a}$; (3) $\dfrac{1-\tan^2 a}{1+\tan^2 a}$;
(4) $\dfrac{\sin 4a}{1-\cos 4a} \cdot \dfrac{\cos 2a}{1+\cos 2a}$;
(5) $\cos^2 a + \cos^2 b - \sin^2(a + b)$;

(h) (1) $\sin(\pi/24)\cdot\cos(\pi/24)\cdot(\cos^2(\pi/24)-\sin^2(\pi/24))$;

(2) $2\cot 2\alpha\left(\frac{\sin\alpha}{\cos\alpha-\sin\alpha}+\frac{\sin\alpha}{\cos\alpha+\sin\alpha}\right)$;

(3) $\frac{2\sin\alpha+\sin 2\alpha}{2\cos\alpha+\sin 2\alpha}\cdot\frac{1-\cos\alpha}{1-\sin\alpha}$.

3.114. Prove the validity of the following equalities:

(a) (1) $\sin 18^\circ\cdot\cos 36^\circ = 1/4$;

(2) $\sin 40^\circ\cdot\sin 50^\circ = (1/2)\cos 10^\circ$;

(3) $\sin^4 a-\cos^4 a=-\cos 2a$; (4) $\frac{\sin 3a}{\cos a}+\frac{\cos 3a}{\sin a}=2\cot 2a$;

(5) $\sin^4(3\pi/8)-\cos^4(3\pi/8)=\sqrt{2}/2$;

(b) (1) $16\cos 20^\circ\cdot\cos 40^\circ\cdot\cos 60^\circ\cdot\cos 80^\circ = 1$;

(2) $\frac{1}{1-\tan a}-\frac{1}{1+\tan a}=\tan 2a$;

(3) $\frac{\cos^3 a-\sin^3 a}{1+\sin a\cos a}=\cos a-\sin a$;

(4) $\sin^2\alpha+\sin^2(120^\circ+\alpha)+\sin^2(120^\circ-\alpha)=3/2$;

(5) $4(\cos^3 20^\circ+\cos^2 40^\circ)=3\sqrt{3}\cos 10^\circ$;

(c) (1) $\sin(2\arcsin x)=2x\sqrt{1-x^2}$, $|x|\leqslant 1$;

(2) $\cos(2\arccos x)=2x^2-1$, $|x|\leqslant 1$;

(3) $\tan(2\arctan x)=\frac{2x}{1-x^2}$, $x\neq\pm 1$;

(4) $\sin(2\arctan x)=\frac{2x}{1+x^2}$;

(5) $\cos(2\arctan x)=\frac{1-x^2}{1+x^2}$;

(d) (1) $(1/2)(\tan\alpha-\cot\alpha)\sin^2\alpha=1-2\cos^2\alpha$;

(2) $\frac{1-2\sin^2\alpha}{1+\sin 2\alpha}=\frac{1-\tan\alpha}{1+\tan\alpha}$; (3) $\cos 4\alpha-\sin 4\alpha\cot 2\alpha=-1$;

(4) $\frac{\cos\alpha+\sin\alpha}{\cos\alpha-\sin\alpha}-\frac{1}{\cos 2\alpha}=\tan 2\alpha$.

3.115. Find the period of each of the given functions:

(1) $y=\sin x\cdot\cos x$; (2) $y=\cos^2 x-\sin^2 x$;

(3) $y=\frac{\cos x}{1-\sin x}$; (4) $y=\sin^4 x-\cos^4 x$;

(5) $y=\cos^4 x+\sin^4 x$; (6) $y=\cos^6 x+\sin^6 x$;

(7) $y=\sin^2 x+\cos^2 x-\cos x$.

3.116. Find the greatest and the least value of the following functions:

(a) (1) $y = 2\cos 2x + \sin^2 x$;
(2) $y = \sin^2(\pi/4 - x) + (\sin x - \cos x)^2$;

(b) (1) $y = \sin(x + \pi/4)\cdot\sin(x - \pi/4)$;
(2) $y = \dfrac{1}{1+\tan x\cdot\tan 2x}$;

(c) (1) $y = \sin^6 x + \cos^6 x$; (2) $y = \sin^4 x + \cos^4 x$;

(d) (1) $y = \cos^4 x - \sin^4 x$; (2) $y = 1 - 8\sin^2 x\cos^2 x$;
(3) $y = 8\cos^2 x + 15\sin^2 x$; (4) $y = 3\sin^2 x + 4\cos^2 x$.

The Half-angle Formulas

3.117. (1) Given: $\cos a = -7/25$; $\pi < a < 3\pi/2$. Find: (a) $\sin(a/2)$; $\cos a/2$; $\tan(a/2)$; (b) $\sin(a/4)$; $\cos(a/4)$; $\tan(a/4)$.
(2) Given: $\sin\alpha = 3/5$; $\pi/2 < \alpha < \pi$. Find: $\sin(\alpha/2)$; $\cos(\alpha/2)$; $\tan(\alpha/2)$.

3.118. Prove that:

(1) $\dfrac{1-\cos a}{\sin a} = \tan(a/2)$; (2) $\dfrac{\sin a}{1+\cos a} = \tan(a/2)$;
(3) $\dfrac{2\tan(a/2)}{1+\tan^2(a/2)} = \sin a$; (4) $\dfrac{\tan^2(a/2)+1}{1-\tan^2(a/2)} = \sec a$.

3.119. Simplify:

(a) (1) $\dfrac{1+\cos 2a}{1-\cos 2a}$; (2) $\dfrac{1-\sin 2a}{1+\sin 2a}$;
(3) $\sqrt{2(1+\cos 2a)}$, $\pi/2 < a < \pi$;
(4) $\sqrt{0.5 - 0.5\cos 4a}$, $\pi/4 < a < \pi/2$;

(b) (1) $\dfrac{\cos a}{\sqrt{1-\cos 2a}} - \dfrac{\sin a}{\sqrt{1+\cos 2a}}$, $0 < a < \pi/2$;
(2) $2\sin^2(a/2) + \cos a$; (3) $2\cos^2(a/2) - \cos a$;
(4) $\cos^2 a - \sin 4a\cdot\cot 2a$;

(c) (1) $\sqrt{\dfrac{1+\cos 2a}{1-\cos 2a}}$; (2) $\sqrt{\dfrac{\sin(3\pi/2-a)+1}{\sin(\pi/2+a)+1}}$;
(3) $\sqrt{1+\cos 8a}$; (4) $\sqrt{1-\cos(a/4)}$;

(d) (1) $\dfrac{1-\sin(\pi/2+2\alpha)}{1+\cos(2\pi-2\alpha)}$;

(2) $\sin^2(\pi/2+2a)+\cos(\pi/2+4a)\cos(\pi+2a)$;

(3) $2\sin^2(\pi+a)+\sin(\pi/2+2a)$;

(4) $\dfrac{1+\sin(\pi/2+2a)}{1-\cos(2\pi-2a)}$.

3.120. Prove the following equalities:

(a) (1) $\dfrac{2\sin a-\sin 2a}{2\sin a+\sin 2a}=\tan^2\dfrac{a}{2}$;

(2) $\dfrac{1+\sin 2b-\cos 2b}{1+\sin 2b+\cos 2a}=\tan b$;

(3) $4\cos^2(\pi/4-c/2)+\sqrt{4\sin^4 c+\sin^2 2c}=2,\ \pi<c<3\pi/2$;

(4) $4\sin^2(\alpha/4)-$
$\sqrt{(\sin^2(\alpha/2)+\sin(3\alpha/2))^2+(\cos(\alpha/2)+\cos(3\alpha/2))^2}=2,$
$\pi<\alpha<3\pi/2$;

(b) (1) $\dfrac{1}{1+\tan x\tan 2x}=\cos 2x$;

(2) $\dfrac{3-4\cos 2\alpha+\cos 4\alpha}{3+4\cos 2\alpha+\cos 4\alpha}=\tan^4\alpha$;

(3) $\dfrac{\sin^4\alpha-\cos^4\alpha+\cos^2\alpha}{2(1-\cos\alpha)}=\cos^2(\alpha/2)$;

(4) $\dfrac{\sin(\pi-\alpha)}{\sin\alpha-\cos\alpha\cdot\tan(\alpha/2)}+\cos(\pi-\alpha)=1$;

(5) $\dfrac{\sin 2a}{1+\cos 2a}\cdot\dfrac{\cos a}{1+\cos a}=\tan(a/2)$;

(c) (1) $2\operatorname{cosec}4\alpha-\cot 2\alpha=\tan 2\alpha$;

(2) $\dfrac{\cos 4\alpha+1}{\cot\alpha-\tan\alpha}=(1/2)\sin 4\alpha$;

(3) $\dfrac{1-\cos 4\alpha}{\sec^2 2\alpha-1}+\dfrac{1+\cos 4\alpha}{\operatorname{cosec}^2 2\alpha-1}=2$;

(4) $\dfrac{4\sin^4(\alpha/4)}{\cos^2 4\alpha-\cos^2(\alpha/2)+\sin^2 4\alpha}=\tan^2(\alpha/4)$;

(5) $\dfrac{1+\sin 2\alpha+\sin(3\pi/2-2\alpha)}{1+\sin 2\alpha-\sin(3\pi/2+2\alpha)}=\tan\alpha$.

3.121. Find the periods of the given functions:

(1) $y=2\sin^2 x-\cos 2x$; (2) $y=\sin 2x(\cos x-2\cot 2x)$;

(3) $y=\dfrac{\sin x+\cos x-1}{\sin x-\cos x+1}$; (4) $y=\sin^2 2x+\cos 2x$;

(5) $y=\dfrac{\tan^2(\pi/4+x)-1}{\tan^2(\pi/4+x)+1}$.

§ 93. The Transformation Formulas

Transforming the Product of Trigonometrical Functions into a Sum or Difference

3.122. Represent in the form of a sum:

(a) (1) $\cos 5^\circ \cdot \cos 15^\circ$; (2) $\cos (3\pi/5) \cdot \sin (\pi/5)$; (3) $\sin (2x - \pi/3) \cdot \sin (x + \pi/3)$;

(b) (1) $2 \cos x \cdot \cos (x + 1)$; (2) $4 \cos 25^\circ \cdot \cos 35^\circ \cdot \cos 15$; (3) $4 \sin 12^\circ \cdot \sin 14^\circ \cdot \sin 16^\circ$; (4) $4 \sin^4 a$; (5) $16 \sin^2 a \times \cos^3 a$;

(c) (1) $\sin^2 3\alpha$; (2) $2 \cos^2 (a - \pi/4)$; (3) $4 \cos x \times \sin^2 (x/2)$;

(d) (1) $32 \sin^5 b \cdot \cos^3 b$; (2) $4 \cos a \cdot \cos 3a \cdot \cos 4a$; (3) $2 \sin a \cdot \sin 2a \cdot \sin 3a$; (4) $\sin 2a \cdot \sin (a + b)$.

3.123. Compute without using tables:

(1) $\cos (3\pi/8) \cdot \cos (\pi/8)$; (2) $\sin 105^\circ \cdot \sin 75^\circ$; (3) $\cos (\pi/24) \cdot \sin (5\pi/24)$; (4) $\sin 14^\circ \cdot \sin 76^\circ - \cos 12^\circ \times \sin 16^\circ + 0.5 \sin 4^\circ$.

3.124. Prove the following identities:

(1) $\sin (3\pi/10) \cdot \sin (\pi/10) = 1/4$;

(2) $\cos 10^\circ \cdot \cos 30^\circ \cdot \cos 50^\circ \cdot \cos 70^\circ = 3/16$;

(3) $16 \sin 20^\circ \cdot \sin 40^\circ \cdot \sin 60^\circ \cdot \sin 80^\circ = 3$;

(4) $8 \cos 10^\circ \cdot \cos 20^\circ \cdot \cos 40^\circ = \cot 10^\circ$;

(5) $\sqrt{1+\cos\alpha} + \sqrt{1-\cos\alpha} = 2 \sin (\pi/4 + a/2)$ if $0 < a < \pi/2$;

(6) $(\sin a + \sin b)^2 + (\cos a + \cos b)^2 = 4 \cos^2 \frac{a-b}{2}$.

Transforming the Sum and Difference of Trigonometrical Functions into a Product

3.125. Transform into a product:

(a) (1) $\sin (\pi/10) + \sin (\pi/12)$; (2) $\sin 46^\circ + \cos 50^\circ$; (3) $\sin (\pi/10) - \cos (\pi/5)$; (3) $\cos (\pi/8) - \cos (\pi/18)$;

(b) (1) $\tan (5\pi/24) - \tan (7\pi/24)$; (2) $\tan 12^\circ + \cot 12^\circ$; (3) $\sin 3a - \sin 5a$; (4) $\tan (3\pi/5) - \cot (\pi/5)$;

(5) $\cos 36^\circ - \sin 16^\circ$; (6) $1 + \cos 18^\circ$; (7) $1 - \sin (\pi/4)$;

(c) $\cos (a + b) - \cos (a - b)$; (2) $1 + \cos 3a$;
(3) $1 - \sin 2a$;

(d) (1) $1 + \sin x + \cos x$; (2) $\sin a - \sin (a - 60^\circ)$;
(3) $\sin \pi\alpha + \tan \pi\alpha$;

(e) (1) $1 \pm \tan a$; (2) $\frac{1-2\cos a+\cos 2a}{1+2\cos a+\cos 2a}$;

(3) $\frac{1-2\sin a-\cos 2a}{1+2\sin a-\cos 2a}$;

(4) $\cos a + \sin 2a + \cos 3a + \sin 4a$; (5) $1 \pm \cot b$.

3.126. Transform the given sum into a product by introducting an auxiliary angle:

(a) (1) $1/2 + \cos a$; (2) $\sqrt{3} + \tan a$; (3) $\sqrt{2} - 2 \sin 15^\circ$;
(4) $3 - 4 \cos^2 a$;

(b) (1) $3 - \tan^2 a$; (2) $1 - 4 \sin^2 a$;
(3) $1 - \sqrt{2} \cos a + \cos 2a$;

(c) (1) $2 \sin^2 a + \sqrt{3} \sin 2a - 1$; (2) $\sin x + \sqrt{3} \cos x$
(3) $\sqrt{3} \sin x - \cos x$;

(d) (1) $a \sin x + b \cos x$; (2) $2 \sin^2 a - 1$;
(3) $1 - 3 \tan^2 a$;

(e) (1) $1 + \sin a - \cos 2a$; (2) $1 + 2 \sin a$;
(3) $1 + \cos a + \cos 2a$; (4) $2 \cos^2 2\alpha + \sqrt{3} \sin 4\alpha - 1$.

3.127. Find the conditions of equality of the given trigonometrical functions: (a) $\sin a = \sin \beta$; (b) $\cos a = \cos \beta$; (c) $\tan a = \tan \beta$; (d) $\cot a = \cot \beta$.

3.128. Simplify:

(a) (1) $2 \sin 10^\circ \cdot \sin 40^\circ + \cos 50^\circ$;
(2) $\sin \alpha (1 + 2 \cos 2\alpha)$;

(b) (1) $\sin 2\alpha + 2 \sin (5\pi/12 - \alpha) \cos (5\pi/12 + \alpha)$;
(2) $\sin^2 \alpha - \sin (a + \pi/3) \sin (\alpha - \pi/3)$;

(c) 1) $\cos 2a \cdot \cos^2 a - (1/4) \cos 4a - (1/2) \cos 2a$;
(2) $\cos 4 \cdot \cos 6 - \sin 1 \cdot \sin 3$; (3) $\frac{1}{2 \sin 10^\circ} - 2 \sin 70^\circ$;

(d) (1) $\dfrac{\sin\alpha-2\sin 2\alpha+\sin 3\alpha}{\cos\alpha-2\cos 2\alpha+\cos 3\alpha}$;

(2) $\dfrac{4\sin^2(\alpha/2)\cdot\sin 3\alpha}{\cos 2\alpha-\cos 4\alpha}$; (3) $\dfrac{\cos 3\alpha\,(\sin 5\alpha-\sin\alpha)}{1+\cos 6\alpha}$.

3.129. Prove the following identities:

(1) $4\cos(a/2)\cdot\cos a\cdot\sin(3a/2)=\sin a+\sin 2a+\sin 3a$;
(2) $4\cos(a/2)\cdot\cos a\cdot\sin 5a/2=\sin a+\sin 2a+\sin 3a+\sin 4a$;
(3) $\tan 20^\circ\cdot\tan 40^\circ\cdot\tan 60^\circ\cdot\tan 80^\circ=3$;
(4) $\cos(\pi/5)+\cos(3\pi/5)=1/2$;
(5) $\cos 2\alpha-\cos 3\alpha-\cos 4\alpha+\cos 5\alpha=-4\sin\alpha/2\times\sin\alpha\cdot\cos 7\alpha/2$;
(6) $\sin^2\alpha+\cos(\pi/3-\alpha)\cos(\pi/3+\alpha)=1/4$;
(7) $\dfrac{\sin\alpha+\sin 2\alpha-\sin(\pi+3\alpha)}{2\cos\alpha+1}=\sin 2\alpha$.

3.130. Prove the inequalities:

(a) if $-\pi/2<a<\pi/2$ and $-\pi/2<b<\pi/2$, then $\cos\dfrac{a+b}{2}\geqslant\dfrac{\cos a+\cos b}{2}$;

(b) if $0\leqslant a\leqslant\pi$ and $0\leqslant b\leqslant\pi$, then $\sin\dfrac{a+b}{2}\geqslant\dfrac{\sin a+\sin b}{2}$.

3.131. (a) Find the least values:

(1) $y=\sin(x+\pi/8)\cos(x-\pi/24)$;
(2) $y=\cos(3x+2\pi/5)\cos(3x+\pi/15)$;
(3) $y=\sin(x-\pi/3)\sin(x+\pi/3)$.

(b) Find the periods of the given functions:

(1) $y=(2\cos 2x-1)\cos x$; (2) $y=\sin^3 x$;
(3) $y=\sin x\cos 2x$; (4) $y=\sin(2x+\pi/6)\cos(2x-\pi/3)$;
(5) $y=\cos(x+\pi/4)\cos(x-\pi/6)$.

§ 94. Trigonometrical Equations and Inequalities. Systems of Trigonometrical Equations

3.132. Solve the following equations:

(a) (1) $\sin 3x = -1$; (2) $\sin(5x - \pi/4) = 1$; (3) $\sin 2x = -0.7524$;

(b) (1) $\sin(x + 18^\circ) = 0.7431$; (2) $\cos(x - 20^\circ) = 1$; (3) $\cos(x/2 + 10^\circ) = \sqrt{2}/2$;

(c) (1) $\cos 3x = -0.2586$, (2) $\cos(x/2) = 0.9387$; (3) $\tan(x + \pi/4) = 1$;

(d) (1) $\tan 3x = -3.230$; (2) $\tan(x - \pi/3) = -\sqrt{3}$; (3) $\cot 3x = 0.5670$;

(e) (1) $\cot(x/2) = 0$; (2) $\cot(x - \pi/4) = \sqrt{3}$; (3) $\cot(x/4) = -3$;

(f) $\tan(\pi/4 - x) = \sqrt{3}/3$; (2) $\cot(2x - 15^\circ) = -\sqrt{3}$; (3) $\cot(x + 30^\circ) = -1$;

(g) (1) $\dfrac{\cos x - \sqrt{5}}{\cos x - 10} = \dfrac{\cos x}{\cos x + \sqrt{5}}$;

(2) $(\sin x + \cos x)^2 + (\sin x - \cos x)^2 = 1 - \cos 2x$;

(3) $2 \tan x = (1 + \tan^2 x)\cos x$.

3.133. Solve the following equations:

(a) (1) $2 \sin^2 x - \sin x = 0$;

(2) $\tan^2 x + \sqrt{3} = (1 + \sqrt{3}) \tan x$; (3) $3 \sin x = 2 \cos^2 x$;

(b) (1) $\sin x + \cos^2 x = 1/4$; (2) $\sin^2 x - \sin^4 x + \cos^4 x = 1$;

(3) $\tan^3 x = \tan x$;

(c) (1) $2 \tan x + 3 \cot x = 5$; (2) $\sin^2 x = 1 + \cos^2 x$;

(3) $3 \sin^2 x = \cos^2 x$;

(d) $\cot^2 x + 4 \cot x = 0$; (2) $2 \tan^4 3x - 3 \tan^2 3x + 1 = 0$;

(3) $\cos^4 2x + 6 \cos^2 2x = 25/16$;

(e) (1) $8 \sin^2 x - 2 \cos x = 5$; (2) $\cos x (2 \cos x + 1) = 1$;

(3) $\cos x + 1/\cos x = 2$.

3.134. Solve the given homogeneous equations:

(a) (1) $3\cos^2 x - \sin^2 x - 2\sin x \cdot \cos x = 0$;

(2) $\cos^2 x + 3\sin^2 x + 2\sqrt{3}\sin x \cdot \cos x = 1$;

(3) $4\sin x + 6\cos x = \frac{1}{\cos x}$;

(4) $5\sin^2 x + 3\sin x \cdot \cos x - 3\cos^2 x = 2$;

(b) (1) $5\sin^2 x - 3\sin x \cdot \cos x - 2\cos^2 x = 0$;

(2) $2\sin^2 x - 5\sin x \cdot \cos x - 8\cos^2 x = -2$;

(3) $6\sin^2 x + \sin x \cdot \cos x - \cos^2 x = 2$;

(c) (1) $\sin^2 x - 2\sin x \cdot \cos x = 3\cos^2 x$;

(2) $\sin^2 3x = 3\cos^2 x$; (3) $4\sin x \cdot \cos(\pi/2 - x) + 4\sin(\pi + x)\cos x + 2\sin(3\pi/2 - x)\cos(\pi + x) = 1$;

(d) (1) $2\sin x \cdot \cos(3/2\pi + x) - 3\sin(\pi - x)\cos x + \sin(\pi/2 + x)\cos x = 0$;

(2) $2\sin x \cdot \cos^2(\pi/2 - x) + 3\cos^2(\pi/2 + x)\cos x - 5\cos^2 x \sin(\pi/2 + x) = 0$; (3) $\sin^2 2x \cdot \cos(3\pi/2 - 2x) + 3\sin 2x \cdot \sin^2(3\pi/2 + 2x) + 2\cos^3 2x = 0$;

(e) (1) $3\sin x - 7\cos x = 0$;

(2) $\sin(x - \pi/6) + \cos(x - \pi/6) = 0$;

(3) $\sin(x + \pi/4) + \sqrt{3}\cos(x + \pi/4) = 0$; (4) $\sin 2x = \cos 2x$;

(5) $\cos^2 x - 3\sin x \cos x = \sin 3\pi/2$.

3.135. Solve the following equations:

(a) (1) $\sin x - \cos x - 4\cos^2 x \cdot \sin x = 4\sin^3 x$;

(2) $4\sin^2 x \cdot \cos x - 4\sin x \cdot \cos^2 x + \cos^3 x = 0$;

(3) $\sin x \cdot \cot x + \cos x \cdot \tan x = 0$;

(b) (1) $\sin^3 x(1 - \cot x) - \cos^3 x(1 - \tan x) = 0$;

(2) $2\sin x + \cos x \cdot \cot x + 2 = 0$;

(3) $\sin x \cdot \cot x + 2 = \cos x + 2\sin x$;

(c) (1) $\sin^2 x + \cot^2 x = 1$;

(2) $2(1 + \sin^6 x + \cos^6 x) - 3(\sin^4 x + \cos^4 x) - \cos x = 0$;

(3) $1 - \frac{\sin^2 x}{1 + \cot^2 x} - \frac{\cos^2 x}{1 + \tan^2 x} = 0$;

(d) (1) $\frac{\sin x}{1+\cos x} = 2 - \cot x$;

(2) $4 \sin x (\sin x + \sqrt{7} \cot x) = 7$;

(3) $2 \cos x (\cos x - 2\sqrt{2} \tan x) = 5$;

(e) (1) $2 \tan^2 x + 4 = \frac{5}{\cos x}$; (2) $\tan^2 x = \frac{1-\cos x}{1-\sin x}$;

(3) $\tan^2 5x = \cos 3x - 1$;

(4) $\sin x \cos x + \sin x + \cos x + 1 = 0$.

3.136. Solve the following inequalities:

(a) (1) $|\sin x| > |\cos x|$; (2) $|\sin x| \leqslant |\cos x|$;

(3) $\sin x > 1/2$.

(b) (1) $\sin < \sqrt{3}/2$; (2) $\cos x > -1/2$; (3) $\cos x < \sqrt{3}/2$;

(c) (1) $\tan x \geqslant \sqrt{3}/3$; (2) $\tan x < 2/3$; (3) $6 \cos^2 x - 11 \cos x + 4 > 0$;

(d) (1) $|\tan 4x| > 1/2$; (2) $|\tan x| < 1/5$;

(3) $|\tan x| > 4/3$;

(e) (1) $6 \sin^2 2x + 5 \sin 2x + 1 \leqslant 0$;

(2) $4 \sin^2 x - 8 \sin x + 3 \geqslant 0$; (3) $2 \cos^2 2x - \cos 2x - 1 \leqslant 0$.

3.137. Solve the following equations:

(a) (1) $\sin(\pi/2 + x) + \cot(2\pi - x) = 0$;

(2) $\cot(3\pi/2 + x) - \cos(3\pi/2 - x) = 0$;

(3) $2 \cot(\pi - x) - \cot 5\pi/4 - \cot(12\pi - x) = 0$;

(4) $\sin(30^\circ + x) - \sin(210^\circ + x) = 2 \cdot \sin 495^\circ$;

(b) (1) $\cos 600^\circ + \cos(x - 200^\circ) + \sin(x + 250^\circ) = \sin 870^\circ$;

(2) $2 \cot(180^\circ - x) - \cot 405^\circ - \cot(360^\circ - x) = 0$;

(3) $\cos(\pi/2 + x) + \tan(2\pi + x) = 0$;

(4) $\sin(x - \pi/3) - \cos(x + \pi/6) = 0$;

(c) (1) $\sin^2(90^\circ + x) + \sin(180^\circ + x) + 1 = 0$;

(2) $\sin^2(270^\circ - x) + 2 \sin x - 1 = 0$;

(3) $2\cos^2(\pi + x) - 3\cos(\pi/2 - x) = 0$;

(4) $3\tan^2(\pi + x) - \frac{1}{\cos^2(\pi - x)} = 1$.

3.138. Solve the following equations:

(a) (1) $\sin 2x \cdot \sin x + \cos 3x = 0$;
(2) $\sin 1.5x = \sin 0.5x \cdot \cos x$;
(3) $\cos(\pi/4 + 2x)\cos(\pi/3 + x) + \cos(\pi/4 - 2x)\cos(\pi/6 - x) = 1$;
(4) $\cos(2\pi/3 + x) + \cos(2\pi/3 - x) = 1$;
(5) $\cos x = \cos 2x \cdot \cos 3x$;

(b) (1) $\frac{\sqrt{3} + \tan x}{1 - \sqrt{3}\tan x} = 1$;
(2) $\tan(\pi/12) - \tan x = 1 + \tan(\pi/12) \cdot \tan x$;
(3) $(\sin 2x + \sqrt{3}\cos 2x)^2 - 5 = \cos(\pi/6 - 2x)$;
(4) $(\cos 2x + (\cos x + \sin x)^2)(\tan x + \cot x) = 0$;
(5) $\cos(\pi + x)\sin(3\pi/2 - 3x) - \cos(3\pi/2 + x)\sin(3\pi - 3x) = \sin 4x$;

(c) (1) $\tan(\pi/4 - x) = \frac{\cos x}{\sin x + \cos x}$;
(2) $\tan 3x - \tan x = \tan 2x$;
(3) $\sin(x + \pi/4) + \cos(x + \pi/4) = 1$;
(4) $\sin 2x \cdot \sin 6x - \cos 2x \cdot \cos 6x = \sqrt{2}\sin 3x \cdot \cos 8x$.

3.139. Solve the following equations:

(a) (1) $\sin^4 x - \cos^4 x = 1/2$; (2) $\sin^4 x + \cos^4 x = 5/8$;
(3) $\sin^3 x \cdot \cos x - \cos^3 x \cdot \sin x = \sqrt{2}/8$;
(4) $4\sin^4 x + \sin^2 2x = 2$;

(b) (1) $\sin 2x - \frac{\sin 4x}{4\cos^2 x} = 1$;
(2) $5\cos 2x = 4\sin x$; (3) $1 - \cos x = 2\sin(x/2)$;
(4) $\sin 2x = \tan^2 x\,(1 + \cos 2x)$;

(c) (1) $1 - \cos x = \sin x \cdot \sin(x/2)$;
(2) $2\sin(\pi/4 + x)\sin(\pi/4 - x) + \sin^2 x = 0$;
(3) $\sin x + 2\cos x = 1$; (4) $\sqrt{3}\sin x + \cos x = 1$;

(d) (1) $4 \sin x + 3 \cos x = 5$; (2) $\sin x + \cos x = \cos 2x$;
(3) $\sin x + 7 \cos x = 2$;
(4) $5 (\sin x + \cos x)^2 = 12 (\sin x + \cos x) - 7$;

(e) (1) $3 (1 - \sin 2x) = 1 + \cos 2x$;
(2) $\sin x + \cos x = 1 + \sin 2x$; (3) $\sin 4x + \cos 3x = \sqrt{2}$;
(4) $1 - \sin 2x = \cos x - \sin x$;

(f) (1) $12 \sin x + 4\sqrt{3} \cos (x + \pi) = 8\sqrt{3}$;
(2) $\sin^3 x (1 + \cot x) + \cos^3 x (1 + \tan x) = \cos 2x$;
(3) $(\sin x + \cos x)^2 - 2 \sin (\pi/4 + x) \sin (\pi/4 - x) = 0$;
(4) $\sqrt{1 + 2 \sin x \cos x} = \sin x + \cos x$;

(g) (1) $\cos (4x + 3\pi) + 2 \cos (3\pi/2 + 2x) \sin (\pi + 2x) + \cos^2 (\pi/2 + x/2) = 0$;
(2) $\sin x + \sin 2x = \tan x$; (3) $\frac{1 + \tan x}{1 - \tan x} = (\sin x + \cos x)^2$;
(4) $4 \sin^4 x + 12 \cos^3 x = 7$.

3.140. Solve the following inequalities:
(1) $5 \sin^2 x + \sin^2 2x > 4 \cos 2x$;
(2) $\sin^6 x + \cos^6 x > 5/8$.

3.141. Solve the following equations:

(a) (1) $\cos x = \cos 5x$; (2) $\sin x + \sin 2x = 0$;
(3) $\tan 5x + \tan 3x = 0$; (4) $\cos 15x = \sin 5x$;

(b) (1) $\sin x = \sin (x + 1)$; (2) $\tan 3x = \cot 5x$;
(3) $\cos^4 x - \sin^4 x = \sin 3x$; (4) $\sin (x + \pi/5) - \sin (x + 2\pi/15) = 0$;

(c) (1) $\sin 4x \cdot \cos 2x = \sin 5x \cdot \cos x$; (2) $\cos 3x = \sin x$;
(3) $\sin (\pi \cos x) - \cos (\pi \sin x) = 0$.

3.142. Solve the following equations:

(a) (1) $\sin x \cdot \sin 7x = \sin 3x \cdot \sin 5x$;
(2) $\cos x \cdot \sin 7x = \cos 3x \cdot \sin 5x$;
(3) $\cos (x + \pi/3) \cos (x - \pi/3) + 1/4 = 0$;
(4) $\sin (x + \pi/3) \cos (x - \pi/6) = 1$;
(5) $\sin x \cdot \sin (x + \pi/3) \sin (x + 2\pi/3) = 1/4$;

(6) $\sin x \cdot \sin 2x \cdot \sin 3x = (1/4) \sin 4x$;
(7) $\sin 3x + \sin x = 2 \sin 2x$;
(8) $1 - \sin 5x = (\cos (3x/2) - \sin (3x/2))^2$;
(9) $2 \sin 17x + \sqrt{3} \cos 5x + \sin 5x = 0$;
(10) $\frac{\sin x}{\sin 2.5x} = \frac{\cos 0.5x}{\cos 2x}$;

(b) (1) $2 \sin 2x \cdot \sin 4x - \cos 2x = \sin 3x$;
(2) $\sqrt{3} \cos (\pi/2 - 2x) + \cos 5x = \cos 9x$;
(3) $\frac{\cos^2 3x}{\tan x} + \frac{\cos^2 x}{\tan 3x} = 0$; (4) $\cos 3x + \cos (5x/2) = 2$;
(5) $2 (\cos 4x - \sin x \cos 3x) = \sin 4x + \sin 2x$;
(6) $\sin 9x + \sqrt{3} \cos 7x = \sin 7x + \sqrt{3} \cos 9x$;
(7) $\sin x + \sin 2x + \sin 3x = \cos x + \cos 2x + \cos 3x$;
(8) $\sin^2 x + \sin^2 2x + \sin^2 3x = 3/2$;
(9) $\tan x \cdot \tan (x + 60^\circ) \tan (x + 120^\circ) = \sqrt{3}$;
(10) $\cos 7x \cdot \cos 5x - \sin 4x \cdot \sin 2x = \sin^2 3x$.

3.143. Solve the following equations:

(a) (1) $\tan (\pi/3 + x) + \cot (\pi/6 - x) = 2/\sqrt{3}$;
(2) $\cos (\pi/4 + x) \sin (\pi/4 - x) = 0.75$;
(3) $\cos (\pi/2 + 5x) + \sin x = 2 \cos 3x$;
(4) $\cos 6x = 2 \sin (3\pi/2 + 3x)$;

(b) (1) $\cos 4x \cdot \cos (\pi + 2x) - \sin 2x \cdot \cos (\pi/2 - 4x) = (\sqrt{2}/2) \sin 4x$; (2) $\sin^2 2x \cdot \cos (3\pi/2 - 2x) + 3 \sin 2x \times \sin^2 (3\pi/2 + 2x) + 2 \cos^3 2x = 0$;
(3) $2 \sin x \cdot \cos (3\pi/2 + x) - 3 \sin (\pi - x) \cos x + \sin (\pi/2 + x) \cos x = 0$;
(4) $1 - \cos (\pi + x) - \sin \frac{3\pi + x}{2} = 0$;

(c) (1) $\sin (\pi/2 + 2x) \cot 3x + \sin (\pi + 2x) - \sqrt{2} \cos 5x = 0$;
(2) $\tan (5\pi/6 - x) \cot (x - \pi/3) = 3$;
(3) $\cos (\pi/3 - x) + \sin (x + \pi/6) = 1$;
(4) $2 \cos (\pi/8 - x) \cot (3\pi/8 + x) + \sqrt{3} = 0$;

(d) (1) $\cot^2(\pi - x) - 3\tan(x - \pi/2) - 4 = 0$;

(2) $\cos(x - \pi/3) = \sin(x + \pi/6)$;

(3) $\sin(\pi/2 - x) - \cot x = 0$;

(4) $2\tan(\pi/4 - x) + \cot(\pi/4 + x) = 1$.

3.144. Solve the following systems of equations:

(1) $\begin{cases} \sin x \cdot \sin y = 1/4\sqrt{2}, \\ \tan x \cdot \tan y = 1/3; \end{cases}$ (2) $\begin{cases} \sin x - 1/\sin x = \sin y, \\ \cos x - 1/\cos x = \cos y; \end{cases}$

(3) $\begin{cases} \sin x + \sin y = \dfrac{1+\sqrt{3}}{2}, \\ \cos x + \cos y = \dfrac{1+\sqrt{3}}{2}; \end{cases}$ (4) $\begin{cases} \sin x + \cos y = 0, \\ \sin^2 x + \cos^2 y = 1/2, \\ 0 < x < \pi,\ 0 < y < \pi; \end{cases}$

(5) $\begin{cases} \sin x + \sin y = -\sqrt{2}/2, \\ x + y = \pi/2; \end{cases}$ (6) $\begin{cases} x - y = 5\pi/3, \\ \sin x = 2\sin y; \end{cases}$

(7) $\begin{cases} \sin^2 x - \sin^2 y = 0.75, \\ x - y = 60°; \end{cases}$ (8) $\begin{cases} \sin x \cdot \sin y = 0.75, \\ \tan x \cdot \tan y = 3; \end{cases}$

(9) $\begin{cases} \cos x + \cos y = 3/2, \\ x - y = 60°; \end{cases}$ (10) $\begin{cases} \sin x \cdot \sin y = 0.25, \\ x + y = \pi/3. \end{cases}$

3.145. Solve the following equations:

(a) (1) $\tan x \cdot \tan(x + 1) = 1$; (2) $\sin \pi\sqrt{x} + \sin \pi x = 0$;

(3) $\tan(2x + 1)\cot(x + 1) = 1$; (4) $\sin(1 + 3x) + \cos 3x = 0$;

(b) (1) $\dfrac{1}{\cos\sqrt{x}} + \dfrac{1}{\sin\sqrt{x}} = 2\sqrt{2}$;

(2) $\sin(\pi \tan x) = \cos(\pi \tan x)$; (3) $\cot(\pi \cot x) = \tan(\pi \tan x)$; (4) $\sin x + \sin 1 = \sin(x + 1)$;

(c) (1) $\sin(\pi\sqrt{8} \cdot \cos x) = \cos(\pi\sqrt{8} \cdot \sin x)$;

(2) $\sin x^2 = \sin 8x$; (3) $\sin x + \sin 2 = \sin(x + 2)$;

(4) $\tan 2x \cdot \cot(x + 1) = 1$;

(d) (1) $\cot(\sin x) = 1$; (2) $\tan(x^2 - x)\cot 2 = 1$;

(3) $\cos x \cdot \cot^2(x/2) = 3/2$; (4) $(\tan x)^{\cos x} = (\cot x)^{\cos x}$.

3.146. Solve graphically:

(a) (1) $2 \sin x = \tan x + 0.5$; (2) $\tan x + 1 = -1 \cdot 5x$; (3) $2 \cos x = x^2 - 2$;

(b) (1) $3 \sin x = x^{-1}$; (2) $\tan x = \sin 3x - 1$; (3) $\tan x = 3$;

(c) (1) $\sin |x| = 1/2$; (2) $\sin x + \cos x = 1/3$; (3) $\tan x = x$;

(d) (1) $\sec x = \cot x$; (2) $\sin x = |x|$, (3) $\cos x + x = 0$.

§ 95. Solving Triangles

Solving Right Triangles

3.147. Solve the following right triangles given: (1) the hypotenuse $c \approx 9.35$ and acute angle $A \approx 65°10'$; (2) the leg $a = 6.37$ and acute angle $A = 4°20'$; (3) the hypotenuse $c = 697$ and leg $a = 528$; (4) the legs: $a = 261$ and $b = 380$.

3.148. Solve an isosceles triangle. Designation: $a = c$, the lateral side; b, the base; $A = C$, the base angles; B, the vertex angle; h_a, the altitude dropped onto the side a; S, the area of the triangle.

(a) $A = 65°$; $h_a = 20$; (b) $a = 8.76$; $b = 13.96$; (c) $B = 73°14'$; $S = 4504$.

3.149. Compute the area of a rhombus given its side $a = 7.5$ cm and acute angle $\alpha = 22°12'$.

3.150. Given: the bases a and b of a trapezoid, one of its lateral sides c, and the acute angle α adjacent to it. Determine the area of the trapezoid.

3.151. A chord is equal to 3/4 of the diameter of a circle. Determine the degree measure of the smaller arc subtended by this chord.

Solving Scalene (Oblique) Triangles

Designation: S, the area of a triangle; R, the radius of the circumscribed circle; r, the radius of the inscribed circle; a, b, the sides of the triangle; A, B, and C, the angles of the triangles: $A = \alpha$, $B = \beta$, $C = \gamma$, their sizes.

3.152. Prove that: (1) $\sin\frac{\alpha+\beta}{2}=\cos\frac{\gamma}{2}$; (2) $\tan\frac{\alpha+\beta}{2}=\cot\frac{\gamma}{2}$; (3) $\sin(\alpha+\beta)=\sin\gamma$; (4) $\cos(\alpha+\beta)=-\cos\gamma$.

3.153. Prove the following equalities:

(1) $\sin\alpha+\sin\beta+\sin\gamma=4\cos(\alpha/2)\cos(\beta/2)\cos(\gamma/2)$;

(2) $\cos\alpha+\cos\beta+\cos\gamma=1+4\sin(\alpha/2)\sin(\beta/2)\times\sin(\gamma/2)$;

(3) $\cot(\alpha/2)+\cot(\beta/2)+\cot(\gamma/2)=\cot(\alpha/2)\cot(\beta/2)\times\cot(\gamma/2)$;

(4) $\sin 2\alpha+\sin 2\beta+\sin 2\gamma=4\sin\alpha\sin\beta\sin\gamma$;

(5) $\sin 3\alpha+\sin 3\beta+\sin 3\gamma=-4\cos(3\alpha/2)\cos(3\beta/2)\times\cos(3\gamma/2)$;

(6) $\tan(\alpha/2)\tan(\beta/2)+\tan(\beta/2)\tan(\gamma/2)+\tan(\alpha/2)\tan(\gamma/2)=1$.

3.154. Find the area of the triangle if:

(1) $a=40$; $b=50$; $C=50°$;

(2) $b=64$; $c=82$; $A=65°$;

(3) $a=32$; $b=53$; $C=61°$; (4) $a=95$; $B=35°$; $C=42°$.

3.155. Compute the radius of the circle circumscribed about a triangle if: (1) $a=28$; $A=72°$; (2) $b=74$ and $B=34°$.

3.156. The lateral side of an isosceles triangle is equal to a, the vertex angle is β. Determine the radii of the inscribed and circumscribed circles.

3.157. The base of an isosceles triangle is b, the base angle is α. Determine the perimeter of the triangle.

3.158. The greater base of a trapezoid serves as the diameter of the circumscribed circle whose radius is R. The acute angle of the trapezoid is equal to α. Find the area of the trapezoid.

3.159. The area of an isosceles triangle is equal to 120 cm^2, the base is 6 cm. Find the area of the trapezoid.

3.160. Prove that in any triangle the side opposite the angle of 30° is equal to the radius of the circle circumscribed about the triangle.

3.161. The sides of a parallelogram are 4 cm and 5 cm, the angle between them is equal to 52°. Find the diagonals of the parallelogram.

3.162. Solve the following scalene triangles given their basic elements:

(a) (1) $a = 370.0$; $B = 86°30'$; $C = 50°50'$;
(2) $b = 13.02$; $A = 11°46'$; $B = 133°40'$;
(b) (1) $a = 510$; $B = 317$; $C = 76°10'$;
(2) $b = 28$; $C = 42$; $A = 124°$;
(c) (1) $a = 87$; $b = 215.4$; $A = 75°$;
(2) $b = 263.1$; $c = 65$; $B = 70°14'$;
(d) (1) $a = 19$; $b = 34$; $c = 49$;
(2) $a = 172.5$; $b = 113.4$; $c = 120.5$.

3.163. In an isosceles trapezoid, the bases are equal to 10 cm and 12 cm, and the altitude is equal to $4\sqrt{3}$ cm. Find the angles of the trapezoid.

3.164. The sides of a parallelogram are 20 cm and 36 cm. Find the angles of the parallelogram if its area is equal to 360 cm^2.

3.165. Find the value of sine and cosine of the acute angle of an isosceles trapezoid if its bases are 8 cm and 14 cm, and the area is equal to 44 cm^2.

3.166. Two chords are given in a circle: $AB = 8$ cm and $AC = 5$ cm which form an angle $\alpha = 60°$. Determine the radius of the circle.

3.167. (1) Prove that the area of the quadrilateral inscribed in a circle is equal to $(1/2)\,(ab + cd)\sin\alpha$, where a, b, c, and d are the sides of the quadrilateral, and α is the angle between the sides a and b.

(2) Prove that the area of the quadrilateral inscribed in a circle is equal to $S = \sqrt{(p-a)(p-b)(p-c)(p-d)}$, where p is the semiperimeter and a, b, c, d are its sides.

(3) The vertices of an equilateral triangle lie on three parallel lines, the distances between which are a and b, respectively. Determine the length of the side of the triangle.

§ 96. Recapitulation Section

3.168. On the unit circle, plot a point M whose coordinates satisfy the given conditions and write the set of numbers corresponding to the point M:

(1) $y = 0.5$, $x > 0$; (2) $y = -\sqrt{3}/2$, $x > 0$;
(3) $y = -\sqrt{3}/2$, $x < 0$; (4) $x = \sqrt{3}/2$, $y > 0$;
(5) $x = -1/2$, $y > 0$; (6) $x = -1/2$, $y < 0$.

3.169. (a) Is there an angle for which: $\sin\alpha = -4.5$; $\cos\alpha = 2$; $\sin\alpha = 0.7$; $\cos\alpha = 0.517$; $\tan\alpha = -1.5$; $\cot\alpha = 0.5$; $\tan\alpha = 3/5$; $\cot\alpha = -20$?

(b) Determine whether the following equalities are possible for the same angle: (1) $\sin\alpha = 1/2$, $\cos\alpha = 2/3$. (2) $\sin\alpha = 5/13$, $\cos\alpha = 12/13$; (3) $\sin\alpha = 0.8$, $\cos\alpha = 0.6$; (4) $\sin\alpha = \sqrt{2}/2$, $\sin\alpha = \sqrt{3}/2$.

3.170. Determine the sign of each expression:

(1) $\sin 5$; (2) $\cos 5$; (3) $\sin 9.2\pi$; (4) $\cos 9.2\pi$;
(5) $\tan(-78\pi/7)$; (6) $\cot(78\pi/7)$; (7) $\tan 6.75\pi$;
(8) $\cot(-6.75\pi)$.

3.171. Prove the following identities:

(1) $\cos(\alpha + \pi/2)\cos(3\pi + \alpha) + \sin(\alpha + 5\pi/2)\sin(3\pi + \alpha) = 0$;

(2) $\dfrac{\cot(3\pi/2+\alpha)\cos(\alpha+3\pi)}{\cot(3\pi-\alpha)\cos(7\pi/2-\alpha)} = \tan\alpha$;

(3) $\dfrac{\cot(5.5\pi-\alpha)+\tan(\beta-4\pi)}{\cot(\alpha-6\pi)-\tan(\beta-3.5\pi)} = \tan\alpha\cdot\tan\beta$;

(4) $\dfrac{\cos(8\pi+\alpha)}{\tan(3\pi-\alpha/2)-\cot(\pi-\alpha/2)} = (1/2)\sin\alpha$.

3.172. Simplify the given expressions:

(1) $\dfrac{\sin(9\pi/2-2\alpha)+2\sin^2(2\alpha-5\pi/2)-1}{1+\sin(2\alpha+\pi/2)-\sin(4\alpha-\pi/2)+\sin(6\alpha-3\pi/2)}$;

(2) $\dfrac{1-2\sin^2\alpha}{2\tan(5\pi/4+\alpha)\cos^2(\pi/4+\alpha)} - \tan\alpha + \sin(\pi/2+\alpha) - \cos(\alpha-\pi/2)$;

(3) $\sin\alpha\cdot\sin^2(\alpha - 270^\circ)(1 + \tan^2\alpha) + \cos\alpha\cdot\cos^2(\alpha + 270^\circ)(1 + \cot^2\alpha)$;

(4) $\cos^2(5\pi/8 + \alpha) - \sin^2(15\pi/8 + \alpha)$.

3.173. Prove the following identities:

(1) $\dfrac{\sin 24^\circ\cdot\cos 6^\circ - \sin 6^\circ\cdot\sin 66^\circ}{\sin 21^\circ\cdot\cos 39^\circ - \sin 39^\circ\cdot\cos 21^\circ} = -1$;

(2) $\cot 10^\circ \cdot \cot 50^\circ \cdot \cot 70^\circ = \cot 30^\circ$;

(3) $\cot 60^\circ + \tan 60^\circ + \cot 50^\circ + \tan 50^\circ = (8/\sqrt{3}) \cos 20^\circ$;

(4) $\tan 9^\circ + \tan 15^\circ - \tan 27^\circ + \cot 9^\circ + \cot 15^\circ - \cot 27^\circ = 8$;

(5) $\dfrac{\cos 8 - \sin 8 + 1}{\cos 8 + \sin 8 - 1} = \cot 4$;

(6) $\dfrac{1 - \cos 6 + \sin 6}{1 + \cos 6 + \sin 6} = \tan 3$.

3.174. Solve the following equations:

(1) $8 \cos^4 x = 3 + 5 \cos 4x$; (2) $\sin^6 x + \cos^6 x = a$;

(3) $\sin^4 x + \cos^4 x + \sin 2x + a = 0$;

(4) $\cos 4x \cdot \cos(\pi + 2x) - \sin 2x \cdot \cos(\pi/2 - 4x) = (\sqrt{2}/2) \sin 4x$;

(5) $\sin^4 x - 2 \cos^2 x + a^2 \quad 0$;

(6) $\sin^2 x - \sin x \cdot \cos x - (2 - a^2) \cos^2 x = 0$;

(7) $\sin x^2 = -1$.

3.175. Solve the equation $\cos 4x + \dfrac{10 \tan x}{\tan^2 x + 1}$ if $x \in [-3\pi/4, \pi/2]$.

3.176. Solve the given equations:

(1) $|\sin t| + |\cos t| = 1.4$;

(2) $(\sin x + \sqrt{3} \cos x)^2 = 5 + \cos(\pi/3 + 4x)$.

3.177. Show that the equation $2 \cot 2x - 3 \cot 3x = \tan 2x$ has no roots.

3.178. Solve the equation: $(49 \sin x - 49 \cos x)(\sin^3 x - \cos^3 x)^{-1} - 28 (\sin 2x)^{-1} = 0$.

3.179. Find the greatest root of the given equation: (a) $|\cos x| = \dfrac{\sqrt{3}}{2}$ if $x \in (0^\circ, 360^\circ)$; (b) $\sin \pi x = \cos 3\pi x$ if $x \in (0, 0.5)$.

3.180. Find the least root of the equation $\cos^2 x - \sin^2 x - \sin x = 0$ if $x \in (0^\circ, 180^\circ)$.

3.181. Find the solutions of the equation $(1/2)(\sin^4 x - \cos^4 x) = \sin x \cdot \cos x$ if $x \in (0^\circ, 90^\circ)$.

3.182. For what values of p does the equation $3 \sin^2 x + 12 \cos x - 3 = p$ have at least one solution?

3.183. Solve the following equations:

(1) $\cos x \cdot \cot x - \sin x = a \cos 2x$ on $[0, 2\pi]$;

(2) $a \cot x - 1 = \cos 2x$ on $(0, 2\pi)$;

(3) $4 \sin x \cdot \cos^3 x + \sin^3 x - \sin x = 0$ on $[0, \pi]$;

(4) $\sqrt{3} - \cot(\pi/2 - x) = \tan(\pi/3 - x)$ on $(\pi/2, \pi)$.

3.184. Solve the following systems of equations:

(1) $\begin{cases} \sin x \cdot \sin y = \sqrt{3}/4, \\ \cos x \cdot \cos y = \sqrt{3}/4; \end{cases}$ (2) $\begin{cases} \tan x + \tan y = 2, \\ \cos x \cdot \cos y = 0.5; \end{cases}$

(3) $\begin{cases} x - y = 6.5\pi, \\ 3\cos^2 x - 12 \cos y = -4; \end{cases}$ (4) $\begin{cases} x + y = \pi/3, \\ \sin x + \sin y = \sqrt{3}/2. \end{cases}$

3.185. Solve the following inequalities:

(1) $\sin x + \sqrt{3} \cos x > 0$; (2) $\tan^2 x - (1 + \sqrt{3}) \tan x + \sqrt{3} < 0$.

(3) $\sqrt{5 - 2 \sin x} \geqslant 6 \sin x - 1$;

(4) $\sin 2x + \sin x \leqslant 0$;

(5) $\cos 2x + 5 \cos x + 3 \geqslant 0$; (6) $|\cos x| \geqslant \sin^2 x$.

CHAPTER 4

Geometry

§ 97. Angles. Measuring Angles

4.1. Prove that the angle between the bisectors of adjacent angles is a right one.

4.2. Straight lines AB and CD intersect at a point O, and the sum of angles AOD and COB is equal to 220°. Find the size of the angle AOC.

4.3. Find two adjacent angles if one of them is by 20° greater than the other one.

4.4. Find two adjacent angles if one of them is 0.8 of the other.

4.5. Find two adjacent angles if their ratio is 4 : 5.

4.6. One of the adjacent angles is $1\frac{2}{3}$ times of the other. Find these angles.

4.7. A ray emanating from the vertex of a right angle forms with one of its sides an angle of: (a) $(3/8)\,d$, (b) $(7/15)\,d$. Find the angle between the ray and the second side of the right angle.

4.8. One of two adjacent angles is five times the other. Find these angles.

4.9. One of two adjacent angles is $\frac{1}{3}$ of the other. Find the angles.

4.10. Inside an obtuse angle, perpendiculars are erected to its sides from its vertex; the angle between the perpendiculars is equal to $(4/7)\,d$. Find the obtuse angle.

4.11. The sum of a given angle and two angles adjacent to it is equal to $\left(2\frac{3}{8}\right)d$. Find the given angle.

4.12. Given two adjacent angles: one is acute, the other obtuse. A straight line drawn through their vertex perpendicular to their common side is deviated from the other side of the acute angle by $(5/7)\,d$ and from the other side of

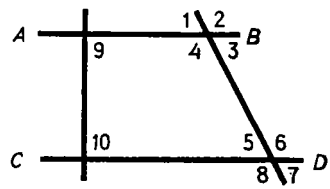

FIG. 7

the obtuse angle by $(3/7)\,d$. Find the given angles and make an accurate drawing.

§ 98. Angles with Parallel and Perpendicular Sides

4.13. Prove that $AB \parallel CD$ if (Fig. 7):

(1) $\angle 1 \cong \angle 5$; (2) $\angle 1 \cong \angle 7$; (3) $\angle 4 \cong \angle 6$;
(4) $\angle 2 + \angle 7 = 180°$; (5) $\angle 3 + \angle 6 = 180°$;
(6) $\angle 1 + \angle 8 = 180°$.

4.14. Two parallel straight lines are intersected by a third straight line. Find the obtained angles if: (a) the ratio of the interior one-sided angles is 7 : 11; (b) the ratio of the exterior one-sided angles is 3 : 5; (c) one of the one-sided interior angles is greater than the other by 33°; (d) one of the exterior one-sided angles is less than the other by 18°24′.

4.15. A perpendicular and two inclined lines are drawn from a point K of a straight line. The sum of the lengths of the inclined lines is equal to 20 cm. Find the lengths of the inclined lines if their projections on the given straight line are congruent.

4.16. Point out the condition and the conclusion in the sentence: "If the given angles are adjacent, then their sum is equal to $2d$".

4.17. Formulate the converse of the theorem: "If one of two adjacent angles is acute, then the other is obtuse".

4.18. Given two angles with mutually perpendicular sides, one of them is $\frac{1}{4}$ of the other. Find each angle.

4.19. Two perpendiculars are drawn to the lateral sides of a triangle through the end points of its base. The perpendiculars intersect at an angle of 130°. Compute the vertex angle of the triangle.

§ 99. Triangles

4.20. (a) Given: $[AD] \cong [BC]$ and $AD \parallel BC$, $\angle 1 \cong \angle 2$. Prove that $\triangle ABC = \triangle ADC$ (Fig. 8); (b) Given: $[AD] \cong [DC]$, $\angle 1 \cong \angle 2$. Prove that $\triangle ABD \cong \triangle CBD$ (Fig. 9).

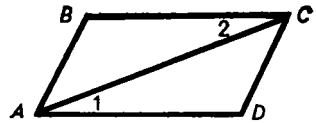

FIG. 8

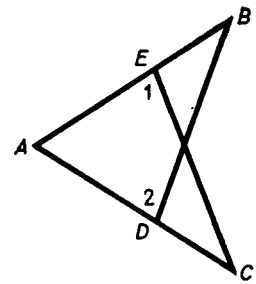

FIG. 11

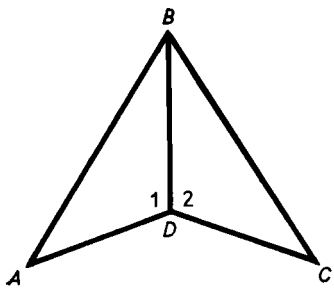

FIG. 9

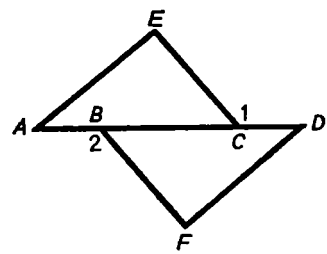

FIG. 12

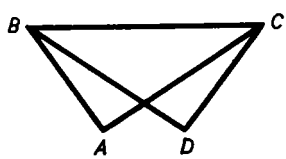

FIG. 10

4.21. (a) Given: $[AB] \cong [DC]$, $\angle ABC \cong \angle DCB$. Prove that $\triangle ABC \cong \triangle DCB$ (Fig. 10). (b) Given: $[AE] \cong [AD]$, $\angle 1 \cong \angle 2$. Prove that $\triangle ABD \cong \triangle ACE$ (Fig. 11); (c) Given: $[AC] \cong [BD]$, $\angle 1 \cong \angle 2$; $\angle A \cong \angle D$. Prove that $\triangle ACE \cong \triangle BDF$ (Fig. 12).

4.22. An equilateral triangle with a perimeter of 45 m is constructed on a lateral side of an isosceles triangle with a perimeter of 40 m. Determine the base of the isosceles triangle.

4.23. The bisectors of the base angles in an isosceles triangle are equal. Prove this.

4.24. (a) Prove that the medians and altitudes drawn to the lateral sides of an isosceles triangle are equal; (b) state the converse and prove it.

4.25. The median drawn to one of the lateral sides of an

isosceles triangle divides its perimeter into two parts 15 and 6 cm long. Find the sides of the triangle.

4.26. The lateral side AB of an isosceles triangle ABC is equal to 14 cm. A perpendicular DE is drawn to it from its midpoint D to intersect the side BC, and the point E is connected to the point A. The perimeter of the triangle AEC is equal to 24 cm. Find AC.

4.27. In an isosceles triangle, the vertex angle is equal to 105°27′. Find the base angle.

4.28. In an isosceles triangle, the base angle is equal to 70°43′. Find the vertex angle.

4.29. Find the point equidistant from all the vertices of a triangle. Is this point always found inside the triangle?

4.30. Prove that a median of an equilateral triangle divides it into two congruent triangles.

4.31. If equal line segments AE and CD are laid off on the lateral sides of an isosceles triangle ABC, then the line segments CE and AD are equal. Prove this.

4.32. In an isosceles triangle, the angle between the base and altitude drawn to a lateral side is equal to $(8/15)\,d$. Find the angles of this triangle.

4.33. Prove that in a right triangle the length of the median drawn to the hypotenuse is equal to half its length.

4.34. The altitudes AD and CE of a triangle ABC intersect at the point M. Find the angle AMC if $\angle BAC = (1/4)\,d$ and $\angle BCA = (5/6)\,d$.

4.35. One side of an isosceles triangle is equal to 25 m, the other being 10 m long. Which of them serves as the base?

§ 100. The Sum of the Angles of a Triangle. Exterior Angles of a Triangle

4.36. Determine the angles of a triangle if their ratio is 1 : 2 : 3.

4.37. The ratio of two angles of a triangle is 5 : 7; the third angle being by $4d/19$ greater than the first angle. Find the third angle.

4.38. Determine the angles of a triangle if one of them constitutes 2/3 of the second angle and 4/5 of the third.

4.39. The angle between the altitude and lateral side of an isosceles triangle is by $d/6$ less than the base angle. Find the angles of the triangle.

4.40. The base angle of an isosceles triangle is 4/5 of the adjacent angle. Find the angles of the triangle.

4.41. One of the acute angles of a right triangle is equal to $2d/3$, and the sum of lengths of the hypotenuse and the lesser leg is equal to 1.8 m. Find the hypotenuse.

4.42. Prove that the bisector of the exterior vertex angle of an isosceles triangle is parallel to the base.

4.43. An exterior angle of a triangle is 1.5 the size of its adjacent angle and $4\frac{1}{3}$ of one of the interior angles. Find the angles of the triangle.

4.44. If a leg of a right triangle is half the hypotenuse, then the angle opposite this leg is equal to 30°. Prove this.

4.45. The exterior angle at the vertex B of a triangle ABC is three times the angle A and by $4d/9$ greater than the angle C. Determine the angles of the triangle.

4.46. Three medians are drawn in an equilateral triangle. Prove that they divide the triangle into six equal triangles.

4.47. An exterior angle of a triangle is equal to 108°, the ratio of the angles not adjacent to this angle is 5 : 4. Find the angles of the triangle.

4.48. The sum of the interior angles of an isosceles triangle together with one of the exterior angles is equal to $21d/18$. Determine the angles of this triangle.

§ 101. The Midline of a Triangle

4.49. The sides of a triangle are equal to 8, 10, and 12 cm. Find the sides of the triangle whose vertices are the midpoints of the sides of the given triangle.

4.50. The perimeter of a triangle is $2p$ cm. The midpoints of the sides are joined in succession. Find the perimeter of the triangle thus obtained.

4.51. The altitude of an equilateral triangle is equal to 6 cm. Find the projection of the given altitude on the other altitude.

4.52. The ratio of the base of a triangle to its midline is 5 : 3. The base is by 6 cm greater than the midline. Find the base of the triangle.

4.53. The vertex angle of an isosceles triangle is equal to 60°. The lateral side is equal to 10 cm. Find the midline.

4.54. The vertex angle of an isosceles triangle is equal to

120°, the altitude $h = 3$ cm. Find the line segment connecting the midpoints of the lateral side and base.

4.55. The ratio of sides of a triangle is 3 : 4 : 6. Joining the midpoints of the sides of this triangle, we get a triangle with a perimeter equal to 5.2 m. Find the sides of the given triangle.

§ 102. The Parallelogram and Its Properties

4.56. One of the angles of a parallelogram is equal to $3d/7$. Find the sizes of the other angles.

4.57. Determine the angles of a parallelogram if one of them is by $3d/11$ greater than the other.

4.58. In a parallelogram $ABCD$, $AB = 9$ cm and is 3/10 of the entire perimeter. Determine all the sides of the parallelogram.

4.59. The ratio of sides of a parallelogram is 3 : 4, its perimeter being equal to 2.8 cm. Determine its sides.

4.60. The bisector of the angle A in a parallelogram $ABCD$ is drawn to intersect the side BC at the point E. Determine the line segments BE and EC if $AB = 9$ cm and $AD = 15$ cm.

4.61. The sides of a parallelogram are equal to 8 and 3 cm; the bisectors of two angles of the parallelogram adjacent to the greater side divide the opposite side into three parts. Find the length of each part.

4.62. Can the diagonal of a parallelogram be equal to its side?

4.63. A straight line is drawn through the point of intersection of the diagonals of a parallelogram. Prove that the line segment of this straight line enclosed between the parallel sides is halved by this point.

4.64. A straight line is drawn in a parallelogram $ABCD$ through the point of intersection of its diagonals and cuts off the line segments $BE = 2$ m and $AF = 2.8$ m on the sides BC and AD, respectively. Determine BC and AD.

§ 103. The Rectangle

4.65. A rectangle is inscribed in a right isosceles triangle so that two of its vertices lie on the hypotenuse and the other on its legs. Determine the sides of the rectangle if

their ratio is 5 : 2 and the hypotenuse of the triangle is equal to 45 cm.

4.66. Prove that any parallelogram with equal diagonals is a rectangle.

4.67. Is there a point in a rectangle which is equidistant from: (a) all of its sides; (b) all of its vertices?

4.68. The perimeter of a rectangle is equal to 24 cm and one side is by 1.4 cm longer than the other. Find the distances from the intersection point of the diagonals to the sides.

4.69. The diagonal of a rectangle makes an angle of $(2/3)\,d$ with the side. Find the angles between the diagonals.

4.70. The perpendicular drawn from a vertex of a rectangle to its diagonal divides the latter in the ratio 1 : 3. Find the angle between the diagonals.

4.71. The distance between the point of intersection of diagonals of a rectangle and the smaller side is by 4 cm greater than the distance between that point and the longer side. The perimeter of this rectangle is 56 cm. Find the sides of this rectangle.

4.72. Prove that the intersection points of the bisectors of all the angles of a rectangle are vertices of a square.

4.73. The perimeter of a rectangle is equal to 12 cm. Find the sum of distances from an arbitrary point of the rectangle to its sides.

§ 104. The Trapezoid

4.74. The midline of a trapezoid is equal to 8 dm and is divided by the diagonal into two line segments whose difference is 2 dm. Determine the bases of the trapezoid.

4.75. Prove that in an isosceles trapezoid: (a) the base angles are equal; (b) the diagonals are equal.

4.76. The smaller base of an isosceles trapezoid is equal to its lateral side, and the diagonal is perpendicular to the lateral side. Find the angles of the trapezoid.

4.77. In an isosceles trapezoid, whose perimeter is 20 m long the lateral side is equal to the midline. Find the lateral side of the trapezoid.

4.78. The greater base of an isosceles trapezoid is equal to 18 cm, the altitude 5 cm, the obtuse angle 135°. Find the midline of the trapezoid.

4.79. In a right-angled trapezoid $ABCD$, $AB \perp AD$ $CD = a$, $\angle C = 120°$, and the diagonal BD bisects the angle D. Find the midline of the trapezoid.

4.80. Three sides of a trapezoid are equal and the diagonal is equal to one of the bases. Find the angles.

4.81. The altitude drawn from the vertex of the obtuse angle of an isosceles trapezoid divides the greater base into two line segments 5 and 2 cm long. Compute the midline of the trapezoid.

§ 105. The Rhombus

4.82. Determine the angles of a rhombus if the altitude drawn from the vertex of the obtuse angle divides the opposite side into two equal parts.

4.83. The perimeter of a rhombus is 8 cm and the altitude 1 cm. Find the angles of the rhombus.

4.84. Find the angles of a rhombus in which: (a) one of the diagonals is equal to its side; (b) the altitude halves the side; (c) the ratio of the angles formed by the diagonals and the side is 11 : 16.

4.85. One of the diagonals of a rhombus is equal to its side. Compute: (a) the angles of the rhombus; (b) the angles formed by the diagonals of the rhombus with its sides.

4.86. Find the angles of a rhombus if: (a) the ratio of the angles formed by the diagonals of the rhombus and its sides is 2 : 3; (b) the altitudes drawn from the vertex of the rhombus form an angle of 30°; (c) the altitudes drawn from the vertex of the rhombus form an angle of 120°.

4.87. If the intersection point of the diagonals of a quadrilateral is equidistant from all of its sides, then this quadrilateral is a rhombus. Prove this.

4.88. Two altitudes are drawn from the vertices of obtuse angles in a rhombus. The altitudes are divided by the point of their intersection in the ratio 1 : 2. Determine the angles of the rhombus.

4.89. The altitude of a rhombus divides its side into the line segments m and n. Find the diagonals of the rhombus.

§ 106. The Square

4.90. The diagonal of a square is 4 m. Its side is the diagonal of another square. Find the length of the side of the latter.

4.91. Given a square whose side equals 1 m. Its diagonal serves as a side of another square. Find the diagonal of the latter.

4.92. A square is inscribed in a right isosceles triangle with the leg 2 m long. Find the perimeter of the square if the triangle and the square have a common angle.

4.93. The sides of a rectangle are equal to 1 cm and 3 cm. Find the diagonals of the square whose vertices lie at the intersection points of the bisectors of the interior angles of the rectangle.

4.94. Given a square whose side is 6 cm. The midpoints of the sides of the square are connected by line segments in succession. Prove that the quadrilateral thus obtained is a square and find its perimeter.

4.95. The length of the diagonal of a square is 4 m; the side of this square serves as the diagonal of another square. Find the perimeter of the second square.

4.96. The leg of a right isosceles triangle is equal to 6 cm. Find the perimeter of the square inscribed in this triangle.

4.97. A square 5 cm on a side is inscribed in a right isosceles triangle. Find the sides of the triangle.

4.98. Prove that if the diagonals of a quadrilateral are equal and serve as bisectors of its angles, then such a quadrilateral is a square.

4.99. How many axes of symmetry does the square have?

4.100. How many distinct displacements, including identical ones, does the square map on itself?

4.101. Prove that a rhombus in which one of the angles is equal to 90° is a square.

4.102. The right angle in a triangle is bisected; from the intersection point of the bisector and hypotenuse straight lines are drawn parallel to the legs. Prove that the quadrilateral formed by these straight lines and legs is a square.

4.103. A square is inscribed in a right isosceles triangle so that two of its vertices lie on the hypotenuse and the other two on its legs. Find the side of the square if the hypotenuse is 3 m long.

§ 107. The Sum of Interior Angles of a Polygon

4.104. Find the sum of interior angles of a convex n-gon if: $n = 6$; $n = 7$; $n = 8$; $n = 12$; $n = 24$; $n = 36$.

4.105. Find the angles of a convex pentagon if their ratio is $1 : 1.5 : 2 : 2.5 : 3$.

4.106. How many sides does a convex polygon have if the sum of its interior angles equals $8d$; $30d$; $48d$; $57d$?

4.107. How many sides does a convex polygon have if the sum of its interior angles together with one of its exterior angles is equal to $23d$?

4.108. How will the sum of interior angles of a polygon change if the number of its sides is increased by 3; 5; 7?

4.109. Find the number of sides of a polygon if the sum of its interior angles is m times the sum of its exterior angles ($m = 2; 3; 4$).

4.110. Find the angles of a quadrilateral if the ratio of the first two angles is $5 : 7$, the third angle is equal to the difference between the first and second, the difference between the third and fourth angles being equal to $(4/11)\,d$.

4.111. What polygon has the same sums of interior and exterior angles?

§ 108. The Circle

4.112. (1) The radius of a circle is 10 cm and a point is located at a distance of 15 cm from the centre. Find its least and greatest distances from the circle.

(2) The radius of a circle is equal to 10 cm and a point is found at a distance of 3 cm from the centre. Find its least and greatest distances from the circle.

4.113. The greatest and least distances from a point lying inside a circle to the circumference are equal to 10 and 4 cm, respectively. Find the radius of the circle.

4.114. The greatest and least distances from a point lying outside a circle to its circumference are equal to 6 and 2 cm, respectively. Find the radius of the circle.

4.115. A diameter and a chord equal to the radius are drawn from a point on a given circle. Find the angle between them.

4.116. Two chords equal to the radius of a circle are drawn from a point on the circle. Find the size of the angle between them.

4.117. Two mutually perpendicular chords are drawn in a circle, each of them is divided by the other into two line

segments 3 and 7 cm long. Find the distance of each chord from the centre of the circle.

4.118. Two mutually perpendicular chords are drawn in a circle at a distance of 1 cm from the centre, each chord is equal to 6 cm. Find the parts into which one chord is divided by the other.

4.119. A chord intersects the diameter at an angle of 30° and divides it into two line segments 2 cm and 6 cm long. Find the distance of the chord from the centre.

4.120. The end points of a diameter are found at a distance of 1.6 and 0.6 m from a tangent line. Determine the diameter.

4.121. Two equal parallel chords are drawn in a circle, the distance between them being equal to the radius of the circle. Find the acute angle between the straight lines joining the end points of the chords.

4.122. The least distance between two concentric circles is 2 cm, while the greatest 16 cm. Determine the radii of these circles.

4.123. Given two concentric circles; in the larger circle, two mutually perpendicular chords are drawn tangent to the smaller circle; either of the chords is divided by the other into two parts 3 and 7 cm long. Find the radius of the smaller circle.

4.124. The ratio of radii of two concentric circles is 7 : 4, the width of the annulus being 12 cm. Determine the radius of the smaller circle.

§ 109. Inscribed Angles

4.125. Compute the angle inscribed in an arc constituting 17/32 of the circumference.

4.126. Three equal circles of radius R touch each other externally. Determine the sides and the angles of the triangle whose vertices are the points of tangency.

4.127. The radius of a circle is equal to 1.4 cm. Find the distance from the centre to the chord which subtends an arc of 120°.

4.128. The chord AB divides a circle into two arcs, the smaller of which is equal to 130°, and the longer is divided by the chord AC in the ratio 31 : 15 beginning from A. Determine BAC.

4.129. A circle is divided in the ratio 7 : 11 : 6, the division points being connected by chords. Find the angles of the triangle thus obtained.

4.130. Through the end point of the chord dividing a circle in the ratio 3 : 5 a tangent line is drawn. Determine the acute angle between the chord and the tangent line.

4.131. A circle is divided by the points A, B, C, and D so that $\overset{\smile}{AB} : \overset{\smile}{BC} : \overset{\smile}{CD} : \overset{\smile}{DA} = 2 : 3 : 5 : 6$ and chords $\overset{\smile}{AC}$ and $\overset{\smile}{BD}$ are drawn intersecting each other at the point M. Determine $\overset{\frown}{AMB}$.

4.132. A diameter AB and chord CD intersect at a point M; $\overset{\frown}{CMB} = 73°$; $\overset{\smile}{BC}$ contains 110°. What is the degree measure of $\overset{\smile}{BD}$?

4.133. A circle is divided by the points A, B, C, and D so that $\overset{\smile}{AB} : \overset{\smile}{BC} : \overset{\smile}{CD} : \overset{\smile}{DA} = 3 : 2 : 13 : 7$. The chord AD and BC are extended to intersect at a point M. Determine $\overset{\frown}{AMB}$.

§ 110. Inscribed and Circumscribed Triangles and Quadrilaterals

4.134. Prove that any trapezoid inscribed in a circle is equilateral.

4.135. A right triangle whose hypotenuse is 26 cm long is circumscribed about a circle of 4 cm radius. Find the perimeter of the triangle.

4.136. The smaller side of a rectangle is 1 m and the acute angle between the diagonals amounts to 60°. Find the radius of the circumscribed circle.

4.137. The side of a rhombus is 8 cm and the acute angle is 30°. Determine the radius of the inscribed circle.

4.138. A circle is inscribed in a rhombus. What are the four arcs into which it is divided by the points at which the circle touches the sides of the rhombus if the acute angle of the rhombus is 37°?

4.139. A trapezoid is circumscribed about a circle. The perimeter of the trapezoid is 12 cm. Determine the midline of this trapezoid.

4.140. An isosceles trapezoid whose acute angle is 30° is circumscribed about a circle. The midline of the trapezoid is 1 m. Determine the radius of the circle.

4.141. A circle is inscribed in a right triangle whose sides are 6, 8, and 10 cm. Straight lines parallel to the sides of the triangle are drawn through the centre of the circle which cut out segments on the sides of the triangle. Find the interior segments on the triangle sides.

4.142. An isosceles triangle with an acute base angle of 30° is circumscribed about a circle 3 cm radius. Determine the sides of the triangle.

4.143. A circle is inscribed in a quadrilateral. The ratio of its three successive sides is 1 : 2 : 5, its perimeter being 60 cm. Find all the sides of the quadrilateral.

§ 111. Regular Polygons

4.144. Compute the interior and exterior angles of a regular n-gon ($n = 3; 4; 5; 6; 8; 10; 12$).

4.145. Determine the number of sides of a regular n-gon if: (a) each of its interior angles is equal to: (1) 60°; (2) 120°; (3) 135°; (4) 140°; (5) 150°; (b) each of its exterior angles amounts to: (1) 24°; (2) 36°; (3) 60°.

4.146. Prove that the central angle of a regular polygon is equal to its exterior angle.

4.147. (a) How many axes of symmetry does a regular polygon have? (b) Does any of regular polygons have the centre of symmetry?

4.148. What is the number of displacements mapping onto itself: (a) a regular pentagon; (b) a regular hexagon; how many: (1) rotations; (2) axial symmetries will be among these displacements?

4.149. Prove that the side of a regular n-gon $a_n = 2R \sin (180°/n)$, where n is the number of sides. Using this formula, compute: (1) a_3; (2) a_4; (3) a_6; (4) a_8; (5) a_{12}.

4.150. Express the radius of the circle circumscribed about a regular polygon in terms of its side a_n if: (1) $n = 3$; (2) $n = 4$; (3) $n = 6$; (4) $n = 8$; (5) $n = 12$.

4.151. Express the side of a regular n-gon in terms of the radius of the inscribed circle if: (1) $n = 3$; (2) $n = 4$; (3) $n = 6$; (4) $n = 8$.

4.152. Given the radii of the circumscribed and inscribed circles, express the side of a regular polygon.

4.153. Express the radius of the inscribed circle in terms of the radius of the circle circumscribed about a regular n-gon if: (1) $n = 3$; (2) $n = 4$; (3) $n = 6$.

4.154. Given the side a of the inscribed regular n-gon, find the side b of the circumscribed regular n-gon.

4.155. Compute the smallest diagonal of a regular n-gon whose side is equal to a ($a = 1$ cm, $n = 5$; $a = 4$ cm, $n = 6$).

4.156. Compute the area of a regular n-gon inscribed in a circle of radius R ($R = 8$ cm; $n = 3$; 4; 6; 8).

4.157. The sides of two polygons having the same name are equal to a and b. What is the ratio of: (a) the perimeters of these polygons, (b) the areas of the polygons?

4.158. Prove that the area of the regular n-gon inscribed in a circle of radius R can be expressed by the formula $S_n = (1/2)\, P_n R \cos(180°/n)$, where P_n is the perimeter of the polygon.

4.159. (a) Prove that $a_8 = R\sqrt{2 - \sqrt{2}}$; $K_8 = (1/2) \times \sqrt{2 + \sqrt{2}}$, where a_8 is the side of the inscribed regular octagon, K_8 the apothem of the inscribed regular octagon, R the radius of the circumscribed circle.

(b) Prove that (1) $a_{12} = R\sqrt{2 - \sqrt{3}}$; (2) $K_{12} = (R/2)\sqrt{2 + \sqrt{3}}$.

4.160. A common chord of two intersecting circles is seen from their centres at angles 90° and 60°. Find the radii of the circles if the distance between their centres is equal to $\sqrt{3} + 1$.

4.161. The side of the triangle inscribed in a circle is equal to a. Compute the area of the square inscribed in this circle.

4.162. A square with side a is cut at the corners so that a regular octagon is formed. Find the area of this octagon.

4.163. Prove that the sum of distances from a point inside an equilateral triangle to its sides is equal to the altitude of the triangle.

§ 112. Proportional Line Segments. Similar Triangles

4.164. The lateral sides AB and CD of the trapezoid $ABCD$ are extended to intersect at a point M. Determine:

(a) BM if $AB = 1.2$ m and $CD/CM = 1/6 : 1/4$;

(b) the line segment CM if $AB = 1$ dm, $CD = 15$ dm, $BM = 9$ dm;

(c) CD if $AB/BM = 17 : 25$ and $CD - CM = 1.6$ m.

4.165. The bases of a trapezoid are 1.8 and 1.2 m; its lateral sides 1.5 and 1.2 m in length are extended to intersect each other. Determine the lengths of the extensions of the lateral sides.

4.166. The ratio of the sides of a triangle is $4 : 5 : 6$; the smaller side is equal to 0.8 m. Determine the other sides.

4.167. The ratio of the sides of a triangle is $2 : 5 : 4$; the perimeter is equal to 55 mm. Determine the sides of the triangle.

4.168. (a) The point M divides the line segment AB in the ratio $AM : MB = 1 : 2$. Find the ratios: $AM : AB$ and $MB : AB$;

(b) The point K divides the line segment AB in the ratio $1 : 3$. Find the ratios $AK : AB$ and $KB : AB$.

4.169. The sides of the angle A are intersected by two parallel straight lines BC and DE (B and D are the points on one side of the angle). Determine:

(a) AE if $AB = 8$ m, $AD = 12$ m, and $AC = 10$ m

(b) AB if $AB + AD = 21$ m, $AC = 12$ m and $AE = 16$ cm;

(c) AD if $AC : AE = 3/11 : 3/5$ and $BD = 12$ dm.

4.170. The bases of an equilateral trapezoid are equal to 4 and 10 cm. The lateral sides are extended by 5 cm to intersect each other. Find the lateral sides of the trapezoid.

4.171. BA and BD are line segments of one side of the angle B; BC and BE are line segments of the other side of the angle. Find out whether the straight lines AC and DE are parallel if:

(a) $BA : AD = 3 : 4$, $BC = 1.2$ m and $BE = 2.8$ m;

(b) $BD : AD = 11 : 8.5$ and $BC = (5/17)\,CE$;

(c) $BA = (7/13)\,BD$, $BC = 2.8$ m and $CE = 2$ m.

3.172. Given a triangle ABC and a line segment DE inside it which is parallel to AC ($D \in AB$, $E \in BC$). Determine DE if:

(a) $AC = 20$ cm, $AB = 17$ cm, and $BD = 11.9$ cm;

(b) $AC = 18$ dm, $AB = 15$ dm, and $AD = 1$ m.

4.173. Given a triangle ABC and a line segment DE inside it which is parallel to AC ($D \in [AB]$ and $E \in [BC]$). Determine:

(a) AD if $AB = 16$ cm, $AC = 2$ dm, and $DE = 15$ cm;

(b) the ratio $AD : BD$ if it is known that $AC : DE = 5/7 : 4/11$.

4.174. In the trapezoid $ABCD$, where $BC \| AD$, with diagonal BD the angles ABD and BCD are equal. Given: $BC = 10$ cm, $DC = 15$ cm, and $BD = 20$ cm. Determine AB and AD.

4.175. In the trapezoid $ABCD$ with diagonal AC the angles ABC and ACD are equal. Determine the diagonal AC if the bases BC and AD are equal to 12 and 27 cm, respectively.

4.176. The ratio of the bases of a trapezoid is 5 : 9, and one of the lateral sides is equal to 16 cm. How long must it be extended to meet the extension of the other lateral side?

4.177. In a triangle with 30 cm base and 10 cm altitude, a right isosceles triangle is inscribed so that its hypotenuse is parallel to the base of the triangle and the vertex of the right angle lies on this base. Determine the hypotenuse.

4.178. The sides of a triangle are equal to 3, 4, and 6 cm. Find the lengths of the sides of a similar triangle with perimeter equal to 58.5 cm.

4.179. The sides of a triangle are 5, 6, and 8 cm. Compute the sides of a similar triangle if its perimeter is equal to 95 cm.

4.180. A rhombus with side m is inscribed in a triangle so that they have one angle in common, and the opposite vertex of the rhombus lies on the side of the triangle and divides this side into line segments of lengths p and q. Find the sides of the triangle.

4.181. The base in an isosceles triangle is 12 cm and the lateral side 18 cm. What line segments must be laid off on the lateral sides to get a trapezoid (by connecting their end points) 40 cm perimeter?

§ 113. Properties of the Bisector of an Interior Angle of a Triangle

4.182. BD is the bisector of the angle B in the triangle ABC. Find: (a) AD and DC if $AB = 10$ m, $BC = 15$ m, $AC = 20$ m; (b) BC if $AD : DC = 8 : 5$ and $AB = 16$ m.

4.183. The sides of a triangle are 51, 85, and 104 cm. A circle with the centre at the larger side touches both smaller sides. Into what parts is the larger side divided by the centre of the circle?

4.184. The altitude of an isosceles triangle is 20 cm, and the ratio of the base to the lateral side is 4 : 3. Determine the radius of the inscribed circle.

4.185. The centre of the circle inscribed in an isosceles triangle divides the altitude in the ratio 12 : 5, and the lateral side is 60 cm. Determine the base of the triangle.

4.186. Inscribed in a triangle is a parallelogram whose angle coincides with that of the triangle. The sides of the triangle enclosing this angle are equal to 20 and 25 cm, and the ratio of the corresponding parallel sides of the parallelogram is 6 : 5. Determine the sides of the parallelogram.

4.187. In a triangle whose base is 30 cm, and the altitude 10 cm, a right isosceles triangle is inscribed so that its hypotenuse is parallel to the base of the given triangle and the vertex of the right angle lies on this base. Determine the hypotenuse.

4.188. The perimeter of a parallelogram is 48 cm, and the ratio of its altitudes is 5 : 7. Determine the lengths of the corresponding sides.

4.189. The sides of a quadrilateral are equal to 10, 15, 20, and 29 dm; the sum of the greatest and least sides of a similar quadrilateral is equal to 26 dm. Determine the sides of the similar quadrilateral.

4.190. The larger sides of two similar polygons are equal to 35 and 14 m, the difference in their perimeters being 60 m. Determine the perimeters of the polygons.

4.191. Construct a triangle similar to a given one if the ratio of similitude is equal to 2/3.

4.192. Construct a quadrilateral similar to a given one if the ratio of similitude is equal to 2.

§ 114. Proportional Line Segments in a Circle

4.193. The annulus formed by two concentric circles is 8 dm wide. The chord of the larger circle tangent to the smaller circle is equal to 4 m. Determine the radii of the circles.

4.194. A tangent to a circle and a secant emanate from the same point and are 20 and 50 cm long, respectively; the secant and the centre of a circle are 8 cm distant from each other. Determine the radius of the circle.

4.195. Determine the distance from the centre of the circle to the point from which a tangent and a secant emanate if they are 4 cm and 8 cm, respectively, and the secant is 12 cm distant from the centre.

4.196. A tangent and a secant are drawn to a circle from the same point. Determine the length of the tangent if it is by 5 cm longer than the external segment of the secant and by the same amount shorter than the internal segment.

4.197. A secant and a tangent are drawn to a circle from the same point. The sum of their lengths is equal to 30 cm, and the internal segment of the secant is by 2 cm shorter than the tangent. Determine the secant and tangent.

4.198. The base of an isosceles triangle is 48 dm and the lateral side 30 dm. Determine the radii of the circumscribed and inscribed circles and the distance between their centres.

4.199. From a point outside a circle a tangent and a secant are drawn. Find the lengths of the tangent and the secant if the tangent is known to be by 2 cm less than the secant and that the external segment of the secant is 4.5 cm.

4.200. Two tangents are drawn to a circle from the same point. The length of each tangent is equal to 12 cm, the distance between the points of tangency being equal to 14.4 cm. Determine the radius of the circle.

4.201. A tangent is drawn to a circle of radius r through one end point of a chord of length $0.5r$, and a secant parallel to the tangent through the other end point. Find the distance between the tangent and secant.

§ 115. Rotation. Central Symmetry. Axial Symmetry

4.202. Given two distinct points O and A. Construct the image of the point A as a result of rotation with respect to O through an angle of: (a) $+45°$; (b) $-60°$; (c) $-135°$; (d) $180°$.

4.203. Onto what figure is a straight line passing through the centre of a rotation mapped?

4.204. Determine whether a line segment, a straight line, a ray, two intersecting straight lines, and a circle have a centre of symmetry.

4.205. Straight lines a and b intersect at the point C. Determine whether the straight lines centrally symmetric to them intersect. If they do, then at what point?

4.206. In symmetry transformation with respect to the point O, the points A and B go to the points A^1 and B^1, respectively, $AB = 5$. What is the length of A^1B^1?

4.207. In symmetry transformation with respect to the point O, the points A, B, and C, not lying on the same straight line, go to the points A^1, B^1, C^1, respectively. Prove that $\triangle ABC = \triangle A^1B^1C^1$.

4.208. D is the midpoint of the side AC of the triangle ABC. The point B^1 is symmetric to the point B with respect to the point D. Prove that the quadrilateral $ABCB^1$ is a parallelogram.

4.209. Find the least angle of rotation round the centre of symmetry at which a square, a rhombus, and a rectangle coincides with itself.

4.210. The points A and B, P and K are symmetric with respect to the point O, $AB = 4$ cm, $PK = 6$ cm. Find AO and PO.

4.211. Construct the line segment symmetric to a given one with respect to a given axis of symmetry.

4.212. Symmetry transformation with respect to the straight line l carries over the points A and B to the points A^1 and B^1, respectively, and $A^1B^1 = 3.5$. Find the length of AB.

4.213. Can a figure have infinitely many axes of symmetry?

4.214. Given straight lines a and b. Construct the straight line l with respect to which the lines a and b are symmetric.

4.215. Prove that if the triangle ABC has two axes of symmetry then it has a third one.

4.216. Prove that points of intersection of two circles are symmetric with respect to the straight line passing through the centres of the circles.

4.217. Given a point $B\ (a,\ b)$. Indicate coordinates of the point symmetric to the point B with respect to: (1) the origin; (2) the x-axis; (3) the y-axis.

4.218. Construct a triangle symmetric to a given one with respect to: (1) a straight line parallel to a side of the triangle; (2) a straight line passing through a vertex of the triangle and perpendicular to a side; (3) the straight line forming an angle of 45° with a side of the triangle.

§ 116. Translation

4.219. Indicate figures which are images of a line segment, a ray, a straight line, a circle, a triangle, and an angle in translation.

4.220. Given a straight line a. Specify a translation by a pair of corresponding points: $A \to A_1$. Construct the image of the straight line a in this translation.

4.221. Points A, B, and C are given on the straight line a. Construct the image of the point C in the translation mapping: (a) the point A onto the point B; (b) the point B onto the point C.

4.222. Given a triangle ABC. Indicate two successive translations to map it into itself.

4.223. Determine whether a translation exists such that: (a) one side of a triangle is mapped onto its other side; (b) one side of a square is mapped onto its other side.

4.224. Given two parallel straight lines. How many translations map one of them into the other?

4.225. Given parallel straight lines a and b and points A and B outside these lines (Fig. 13). Construct a common perpendicular to these lines so as to get a shortest polygonal line $AMNB$.

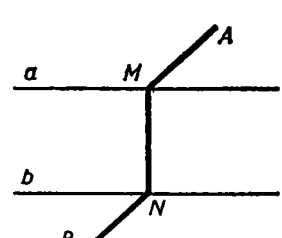

FIG. 13

Hint. The point N is the intersection of the straight line b and the straight line $A'B$, where A' is obtained from the point A by translating it on the line segment MN.

§ 117. Metric Relationships in a Right Triangle

4.226. The ratio of the legs is 5 : 6, and the hypotenuse is 122 cm long. Into what parts is the hypotenuse divided by the altitude?

4.227. The ratio of the lengths of legs is 3 : 2, and the altitude divides the hypotenuse into two line segments one of which is by 2 cm longer than the other. Determine the hypotenuse.

4.228. The ratio of the legs is 3 : 7, and the altitude drawn to the hypotenuse is equal to 42 cm. Determine the segments of the hypotenuse.

4.229. The base of an isosceles triangle is equal to 42 cm, and the median drawn to the lateral side is 5 cm. Find the lateral side.

4.230. A circle is inscribed in an isosceles triangle with the vertex angle of 120° and the lateral side of a. Find the radius of this circle.

4.231. The median of a right triangle drawn from the vertex of the right angle divides the right angle in the ratio 1 : 2. Find the sides of the triangle if this median is m in length.

4.232. The base and lateral side in an isosceles triangle are 5 and 20 cm, respectively. Find the bisector of the base angle.

4.233. The lateral side of an isosceles triangle is equal to 17 cm, and the base 16 cm. Determine the altitude.

4.234. Determine the sides of an isosceles triangle if its altitude is equal to 35 cm, and the ratio of the base to the lateral side is 48 : 25.

4.235. Two tangents are drawn to a circle of radius $R = 7$ cm from the same point at a distance of 25 cm from the centre. Determine the distance between the points of tangency.

§ 118. Areas of Plane Figures

4.236. The perimeter of a right triangle is 24 cm, its area 24 cm^2. Find the area of the circumscribed circle.

4.237. Find the ratio of areas of a square, a regular triangle, and a regular hexagon inscribed in the same circle.

4.238. Find the ratio of areas of squares circumscribed about and inscribed in the same circle.

4.239. Diagonals of two squares are equal to 3 and 4 m. Find the diagonal of the square whose area is equal to the sum of areas of the given squares.

4.240. (a) Determine the sides of a rectangle of 144 m^2 area if their ratio is 4 : 9.

(b) Determine the sides of a rectangle of 3 m^2 area if its perimeter is 74 dm.

4.241. The sides of a rectangle are 72 and 8 m long. Find the side of the equivalent square.

4.242. The diagonal of a rectangle is equal to 305 cm, its area 37.128 cm^2. Find its perimeter.

4.243. The area of a parallelogram amounts to 480 cm^2, its perimeter is 112 cm, the distance between the longer sides is equal to 12 cm. Determine the distance between the smaller sides.

4.244. The lateral side of an isosceles triangle is equal to 10 cm, the area 48 cm^2. Find the base.

4.245. (a) Determine the area of a rhombus with diagonals 72 and 40 cm long.

(b) Determine the altitude of a rhombus with diagonals 16 and 12 cm long.

4.246. Find the area of a parallelogram if one of its sides is equal to 51 cm, and the diagonals 40 and 74 cm.

4.247. The greater base of an isosceles trapezoid is 44 m, the lateral side 17 m, and the diagonal is 39 m. Determine the area of this trapezoid.

4.248. Determine the area of an isosceles trapezoid whose bases are 10 and 26 cm and the diagonals are perpendicular to the lateral sides.

4.249. Compute the area of an isosceles trapezoid whose bases are 42 cm and 54 cm and the angle at the greater base is equal to 45°.

4.250. Determine the area of a trapezoid whose parallel sides are 60 and 20 cm and the nonparallel sides 13 and 37 cm.

4.251. Compute the area of a triangle, given its three sides: (a) 13; 14; 15; (b) 29; 25; 6; (c) 5; $\sqrt{58}$; $\sqrt{65}$; (d) 5; $\sqrt{10}$; $\sqrt{13}$.

4.252. Find the smallest altitude of the triangle with 25, 29 and 36 dm sides.

4.253. Determine the greatest altitude of the triangle with 15, 112, and 113 cm sides.

4.254. The perimeter of a circumscribed polygon is 60 cm, the area 240 cm^2. Find the radius of the inscribed circle.

4.255. The side of a regular hexagon is 84 cm. Compute the side of the equivalent regular triangle.

4.256. Given the length radius R of the circle, determine the areas of inscribed regular octagon and dodecagon.

4.257. Find the area of a rhombus if its altitude is equal to 12 cm and the smaller diagonal to 13 cm.

4.258. The lateral side of an isosceles trapezoid is equal to $14\sqrt{2}$ cm, the angle at the greater base 45°, and the area of the trapezoid 560 cm^2. Find the greater base.

4.259. One square is inscribed in the other so that its vertices are on the sides of the latter and its sides form an angle α with the sides of the latter square. Find the ratio of areas of these squares.

4.260. A rhombus is inscribed in a right triangle with an angle α so that the acute angle of the rhombus coincides with α. Find the ratio of areas of the triangle and rhombus.

4.261. Find the longer base of an isosceles trapezoid with 45° acute angle, $8\sqrt{2}$ cm lateral side, and 480 cm^2 area.

§ 119. The Circumference and Arc Length

4.262. Compute the radius of the circle whose circumference is equal to: (a) 78.5 cm; (b) 12.12 dm.

4.263. Find the circumference of the circle: (a) inscribed in and (b) circumscribed about the regular triangle of 3 cm side.

4.264. Find the circumference of the circle: (a) inscribed in and (b) circumscribed about the square of 4 cm side.

4.265. The circumference of a circle is equal to C. Find the arc length of the sector if the central angle is equal to: (a) 30°; (b) 90°; (c) 150°.

4.266. The central angle of a sector is 36°. Find its arc length if $R_{sector} = 3$ cm.

4.267. Given two concentric circles with circumferences of 167 and 117 cm. Find the width of the annulus.

4.268. The circumference of a circle is by 7 cm longer than the perimeter of a regular inscribed hexagon. Find the circumference.

4.269. The length of a 120° segment of a circle is l. Find the circumference of the circle inscribed in this segment.

4.270. The radius of a circle is equal to 32.6 cm. Compute the arc of the circle of: 5°; 45°; 73°; and 185°.

4.271. A circumference of 68 cm diameter is unbent into an arc of 75 cm radius. Find the degree measure of the arc thus obtained.

§ 120. The Area of a Circle and Its Parts

4.272. Find the radius of a circle whose area is equal to: (a) 706.5 cm^2; (b) 2 cm^2; (c) 17 cm^2; (d) 50 cm^2; (e) 1 m^2; (f) 314 dm^2 if π is assumed to be about 3.14.

4.273. Find the area of the circle if its circumference is equal to: (a) 8 cm; (b) 144π cm; (c) 2π cm.

4.274. Given two concentric circles: the circumference of one of them is 157 m, of the other 94.2 m. Find the area of the annulus enclosed between them if π is assumed to be about 3.14.

4.275. The chord of the larger of two concentric circles is tangent to the smaller circle and has a length a. Compute the area of the annulus.

4.276. Find the area of a circle if the area of the inscribed square is equal to m.

4.277. The area of a circle is Q. Find the area of its sector with an arc of: (a) 30°; (b) 60°; (c) 120°; (d) 270°.

4.278. Compute the area of the segment of radius R if its arc contains: (a) 90°; (b) 60°; (c) 120°.

4.279. Find the areas of the hatched domains shown in Figs. 14, 15, 16, and 17.

4.280. Drawn in a circle of radius R are two parallel chords subtending arcs of 60° and 120°. Compute the area of the part of the circle enclosed between the chords.

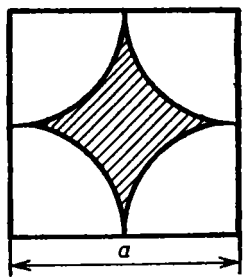

FIG. 14

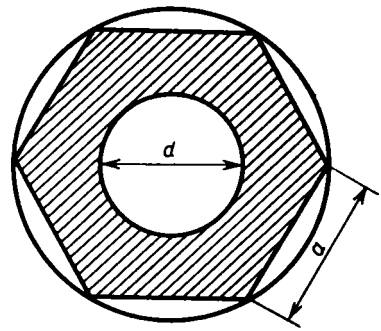

FIG. 17

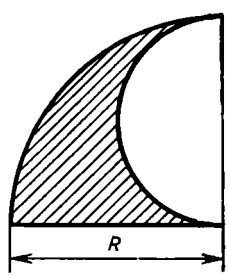

FIG. 15

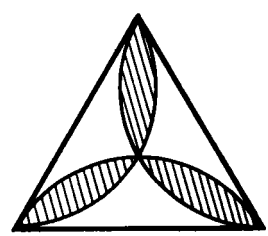

FIG. 18

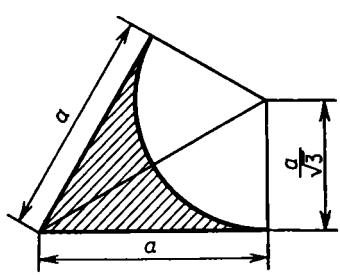

FIG. 16

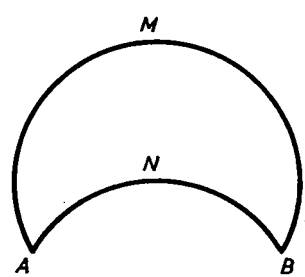

FIG. 19

4.281. Given a rhombus. On its sides as diameters semicircles are constructed bulging inwards. The diagonals of the rhombus are a and b. Determine the area of the figure thus obtained.

4.282. A rhombus with an acute angle of 30° is circumscribed about a circle whose area is Q. Find the area of the rhombus.

4.283. Find the area of the circle inscribed in a right triangle if the projections of its legs on the hypotenuse are 9 and 16 m.

4.284. The perimeter of a right triangle is $2p$ and the hypotenuse is C. Find the area of the circle inscribed in this triangle.

4.285. Each two adjacent vertices in an equilateral triangle are connected by arcs passing through the centre of the triangle (Fig. 18). The side of the triangle is a. Determine the area of the hatched figure.

4.286. Between the points A and B two circular arcs are drawn bulging in the same direction; the arc AMB contains 120°. The distance between the midpoints of the arcs is a. Compute the area of the figure enclosed between the arcs (see Fig. 19).

§ 121. Parallel Straight Lines and Planes

4.287. A line segment $AB = 10$ cm pierces a plane. The point A is 5 cm distant and the point B is 3 cm distant from the plane. Find the projection of the line segment AB on the plane.

4.288. A line segment AB pierces a plane. Its end points are 8 and 2 cm distant from the plane. Find the distance of the midpoint of this line segment from the plane.

4.289. Given (see Fig. 20): $AC = 30$ cm, $BD = 50$ cm. The point O divides the line segment AB in the ratio $3 : 7$. Find the distance from the point O to the plane.

4.290. Given: a trapezoid $ABCD$. Its base AD lies on the plane B, the base BC is 5 cm distant from the plane P, and $AD : BC = 7 : 3$. Find MO (Fig. 21).

4.291. Given: a parallelogram $ABCD$. Its side $AD = 10$ cm and lies on the plane P; $AB = 15$ cm, $B_1D = 10.5$ cm, $AC_1 = 13.5$ cm. Find the diagonals of the parallelogram (Fig. 22).

4.292. Given (see Fig. 23): a right triangle ABC with $\angle C = 90^\circ$, $AB \not\subset P$. The distance of the hypotenuse AB from the plane P is equal to 1 dm. The pro-

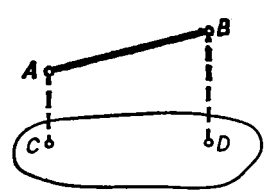

FIG. 20

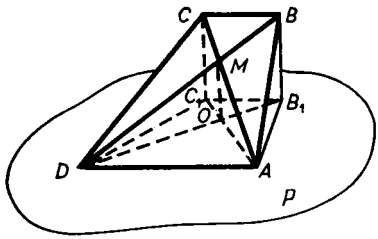

FIG. 21

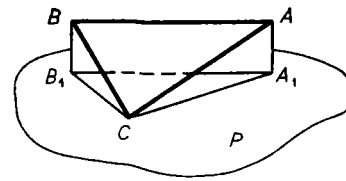

FIG. 23

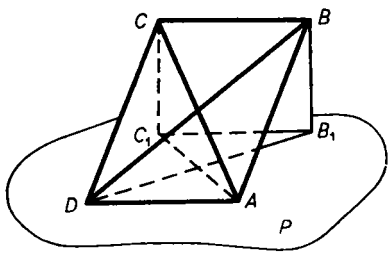

FIG. 22

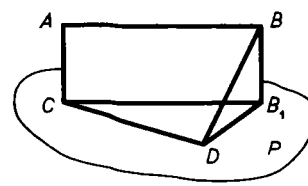

FIG. 24

jections of the legs on the plane P are 5 and 3 dm. Find the projection of the hypotenuse.

4.293. Given (Fig. 24): $AB \parallel$ pl. P, $AC \perp$ pl. P, $AB \perp BD$. Find CD if $AB = a$, $AC = b$, $BD = c$.

4.294. The line segments of two straight lines enclosed between two parallel planes are equal to 51 and 53 cm. The ratio of the line segment projections on one of the planes is 6 : 7. Find the distance between the planes.

4.295. The distance between two parallel planes is 8 dm. A line segment 10 dm long rests with its end points on these planes. Find the projections of the line segment on each of the planes.

§ 122. The Coordinate Method in Space

4.296. $ABCD$ is a parallelogram, E and F are midpoints of the opposite sides AD and BC; O is the intersection point of the diagonals. Taking $\overrightarrow{AB} = e_1$ and $\overrightarrow{AD} = e_2$ as basis

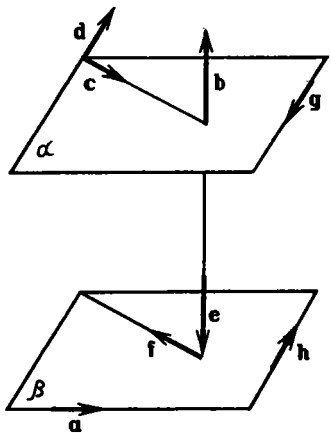

FIG. 25

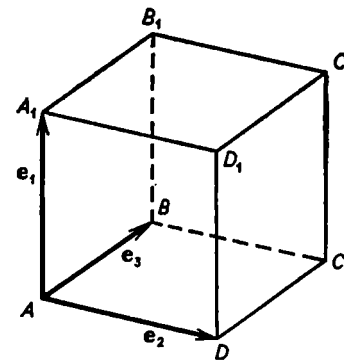

FIG. 26

vectors, determine the coordinates of the following vectors $\overrightarrow{AC}$, $\overrightarrow{OD}$, $\overrightarrow{FC}$, $\overrightarrow{BC}$, $\overrightarrow{EO}$, $\overrightarrow{BD}$, $\overrightarrow{EA}$.

4.297. The plane α is parallel to the plane β. Indicate the collinear and coplanar vectors (Fig. 25).

4.298. In the parallelepiped $ABCDA_1B_1C_1D_1$ vectors are given which coincide with its edges: $\overrightarrow{AA_1} = \mathbf{l}_1$, $\overrightarrow{AD} = \mathbf{l}_2$, and $\overrightarrow{AB} = \mathbf{l}_3$. Construct the following vectors: (a) $\mathbf{l}_1 + \mathbf{l}_2 + \mathbf{l}_3$; (b) $\mathbf{l}_1 + \mathbf{l}_2 + \frac{1}{2}\mathbf{l}_3$; (c) $\frac{1}{2}\mathbf{l}_1 + \mathbf{l}_2 - \mathbf{l}_3$; (d) $-\mathbf{l}_1 - \mathbf{l}_2 + \frac{1}{2}\mathbf{l}_3$ (Fig. 26).

4.299. Compute the coordinates of the point belonging to the y-axis and equidistant from the points A (2, —1, 1) and B (0, 1, 3).

4.300. Find the coordinates of the vector a if: (1) $\mathbf{a} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$; (2) $\mathbf{a} = 8\mathbf{i} + \mathbf{k}$; (3) $\mathbf{a} = 0.5\mathbf{j} - \sqrt{3}\mathbf{k}$; (4) $\mathbf{a} = 7\mathbf{j}$; (5) $\mathbf{a} = \mathbf{i}$; (6) $\mathbf{a} = 0$, (7) $\mathbf{a} = 8\mathbf{k} + \mathbf{i}$.

4.301. Given: $\mathbf{a}\{-2, 3, 0\}$; $\mathbf{b}\{1, -1, 5\}$; $\mathbf{c}\{7, 0, 4\}$. Find: (1) $\mathbf{a} + \mathbf{b}$; (2) $\mathbf{a} - \mathbf{c}$; (3) $\mathbf{a} - \mathbf{b} + \mathbf{c}$; (4) $2\mathbf{a} - \mathbf{b} + 3\mathbf{c}$.

4.302. Given three successive vertices of a parallelogram: A (—3, 2, 0), B (3, —3, 1), C (5, 0, 2). Find its fourth vertex.

4.303. Compute the lengths of the following vectors: (1) $\mathbf{a} = \mathbf{i} - \mathbf{j} + \mathbf{k}$; (2) $\mathbf{b} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$; (3) $\mathbf{c} = \mathbf{j} - \mathbf{k}$; (4) $\mathbf{d} = -2\mathbf{j}$.

4.304. The length of a vector is equal to 3. Compute the coordinates of this vector if they are equal.

4.305. Compute the length of the vector $\mathbf{a} + 8\mathbf{b}$ if: (1) $\mathbf{a}\{1, 2, -1\}$, $\mathbf{b}\{3, 0, -2\}$; (2) $\mathbf{a}\{1, -1, 3\}$, $\mathbf{b} = \{-1, 1, -3\}$.

4.306. Compute the length of the vector $2\mathbf{a} + 3\mathbf{b}$ if: (1) $\mathbf{a}\{1, 1, -1\}$, $\mathbf{b}\{2, 0, 0\}$; (2) $\mathbf{a}\{3, 1, 0\}$, $\mathbf{b}\{0, -1, 1\}$.

4.307. Find the coordinates of the vector $\overrightarrow{AB}$ if: (1) $A(2, -3, 0)$, $B(-1, 2, -3)$; (2) $A(1/2, -4/3, 5/6)$, $B(-3/5, 0, 3/5)$.

4.308. Compute the diagonals of the parallelogram $ABCD$, given the coordinates of three its vertices: $A(1, -3, 0)$, $B(-2, 4, 1)$, $C(-3, 1, 1)$.

4.309. Prove that the points: $A(2, 4, -1)$, $B(1, 1, -3)$, $C(-2, 0, 5)$, and $D(-1, 3, 4)$ are vertices of a parallelogram and compute the cosine of the angle between its diagonals.

4.310. The vector $\mathbf{a}\{x, y, z\}$ forms angles α, β, γ with the basis vectors.

(1) Prove that the unit vector in the direction of the vector $\mathbf{a}$ has the coordinates $(\cos\alpha, \cos\beta, \cos\gamma)$;

(2) prove the equality: $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$;

(3) compute γ if $\alpha = \beta = 60°$.

§ 123. Scalar Product of Vectors in Space

4.311. Given: vectors $\mathbf{a}\{4, -2, -4\}$, $\mathbf{b}\{2, 4, 3\}$, $\mathbf{c}\{0, 1, -1\}$. Compute: $\mathbf{ab}$, $\mathbf{ac}$, $\sqrt{\mathbf{a}^2}$, $(\mathbf{a} - \mathbf{b})(\mathbf{a} + \mathbf{c})$; $(\mathbf{a} - \mathbf{b})^2$.

4.312. Given: vectors $\mathbf{a}\{2, -1, 0\}$, $\mathbf{b}\{1, \sqrt{2}, -5\}$, $\mathbf{c}\{1, 2, 5\}$, $\mathbf{d}\{1, 0, 2\}$. Find the products: $\mathbf{ab}$, $\mathbf{ac}$, $\mathbf{ad}$, $\mathbf{bc}$, $\mathbf{bd}$, $\mathbf{cd}$ and find out what angles are formed by these vectors.

4.313. Find the angle between the vectors: (1) $\mathbf{a}_1\{2, -1, 3\}$ and $\mathbf{b}_1\{1, -4, 3\}$; (2) $\mathbf{a}_2\{2, -2, 1\}$ and $\mathbf{b}_2\{3, 0, -4\}$; (3) $\mathbf{a}_3\{0, -1, 5\}$ and $\mathbf{b}_3\{7, 5, 1\}$; (4) $\mathbf{a}\{2, -3, 1\}$ and $\mathbf{b}\{6, 2, -4\}$; (5) $\mathbf{a}\{1, 1, 0\}$ and $\mathbf{b}\{1, 0, 1\}$; (6) $\mathbf{a} - \mathbf{b}$ and $\mathbf{a} + \mathbf{b}$ if $\mathbf{a}\{1, 2, 1\}$ and $\mathbf{b}\{2, -1, 0\}$; (7) $\mathbf{a}\{-1, 2, -2\}$ and $\mathbf{b}\{6, 3, -6\}$.

4.314. Find the angles of the triangle ABC if $\overrightarrow{AB} = 4\mathbf{i} + 2\mathbf{j}$, $\overrightarrow{AC} = 2\mathbf{i} + 6\mathbf{j}$.

4.315. Show that the quadrilateral with vertices A (4, 0, 8), B (5, 2, 6), C (3, 1, 4), and D (2, —1, 6) is a square.

4.316. Given a triangle ABC: A (1, 1, 1), B (5, 3, 0), and C (2, 0, 1). Find the coordinates and length of the vector $\overrightarrow{BH}$, where H is the foot of the perpendicular dropped from the vertex B on the opposite side.

4.317. Prove that the triangle ABC is a right one with the right angle at the vertex C if the coordinates of its vertices are: A (2, 4, 5); B (—3, 2, 2); C (—1, 0, 3).

4.318. A triangle is specified by the coordinates of its vertices: A (3, —2, 1); B (3, 1, 5); C (4, 0, 3). Compute:
(1) the medians drawn from the vertices A and B;
(2) the distance from the origin to the point of intersection of the medians;
(3) the angles of the triangle ABC.

4.319. For what value of t is $\mathbf{a} \perp \mathbf{b}$ if: $\mathbf{a}\ \{2, t, 4\}$, $\mathbf{b}\ \{-3, 2, 3\}$?

4.320. Prove that the vectors $\mathbf{a}\ \{-3, 3, 0\}$ and $\mathbf{b}\ \{3, 3, 3\}$ are mutually perpendicular.

4.321. For what value of t are the vectors $\mathbf{a}$ and $\mathbf{b}$ mutually perpendicular if $\mathbf{a}\ \{3, t, -5\}$ and $\mathbf{b}\ \{3, -5, 0\}$?

4.322. Evaluate the given expressions:

(1) $3\mathbf{a}\,(\mathbf{a} + \mathbf{b}) - (\mathbf{a} + \mathbf{b})^2$ for $|\mathbf{a}| = 3$, $|\mathbf{b}| = 6$, $(\widehat{\mathbf{ab}}) = 60°$;

(2) $(\mathbf{a} + 2\mathbf{b})\,\mathbf{b} - (\mathbf{a} - \mathbf{b})$ for $|\mathbf{a}| = 2$, $|\mathbf{b}| = \sqrt{2}$, $(\widehat{\mathbf{ab}}) = 45°$.

4.323. The vectors $\mathbf{a}$, $\mathbf{b}\ \{1, 1, 0\}$, and $\mathbf{c}\ \{0, 1, 1\}$ are coplanar. Find the coordinates of $\mathbf{a}$ if its length is equal to $\sqrt{8}$ and it is perpendicular to the vector $\overline{\mathbf{b}} + \overline{\mathbf{c}}$, and $\mathbf{a} \cdot \mathbf{b} > 0$.

4.324. The vectors $\mathbf{a}\ \{3, 1\}$ and $\mathbf{b}$ are of equal length. Find the coordinates of $\mathbf{b}$ if the vector $(\mathbf{b} - 2\mathbf{a})$ has a length equal to $\sqrt{26}$ and makes an acute angle with the y-axis.

4.325. It is known that $\mathbf{a} \cdot \mathbf{c} = 2$ and $(\widehat{\mathbf{ac}}) = 60°$. Find the length of $\mathbf{a}$ if the length of $\mathbf{c}$ is four times the length of $\mathbf{a}$.

4.326. Find the coordinates of the vector $\mathbf{k}$ which is perpendicular to the vector $\mathbf{a}\ \{1, 2\}$ and has a positive abscissa and length equal to that of the vector $\mathbf{b}\ \{-1, -2\}$.

4.327. Find the coordinates of the vector $\mathbf{b}$ collinear

with the vector $\mathbf{a}\{1, -2\}$ if $\mathbf{b} = 5\sqrt{5}$ and $\mathbf{a} \cdot \mathbf{b} > 0$.

4.328. Find the angle between the planes ABC and ABD for A (0, 0, 0), B (1, 1, 1), C (3, 2, 1), and D (3, 1, 2).

4.329. The points M and N are centres of two adjacent faces of a cube. The point A is a vertex of the cube belonging to these faces. Find the angle MAN.

4.330. The angle between two adjacent lateral faces of a regular quadrangular pyramid $SABCD$ with vertex S is equal to α, and the lateral edges are 1 dm each. Find the scalar product $\overrightarrow{AB} \cdot \overrightarrow{AC}$ and the length of the vector $\overrightarrow{AK}$, where K is the midpoint of the edge BC.

4.331. Prove that the sum of the squares of the distances from any point of the circle circumscribed about a regular triangle to its vertices is a constant.

§ 124. The Equation of a Plane

4.332. Set up the equation of the plane passing through the following three points:

(a) A (1, 2, 3), B (−1, 0, 0), C (3, 0, 1);

(b) D (0, 0, 0), E (1, 1, 1), F (2, 2, 2);

(c) K (1, −1, 2), M (2, 1, 2), N (1, 1, 4).

4.333. How are the planes: (a) $3x - 5z + 1 = 0$; (b) $2y - 2 = 0$; (c) $x + y - 5 = 0$; (d) $2x + 3y - 7z = 0$; (e) $8y - 3z = 0$ arranged with respect to the coordinate axes?

4.334. Set up the equation of the plane passing through the point $M(x_1, y_1, z_1)$ and perpendicular to the nonzero vector $\mathbf{n}\{a, b, c\}$.

4.335. Given: points A (3, 2, 5), B (−1, 2, 2), C (7, 0, −9), and D (3/4, 5/6, 6). Indicate which of them belong to the plane $2x - 3y + z - 5 = 0$.

4.336. Write the equation of the plane which passes through the point A (3, 4, 5) and is perpendicular to the vector $\mathbf{n}\{3, -2, 4\}$.

4.337. Set up the equation of a plane given that the point N (3, 5, 2) serves as a foot of the perpendicular drawn from the origin to this plane.

4.338. Write the equation of a plane if it is known that it passes through the origin and is perpendicular to the vector $\mathbf{n}\{-6, 3, 6\}$.

4.339. Write the equation of a plane which passes through the point A (2, 3, -1) and is perpendicular to the straight line passing through the points D (-1, 0, 0) and E (0, 1, -3).

4.340. Compute the distance from the origin to the plane:

(1) $2x - 2y + z - 6 = 0$;

(2) $2x + 3y - 6z + 14 = 0$;

(3) which passes through the points C ($-\sqrt{3}$, 0, 0) and D (0, 0, -1) and is perpendicular to the xy-plane.

4.341. Determine whether two given planes are parallel:

(1) $x + 2y + 3z + 5 = 0$ and $2x + 4y + 6z + 11 = 0$;

(2) $x - 7y + 5z - 1 = 0$ and $2x + y - 3z + 4 = 0$.

4.342. Set up the equation of a plane passing through the point M (1, 3, -1) and parallel to the plane:

(1) $3x + y - z + 5 = 0$;

(2) $x - y + 5z - 4 = 0$.

4.343. Find out whether two given planes are perpendicular:

(1) $2x - 5y + z + 4 = 0$ and $3x + 2y + 4z - 1 = 0$;

(2) $7x - y + 9 = 0$ and $y + 2z - 3 = 0$.

4.344. Set up the equation of a plane passing through the point N (3, 5, 2) and perpendicular to the vector $\mathbf{n} = \{3, 5, 2\}$.

§ 125. The Equation of a Sphere

4.345. Write the equation of the spherical surface having the centre at the point:

(a) O (2, -3, 3) and radius $R = 6$;

(b) O (0, 0, 0) and passing through the point B (6, -2, 3);

(c) A (1, 4, -7) and touching the plane $6x + 6y - 7z + 42 = 0$;

(d) B (6, -8, 3) and touching the z-axis.

4.346. Find the coordinates of the centre and the radius for each of the given spheres:

(a) $x^2 + y^2 + z^2 - 6x + 8y + 2z + 10 = 0$;
(b) $x^2 + y^2 + z^2 + 2x - 4y - 4 = 0$;
(c) $x^2 + y^2 + z^2 - 6x + 10 = 0$;
(d) $x^2 + y^2 + z^2 = 2az$;
(e) $x^2 + y^2 + z^2 - 6x - 16 = 0$.

§ 126. Perpendiculars and Straight Lines Inclined to a Plane

4.347. The distance from the point A to the plane P is equal to 6 cm. The inclined line AB is 10 cm long. Find the projection of AB on the plane P.

4.348. ABC is a right triangle. A perpendicular CD is drawn to the plane of the triangle. The point D is connected to the points A and B. Find the area of the triangle ADB if $CA = 3$ dm, $BC = 2$ dm, and $CD = 1$ dm.

4.349. A perpendicular AK is drawn to the plane of the rectangle $ABCD$ through the point A. The distances from the point K to the points B, C, and D are equal to 6 cm, 7 cm, and 9 cm, respectively. Find the length of the perpendicular AK.

4.350. A and B are points on the plane M; AC and BD are perpendiculars to this plane: $AC = a$ and $BD = b$. Prove that the lines AD and BC intersect and find the distance from the point of their intersection to the plane M.

4.351. An inclined line from the vertex of an angle lying in a plane is drawn to the plane so that it makes equal angles with the sides of the angle. Prove that the projection of this inclined line will serve as the bisector of the given angle.

4.352. From the vertex A of the triangle ABC a straight line AD not lying in the triangle plane is drawn which makes equal acute angles with the sides AB and AC. Into what parts does the projection of the line AD on the plane of the given triangle divide the side BC if $AB = 51$ m, $AC = 34$ m, and $BC = 30$ m?

§ 127. Angles Made by Straight Lines with Planes

4.353. A rectangular parallelepiped of 3 cm $\times$ 4 cm base has the altitude equal to 5 cm. Find its diagonal and the angle made by the diagonal with the plane of the base.

4.354. The diagonal of a rectangular parallelepiped forms an angle of 45° with the plane containing its base. The sides of the base are 120 and 209 cm. Find the altitude of the parallelepiped.

4.355. The length of an inclined line is a; it makes an angle α with the plane of projection. Find the projection of the inclined line if the angle α is equal to: (a) 45°; (b) 60°; (c) 30°.

4.356. Two equal inclined lines are drawn from the point A to the plane P. The angle between the lines is 60°, while the angle between their projections is 90°. Find the angles between each inclined line and its projection.

4.357. A point M is found at a distance of a cm from a plane. Two equal inclined lines MA and MB are drawn from the point M at angles of 45° to the plane, the angle between the lines being equal to 60°. Find AB.

4.358. A line segment whose length is 10 cm pierces a plane; its end points are located at distances of 3 and 2 cm from the plane. Find the angle between the given line segment and the plane.

4.359. From a point located at a distance a from a plane, two inclined lines are drawn at angles of 45° to the plane, the angle between the lines being equal to 60°. Find the distance between the end points of the inclined lines.

4.360. From a point located at a distance a from a plane, two inclined lines are drawn at angles of 45° and 30° to the plane, the angle between the lines being equal to 90°. Find the distance between the end points of the inclined lines.

§ 128. Dihedral Angles and Perpendicular Planes

4.361. Given a dihedral angle of 45°. A point is given on one of the faces at a distance a from the other. Find the distance of this point from the edge of the dihedral angle.

4.362. Two isosceles triangles have a common base 16 cm long. The planes of the triangles form an angle of 60°. The lateral side of one of the triangles is equal to 17 cm; the lateral sides of the other triangle are mutually perpendicular. Find the distance between the vertices of the triangles.

4.363. The legs of a right triangle are 7 and 24 cm.

A plane is drawn through the hypotenuse of the triangle to form an angle of 30° with the plane of the triangle. Find the distance from the vertex of the right angle to this plane.

4.364. Inside a right dihedral angle, a point is taken at distances of 12 and 16 cm from its faces. Find the distance of this point from its edge.

4.365. Through the hypotenuse AB of a right triangle ABC a plane α is drawn to form a dihedral angle of 60° with the plane of the triangle. Find the distance from the vertex C of the triangle to the plane α if the legs of the given triangle are 6 and 8 dm.

4.366. From a point taken on the edge of a right dihedral angle two straight lines lying on the faces are drawn which are inclined to the edge at an angle of 45° each. Find the angle between the straight lines.

§ 129. Polyhedral Angles

4.367. Each plane angle in a trihedral angle is equal to 60°; on one of the edges a line segment 3 cm in length is laid off from the vertex and from its end point a perpendicular is dropped on the opposite face. Find the length of this perpendicular.

4.368. In a trihedral angle $SABC$: $\widehat{BSC} = 90°$; $\widehat{ASB} = \widehat{ASC} = 60°$, and $SA = a$. Find the distance from the point A to the plane BSC. Prove that the edge SA makes an angle of 45° with the plane BSC.

4.369. In a trihedral angle, one plane angle is a right one, two other containing 60° each. Prove that the plane cutting off three equal line segments from the edges is perpendicular to the plane of the base.

4.370. In a trihedral angle, two plane angles contain 60° each, the third plane angle being a right one. Find the angle between the plane of the right angle and the opposite edge.

4.371. In a trihedral angle, two dihedral angles are of 45° each. The plane angle between them is equal to 90°. Prove that the third plane angle is of 120°.

4.372. In a trihedral angle, all the plane angles are right ones. Prove that its dihedral angles are also right angles.

4.373. How many faces does a convex polyhedral angle have if each of its plane angles contains: (1) 90°; (2) 60°; (3) 45°; (4) 30°.

4.374. If the plane angle at the vertex of a regular quadrangular pyramid is equal to 60°, then the opposite lateral edges are mutually perpendicular. Prove this.

§ 130. The Prism. The Area of Its Surface

4.375. The lateral edge of a right parallelepiped is equal to a; the sides of the base are b and c; one of the diagonals of the base is d. Find the diagonals of the parallelepiped if: (a) $a = 5$ m, $b = 6$ m, $c = 8$ m, $d = 12$ m; (b) $a = 9$ cm, $b = 7$ cm, $c = 11$ cm, $d = 14$ cm.

4.376. Find the diagonals of a right parallelepiped in which each of the edges is of length a, the acute angle of the base being equal to 60°.

4.377. In a right parallelepiped the sides of the base 3 and 4 cm long form an angle of 60°, and the length of the lateral edge is a mean proportional quantity between the sides of the base. Find the diagonals of this parallelepiped.

4.378. The edge of a cube is of length a. Find the distance from a vertex of the cube to its diagonal.

4.379. The surface area of a cube amounts to 24 m². Find its edge.

4.380. The base of a right parallelepiped is a rhombus. The diagonals of the rhombus are 6 and 9 cm, the diagonal of the lateral face 13 cm. Find the area of the total surface of the parallelepiped.

4.381. The diagonal of a regular quadrangular prism is equal to 9 cm, and the area of the total surface amounts to 144 cm². Determine the side of the base and the lateral edge.

§ 131. The Pyramid. The Area of Its Surface

4.382. The perimeter of the base of a regular triangular pyramid is equal to $30\sqrt{3}$ cm, the lateral edge being 26 cm long. Find the altitude of the pyramid.

4.383. Find the side of the base of a regular quadrangular pyramid if its altitude is 6 dm, the slant height being equal to 6.5 dm.

4.384. The base of a pyramid is a parallelogram whose sides are 3 and 7 cm long, one of the diagonals being equal

to 6 cm. The altitude of the pyramid passes through the point of intersection of the diagonals and has a length of 4 cm. Find the lateral edges of the pyramid.

4.385. The base of a pyramid is an isosceles triangle in which the base is 12 cm and the lateral side 10 cm. The lateral faces form congruent dihedral angles of 45° with the base. Find the altitude of the pyramid.

4.386. The base of a pyramid is a right triangle with legs 6 and 8 cm. Each lateral edge of the pyramid has a length equal to 13 cm. Find the altitude of the pyramid.

4.387. Find the lateral edge of a regular hexagonal pyramid in which the side of the base is equal to a, and the lateral edges are inclined to the plane of the base at an angle α.

4.388. The side of the base of a regular quadrangular pyramid is 14 cm, the lateral edge being equal to 10 cm. Find the area of the diagonal section.

4.389. The side of the base of a regular n-gonal pyramid is a, and the altitude is H. Find the total surface area of the pyramid if: (1) $n = 3$; (2) $n = 4$; (3) $n = 6$.

4.390. The base of a pyramid is a right triangle with legs $3a$ and $4a$ long. Each lateral face is inclined to the plane of the base at an angle of 60°. Find the total surface area of the pyramid.

4.391. The sides of the base of a triangular pyramid are 6, 10, and 14 cm. The planes of the lateral faces are inclined to the base at an angle of 60°. Find the total surface area of the pyramid.

4.392. The area of the lateral surface of a regular quadrangular pyramid amounts to 14.76 m^2, the total surface area being equal to 18 m^2. Find the side of the base and the altitude of the pyramid.

4.393. The side of a square which serves as the base of a pyramid is a. Two adjacent lateral faces are perpendicular to the base, the other two being inclined to the base at an angle of 60°. Find the total surface area of the pyramid.

4.394. The area of the lateral surface of a regular triangular pyramid is equal to $96\sqrt{3}$ cm^2, the total surface area amounting to $112\sqrt{3}$ cm^2. Find the side of the base and the altitude of the pyramid.

4.395. The slanted height of a hexagonal pyramid is m. The dihedral angle at the base is equal to 60°. Find the total surface area of the pyramid.

4.396. The base of a pyramid is an equilateral trapezoid whose diagonal is of length l and forms an angle α with the greater base. All the lateral faces of the pyramid are inclined to the plane of the base at an angle β. Find the total surface area of the pyramid.

4.397. Find the side of the base of a regular triangular pyramid whose total surface area is Q, the angle between the lateral face and the base being equal to φ.

4.398. The lateral edge of a regular triangular pyramid of length a forms an angle α with the plane of the base. Find the surface area of the pyramid.

4.399. Find the area of the lateral surface of a regular triangular pyramid if the plane angle at its vertex is φ, and the area of the base is equal to Q.

4.400. Find the altitude of a regular quadrangular pyramid whose total surface area is S, the plane angle at the vertex being equal to α.

4.401. The base of a pyramid is an equilateral triangle whose side is a. One of the lateral sides is also an equilateral triangle which is perpendicular to the plane of the base. Find the total surface area of the pyramid.

4.402. The base of a pyramid is a rhombus with side a and acute angle α. All the lateral sides are inclined to the plane of the base at an angle β. Find the total surface area of the pyramid.

4.403. The base of a pyramid is a rhombus whose diagonals are 12 and 16 cm. The altitude of the pyramid passes through the intersection point of the diagonals of the rhombus and has a length of 6.4 cm. Find the total surface area of the pyramid.

§ 132. The Frustum of a Pyramid. Its Surface Area

4.404. The altitude of a frustum of a regular quadrangular pyramid is equal to 7 cm. The sides of the bases are 10 and 2 cm. Find the lateral edge of the frustum.

4.405. The sides of the bases of a frustum of a regular triangular pyramid are 4 and 1 cm, the lateral edge 2 cm. Find the altitude of the frustum.

4.406. Determine the altitude of a frustum of a regular n-gonal pyramid if the lateral edge is equal to c, the side

of the lower base is a, and the side of the upper base is b ($n = 3$; $n = 4$; $n = 6$).

4.407. The altitude of a frustum of a regular quadrangular pyramid is equal to 63 cm, the slant height 65 cm, the ratio of the sides of the bases is 7 : 3. Find the areas of the bases.

4.408. The altitude of a frustum of a regular quadrangular pyramid is 2 cm; the sides of the bases are equal to 3 and 5 cm. Find the diagonal of the frustum.

4.409. Find the areas of the bases of a frustum of a regular quadrangular pyramid if its altitude is 7 cm, the lateral edge 9 cm, the diagonal 11 cm.

4.410. In a frustum of a regular triangular pyramid, the sides of the bases are 2 and 6 cm. The lateral face forms an angle of 60° with the greater base. Find the altitude of the frustum.

4.411. Find the total surface area of a frustum of a regular n-gonal pyramid for $n = 3$, $n = 4$, and $n = 6$ if h is the altitude, a the side of the upper base, and b the side of the lower base ($b > a$).

4.412. In a frustum of a regular pyramid, the sides of the bases are 8 and 2 m and the altitude of the frustum 4 m. Find the total surface area of the frustum.

4.413. In a frustum of a regular quadrangular pyramid, the altitude is equal to 12 cm, the difference between the sides of the bases is 10 cm, and the total surface area amounts to 512 cm^2. Find the sides of the bases.

§ 133. The Cylinder. Its Surface Area

4.414. A plane is drawn in a cylinder parallel to its axis. This plane cuts off an arc of 120° from the circle of the base. The axis of the cylinder is equal to 10 cm. The distance from the axis to the cutting plane is 2 cm. Find the area of the section.

4.415. The ratio of areas of a cylinder base and its axial section is $\pi : 4$. Find the angle between the diagonals of the axial section.

4.416. The area of the axial section of a cylinder is equal to 8 m^2. Find the area of the section parallel to the axis and 1 m distant from it.

4.417. The altitude of a cylinder is by 10 cm longer than the radius of the base, the total surface area amounting to

144π cm^2. Find the radius of the base and the altitude of the cylinder.

4.418. The area of the axial section of a cylinder is equal to Q. Find the area of the lateral surface of the cylinder.

4.419. The radius of the base of a cylinder is 2 cm, the altitude being equal to 7 cm. Find the radius of the circle whose area is equal to the total surface area of the cylinder.

4.420. The area of the base in a cylinder is Q and the area of the axial section is M. Find the total surface area of the cylinder.

4.421. Find the total surface area of the cylinder circumscribed about a cube. The edge of the cube is a, the vertices of the cube lying on the circles in the bases of the cylinder.

4.422. Inscribed in a cylinder is a right prism whose base is a right triangle with an acute angle α. The altitude of the base drawn to the hypotenuse is H and is equal to the altitude of the prism. Find the total surface area of the cylinder.

4.423. Find the area of the total surface of a cylinder whose axial section is a square, and the side of the square equals H.

§ 134. The Cone. Its Surface Area

4.424. The axial section of a cone is an equilateral triangle. The radius of the base equals R. Find the area of the section drawn through two generatrices, the angle between which is equal to 30°.

4.425. The altitude of a cone is equal to the radius R of the base and a plane is drawn through the vertex which cuts off an arc of 90° from the circle of the base. Find the area of the obtained section.

4.426. The area of the base of a cone is equal to 25π dm^2. The generatrix of the cone is by 1 dm longer than the altitude. Find the total surface area of the cone.

4.427. The area of the lateral surface of a cone is equal to S and its total surface area amounts to P. Find the angle between the altitude and generatrix.

4.428. The greatest angle between two generatrices of a cone is equal to 60°. Find the ratio of the area of the lateral surface to the area of the cone base.

4.429. A regular quadrangular pyramid is inscribed in

an equilateral cone. Find the ratio of the lateral surfaces of the cone and pyramid.

4.430. The altitude of a cone is equal to H, the lateral surface area is π m^2. Find the radius of the base.

§ 135. The Frustum of a Cone. Its Surface Area

4.431. The radii of the bases in a frustum of a circular cone equal 3 and 7 dm and the generatrix 5 dm. Find the area of the axial section.

4.432. The radii of the bases in a frustum of a cone are equal to 11 and 16 cm and the generatrix is 13 cm. Find the distance from the centre of the smaller base to the circumference of the larger base.

4.433. The generatrix of a frustum of a cone is $2a$ and the generatrix is inclined to the plane of the base at an angle of 60°. The radius of one base is twice the radius of the other. Find the radii of the bases.

4.434. The generatrix of a frustum of a cone is inclined to the plane of the greater base at an angle α; the radii of the bases are equal to r and R. Find the area of the lateral surface of the frustum.

4.435. In a frustum of a cone, the diagonals of the axial section are mutually perpendicular, and the generatrix, whose length is l, forms an angle α with the plane of the lower base. Determine the areas of the lateral and total surface of the frustum.

4.436. The altitude of a frustum of a cone is h, the generatrix makes an angle α with the plane of the greater base and is perpendicular to the diagonal of the axial section. Find the lateral surface area of the frustum.

4.437. The generatrix of a frustum of a cone is l and is inclined to the lower base at an angle α; the diagonal of the axial section makes an angle β with the base. Find the area of the lateral surface of the frustum.

4.438. The radii of the bases in a frustum of a cone are 20 and 10 cm. The generatrix is inclined to the plane of the base at an angle of 45°. Find the altitude and generatrix of the frustum.

4.439. The length of the generatrix in a frustum of a cone is 30 cm, the radius of the lower base 34 cm, and the altitude 24 cm. Find the radius of the upper base.

4.440. The radii of the bases in a frustum of a cone are R and r, its area of the lateral surface is equal to the sum of the areas of the bases. Find the altitude of the frustum.

4.441. The area of the lateral surface of a frustum of a cone is equal to S, the radii of the bases are R and r. Determine the lateral surface area of the entire cone.

§ 136. Volumes of Prisms and Cylinders

4.442. Find the volume of a cube: (a) given its diagonal; (b) given its surface area.

4.443. (a) If each edge of a cube is increased by 2 cm, then its volume will increase by 98 cm^3. Find the edge of the cube; (b) if each edge of a cube is increased by 1 m, then its volume will increase eight-fold. Find the edge of the cube.

4.444. The dimensions of a rectangular parallelepiped are 15, 50, and 36 m. Find the edge of the equivalent cube.

4.445. The diagonal of a rectangular parallelepiped is equal to a and forms an angle of 30° with one face and angle of 45° with the other. Find the volume of the parallelepiped.

4.446. The sides a and b of the base in a right parallelepiped make an angle of 30°. The area of the lateral surface of the parallelepiped is S. Find its volume.

4.447. The sides 8 and 15 cm long of the base in a right parallelepiped form an angle of 60°. The smaller diagonal of the parallelepiped makes an angle of 30° with the plane of the base. Find the volume of the parallelepiped.

4.448. The base of a right parallelepiped is a rhombus whose area is equal to Q; the areas of the diagonal sections are M and N. Find the volume of the parallelepiped. Compute it for $Q = 1$ m^2, $M = 3$ m^2, and $N = 6$ m^2.

4.449. The diagonal of a regular quadrangular prism is 3.5 m and that of the lateral face 2.5 m. Find the volume of the prism.

4.450. The base of a prism is a regular triangle inscribed in a circle of radius R; its lateral faces are squares. Find the volume of the prism.

4.451. In a regular hexagonal prism, the area of the greatest diagonal section is equal to 4 m^2, and the distance between the two opposite lateral faces 2 m. Find the volume of the prism.

4.452. The area of the base of a right triangular prism equals 4 cm^2, the areas of the lateral faces being equal to 9, 10, and 17 cm^2. Find the volume of the prism.

4.453. Find the volume of a regular n-gonal prism in which the greater diagonal is l and makes an angle α with the plane of the base.

4.454. The base of a right prism is a right triangle whose area is S and the acute angle α. The area of the greater lateral face is Q. Find the volume of the prism.

4.455. The greater diagonal of a regular hexagonal prism is d and makes an angle of 30° with the lateral edge of the prism. Find the volume of the prism.

4.456. The area of the lateral surface of a cylinder (in sq. units) and its volume (in cu. units) are expressed by the same number. Find the diameter of the cylinder.

4.457. Find the ratio of volumes of two cylinders if the diameter and altitude of one cylinder are equal respectively to 0.2 and 0.6 m, and those of the other to 0.3 and 0.3 m.

4.458. The lateral surface of a cylinder can be unrolled to form a square with side a. Find the volume of the cylinder.

4.459. The altitude of a cylinder is equal to H. Its lateral surface can be unrolled to form a figure whose diagonal makes an angle of 60° with the generatrix. Find the volume of the cylinder.

4.460. The unrolled lateral surface of a cylinder has a diagonal which is equal to d and makes an angle α with the base. Find the volume of the cylinder.

4.461. Find the volume of a cylinder inscribed in a regular hexagonal prism whose edge is equal to a.

4.462. A chord a is drawn in the base of a cylinder. The corresponding central angle equals α. The altitude of the cylinder is H. Find its volume.

4.463. A cylinder is inscribed in a rectangular parallelepiped whose diagonal makes angles α and β with the adjacent sides of the base. Find the ratio of volumes of the parallelepiped and cylinder.

4.464. In the base of a cylinder whose axial section is a square, a regular n-gon is inscribed, its side being equal to a. Determine the volume of this cylinder.

§ 137. The Volume of a Pyramid

4.465. Compute the volume of a regular pyramid if it is: (a) triangular; (b) quadrangular; (c) hexagonal and the side of the base is a and the lateral edge is b.

4.466. The side of the base in a regular quadrangular pyramid is equal to a; the dihedral angles at the lateral edges contain 120° each. Find the volume of the pyramid.

4.467. The lateral edge of a regular quadrangular pyramid equals a, the area of the lateral face S, and the dihedral angle at the lateral edge contains 150°. Find the volume of the pyramid.

4.468. The base of a quadrangular pyramid is a rectangle with the diagonal b and an angle of 60° between the diagonals. Each of the lateral edges makes an angle of 45° with the plane of the base. Find the volume of the pyramid.

4.469. The side of the base of a regular triangular pyramid equals 1 cm and the area of the lateral surface 3 cm^2. Find the volume of the pyramid.

4.470. The base of a pyramid is a rhombus whose side is 15 dm. The lateral faces are inclined to the plane of the base at an angle of 45°. The greater diagonal of the base is equal to 24 dm. Find the volume of the pyramid.

4.471. Find the volume of a regular quadrangular pyramid whose lateral edge is l and the plane angle at the vertex is α.

4.472. Find the volume of the total surface area of a regular quadrangular pyramid, the side of whose base has a length a, and the angle of inclination of the lateral face to the plane of the base amounts to 60°.

4.473. The greatest diagonal section of a regular hexagonal pyramid is a right isosceles triangle with a hypotenuse c. Find the volume of the pyramid.

4.474. The base of a pyramid is a right triangle one of whose acute angles is α. Each lateral edge of the pyramid has a length b and makes an angle β with the plane of the base. Find the volume of the pyramid.

4.475. One of the diagonal sections of a regular hexagonal pyramid divides it into two unequal parts. Find the ratio of volumes of these parts.

4.476. The base of a pyramid is a regular triangle with a side a; one of the lateral faces is perpendicular to the plane

of the base, the two other faces making an angle α with the plane of the base. Find the volume and the area of the lateral surface of the pyramid.

4.477. The length of each lateral edge of a pyramid equals 269/32 cm. The base of the pyramid is a triangle whose sides are 13, 14, and 15 cm. Find the volume of the pyramid.

4.478. Two lateral faces of a triangular pyramid are right isosceles triangles whose hypotenuses form an angle α. The length of each hypotenuse is b. Find the volume of the pyramid.

§ 138. The Volume of a Frustum of a Pyramid

4.479. The lateral edge of a frustum of a regular quadrangular pyramid has a length of 3 m; the sides of the bases being 5 and 1 m. Find the volume of the frustum.

4.480. The altitude in a frustum of a triangular pyramid is 10 m; the sides of one base are 27, 29, and 52 m; the perimeter of the other base being equal to 72 m. Determine the volume of the given frustum.

4.481. Find the volume of a frustum of a pyramid if the areas of its bases are equal to 98 and 32 cm^2 and the altitude of the corresponding complete pyramid is 14 cm.

4.482. Compute the volume and the total surface area of a frustum of a regular quadrangular pyramid with the sides of its bases 24 and 12 cm long, its altitude being equal to 8 cm.

4.483. The sides of the bases in a frustum of a regular triangular pyramid are equal to a and b. The dihedral angle at the side of the greater base is α. Find the volume.

4.484. The sides of the bases in a frustum of a regular quadrangular pyramid are a and b $(a > b)$. The acute angle of the lateral face is α. Find the volume of the frustum.

4.485. The sides of the bases in a frustum of a regular n-gonal pyramid are a and b. The lateral edge forms an angle α with the plane of the base. Find the volume.

4.486. Determine the volume of a frustum of a regular quadrangular pyramid if its diagonal is 18 cm and the sides of the bases are 14 and 10 cm.

4.487. The bases of a frustum of a regular pyramid are squares with sides a and b $(a > b)$. The lateral edges are inclined to the plane of the base at an angle of 45°. Find the volume of this frustum.

§ 139. The Volume of a Cone

4.488. The area of the cone base is 9π cm^2, the total surface area 24π cm^2. Find the volume of the cone.

4.489. The ratio of the altitude to the generatrix of a cone is 4 : 5, the volume of the cone 96π cm^3. Find the area of its total surface.

4.490. The generatrix of a cone is l and the circumference of its base circle C. Find the volume of the cone.

4.491. Determine the volume of a cone given the area of its base Q and the area of its lateral surface S.

4.492. The altitude of a cone is 15 m, its volume 320π m^3. Determine the area of its total surface.

4.493. The generatrix of a cone is l. The generatrix makes an angle of 30° with the plane of the base. Determine the volume of the cone.

4.494. The axial section of a cone is a right triangle whose area is equal to 9 m^2. Find the volume of the cone.

4.495. Find the volume of a cone if the area of its lateral surface is equal to 18, and the distance from the centre of the base to the generatrix is 3.

4.496. A cone is inscribed in a regular triangular pyramid. Find the volume of the cone if the edge of the pyramid is l and it is inclined to the plane of the base at an angle of 60°.

4.497. The difference between the directrix and altitude of a cone is equal to 2, the cosine of the angle between them being 0.75. Find the volume of the cone.

§ 140. The Volume of a Frustum of a Cone

4.498. The altitude of a frustum of a cone is equal to 3 cm. The radius of one base is twice the radius of the other, the generatrix is inclined to the plane of the base at an angle of 45°. Find the volume of the frustum.

4.499. The area of the axial section of a frustum of a cone is equal to the difference between the areas of the bases whose radii are r and R. Find the volume of the cone.

4.500. The ratio of the radii of the bases and generatrix in a frustum of a cone is 4 : 11 : 25. The volume of the frustum is equal to 181π m^3. Find the radii of the bases and the generatrix.

4.501. The radius of one base in a frustum of a cone is twice that of the other; the lateral surface is equal to the sum of areas of the bases; the area of the axial section equals 36 cm^2. Find the volume of the frustum.

4.502. The volume of a frustum of a cone is equal to 584π cm^3, the radii of the bases 10 and 7 cm. Determine the altitude.

4.503. The radii of the bases in a frustum of a cone are R and r. The generatrix makes an angle α with the plane of the base. Find the volume of the frustum.

§ 141. The Sphere and Its Properties

4.504. A sphere 41 dm of radius is cut by a plane passing at a distance of 9 cm from the centre. Find the area of the section.

4.505. A plane is drawn through the midpoint of a sphere radius perpendicular to it. Find the ratio of the area of the section thus obtained to the area of the diameter section.

4.506. Three points are given on the surface of a sphere. The rectilinear distances between them are 6 and 10 cm. Find the distance from the centre of the sphere to the plane passing through the points if the radius of the sphere is 13 cm.

4.507. The radii of two spheres are equal to 25 and 29 cm, their centre distance 36 cm. Find the length of the intersection line of their surfaces.

4.508. The sides of a triangle are equal to 13, 14, and 15 cm. Find the distance from the plane of the triangle to the centre of the 5 cm radius sphere touching the sides of the triangle.

4.509. The diagonals of a rhombus are 15 and 20 cm. A sphere of 10 cm radius touches all the sides of the rhombus. Find the distance from the centre of the sphere to the plane of the rhombus.

4.510. Three equal chords are drawn at an angle α to one another from a point on a sphere surface of radius R. Find their lengths.

§ 142. The Surface Area and Volume of a Sphere

4.511. Given a hemisphere of radius R. Find the area of its total surface.

4.512. Find the surface area of a sphere of 5 cm radius.

4.513. The surface area of a sphere is 225π cm^2. Find its volume.

4.514. Given the volume V of a sphere, find its surface area.

4.515. The hypotenuse and legs of a right triangle serve as diameters of three spheres. Find the relationship between the areas of their surfaces.

4.516. Two parallel sections are drawn in a sphere on one side from its centre whose areas are equal to 49π dm^2 and 4π m^2, the distance between them 9 dm. Find the surface area of the sphere.

§ 143. Solids of Revolution

4.517. The area of a rhombus is Q. The rhombus rotates about its side. Find the surface area of the solid of revolution thus obtained.

4.518. A right triangle with legs 15 and 20 cm long rotates about the perpendicular to its hypotenuse drawn through the vertex of the greater acute angle. Find the surface area and volume of the solid of revolution.

4.519. An isosceles triangle with the base a and the vertex angle α rotates about its lateral side. Find the surface area and volume of the solid of revolution.

4.520. Determine the volume and surface area of the solid generated by rotating a rectangle about an axis passing through its vertex perpendicular to the diagonal of length d, which makes an angle α with the side.

4.521. The perimeter of a right triangle is $2p$. One of the acute angles is α. Find the volume of the solid generated by rotating the triangle about the hypotenuse.

4.522. A square rotates about an axis which passes externally through its vertex perpendicular to its diagonal. Find the surface area of the solid obtained if the length of the side of the square is equal to a.

Answers

Chapter 1.
Arithmetic

1.33. (b) (2) 18/25; (3) $2\frac{37}{12}$; (5) $1\frac{1}{55}$. **1.35.** (a) (9) 1/6; (10) 3/4; (b) (6) $7\frac{17}{40}$. **1.36.** (6) $2\frac{7}{8}$ of $2\frac{19}{23}$ is greater than $4\frac{2}{7}$ of $1\frac{19}{30}$. **1.38.** (a) (3) 121; (6) 115; (b) (1) 10/21; (4) 8/11; (5) 6. **1.39.** (1) 16/35; (2) 132; (3) 9.5; (4) 21.4; (5) 266.63; (6) 58.5; (7) 152/261; (8) $5\frac{13}{99}$; (9) 7/40; (10) 37/50; (11) $30\frac{29}{126}$; (12) $15\frac{74}{105}$; (13) 181; (14) 3. (15) 5. **1.40.** (7) $27\frac{5}{6}$; (8) 5. **1.46.** (7) 0; (8) 9.55. **1.48.** (2) 13; (3) 5; (4) 2.936; (6) 18.46. **1.50.** (1) 200; (2) 40.5; (3) 0.25; (4) 8; (5) 2; (6) $8\frac{1}{3}$. **1.51.** (4) 1; (5) 0.2; (6) 0.6. **1.52.** (1) 10; (2) 4; (4) 0.6; (5) 0.8; (7) 13.2; (8) 6.4. **1.61.** (1) 0.82; (2) 23/24; (3) 40/41; (4) 5/6; (5) 11; (6) 1; (7) 1/48; (8) 7.2. **1.63.** (3) $5\frac{1}{6}$; (4) 17.6; (6) 0.1; (7) 4. **1.65.** (2) 7 : 5; (5) 10 : 3; (7) 4 : 1. **1.67.** (a) (3) $\frac{5}{2} : \frac{3}{20} = \frac{3}{10} : \frac{9}{500}$; (6) $\frac{5}{2} : \frac{4}{15} = \frac{45}{16} : \frac{9}{10}$; (b) (2) $16 : 4 = 44 : 11$. **1.68.** (9) 125/1264; (10) $4\frac{7}{27}$; (11) 3.2. **1.69.** (a) (1) 54 and 81; (2) 63 and 72; (b) (1) 1920 and 480; (2) 2100 and 300. **1.70.** (a) 4.5, 3, 1.8, and 9; (3) 6, 10, 0.9, and 1.4. **1.71.** 36, 48, and 60. **1.72.** (b) (3) 72; (6) 0.0855. **1.72.** (3) 30; (5) 50. **1.77.** (1) 25%; (2) 70%; (3) $291\frac{2}{3}$%; (4) 610%; (5) 125%; (6) 16%.

Chapter 2.
Algebra. Elementary Functions and Fundamentals of Analysis

2.9. (a) (1) −0.05; (2) −1.8; (3) 1; (4) −0.01; (6) 1/15. **2.17.** (1) −45; (2) −8.24; (4) 1.2; (5) 0.6. **2.19.** (6) 3; (8) 3; (10) 0.2. **2.22.** (1) 15; (4) 3; (5) 0.2; (6) 1; (7) −1; (8) 3. **2.31.** (c) (1) −35; (2) −28; (3) 0.5; (4) −7; (5) −6.4. **2.33.** (12) $-8a^2b^2 + 8ab - 9ab^2$; (13) $10a^2bc + 10abc^2$; (14) $-2\frac{2}{3}ab^3 + a^3b - 5a^2b$. **2.34.** (8) 0; (9) $3a^2$; (10) $3a^3$; (11) $2a^2$. **2.35.** (a) (1) 9; (3) 3; (4) 0.5; (5) −10; (6) −9; (7) 7. **2.36.** (b) (6) $-3y^2$; (8) $-mn$; (9) x^2y; (10) a^3; (11) $-a^3$. **2.40.** (1) 3.5; (3) (6); (4) 82; (5) 148; (6) 0; (7) −1/27. **2.57.** (1) 4/7; (2) 1. **2.73.** (1) 0.0756; (2) 1.2; (3) 1; (4) 32.4. **2.77.** (1) 500; (2) 200; (3) 7000; (4) 360; (5) 123; (6) 110; (7) 678. **2.84.** (b) (5) 1; (6) 5. **2.94.** (a) (6) 4/13; (7) 5/19; (b) (1) 10; (2) 94.5; (3) 3/64; (4) 810; (5) 1/648; (6) $(c - a)/(a + b + c)$;

(7) $(a+b+5)/(a+5)$; (8) $(b-c-9)/(b+c)$; (c) (1) $(a+5b-c)/(a-c)$; (2) $(6x+z)/(6x-y+z)$; (3) $(ab+6)/(ab-6)$. **2.96.** (b) (1) $(4+c)/(4c-c^2)$; (2) $-2/x$; (3) $(1-5x)/6$; $(10c+15)/(12c-18)$; (5) $(a-2x)/(2ax(a+2x))$; (6) $1/5z$; (7) $(x-2)/(2x(x-3y))$. **2.97.** (4) $(a(a-3))/(n(a-n-3))$; (5) $((t+1)(2p+3))/(2p-3)$; (6) $(3m-1)(4m-3k)/(3k+4m)$. **2.98.** (9) $(4(4-a-x))/(a(a-x)$; (10) $(n(t-n+5)/(t-n)^2$; (11) $(2x+1)(5a-1)/(5a+1)$; (12) $(5y-3z)(5y+2)/(3z-5y)$. **2.99.** (1) $75k^{16}c/8$; (2) $-400a/9$; (3) $-240a$; (4) $-2.7p$; (5) $a/25b^3$; (6) $-x/(9y)$; (7) $-b/(20a)$; (8) 1; (9) $(a^{2n}+b^{2n})/(a^n+b^n)^2$; (10) $(a^{2n}+b^{2n})^2$. **2.100.** (1) $c-1$; (2) $-(y+4)$; (3) $3p+q+2$; (4) $a-2c+1$; (6) $(3y+9)/y^2$. **2.101.** (1) $-1/(x+y-5)$; (2) $(2a+1)/(2a-1)^2$; (3) $1/a$; (4) $a(a+8)/(a+2)$; (5) 1; (6) $1/a$; (7) $1/(2m-1)^2$; (8) 1; (9) $(x^2+3x+9)/(x-a)$; (10) $-1/(a+c)$. **2.110.** (1) 7; (2) 10; (3) 7; (4) -9; (5) 13; (6) 17; (7) -4; (8) 5; (9) -4; (10) 5; (11) 4; (12) 1/6. **2.111.** (2) -4.5; (3) 15; (6) -2; (7) no roots; (8) 21; (9) no roots. **2.112.** (8) a; (9) $1+a^2$; (10) a^2+1; (11) $3(n+1)/4$. **2.121.** (4) 1; (6) 18; (7) 5; (8) 9; (14) 9; (15) 0; (16) 0. **2.124.** (1) 2; (2) $-1/7$. **2.127.** 1/6. **2.132.** (a) (1) $\{4, 3\}$; (2) $\{11, 55\}$; (3) $\{42, 70\}$; (4) $\{6, 21\}$; (5) $\{a+b, b\}$; (b) (2) $\{4, 2\}$; (3) $\{4, 4\}$. **2.133.** (a) (1) $\{8.5, 4.5\}$; (2) $\{3, 2\}$; (4) $\{1/2, 1/3\}$; (5) $\{3, -2\}$; (6) $\{1, 2/7\}$; (b) (1) $\left\{\frac{a+b}{2}, \frac{a-b}{2}\right\}$; (2) $\left\{\frac{3a+8}{a^2+2}, \frac{3-4a}{a^2+2}\right\}$; (3) $\left\{\frac{7}{a}, \frac{9}{b}\right\}$. **2.134.** (1) $\{1, 0\}$; (2) $\{8, 4\}$; (3) $\{5, 3\}$; (4) $\{1, 1\}$; (5) $\{2, 1\}$; (6) $\{2, 1\}$. **2.135.** (1) $\{0.5, 0.5\}$; (2) $\{3, 2\}$; (3) $\{2, 5\}$; (4) $\{-2/3, 0\}$; (b) (1) $\{6, -1, 1\}$; (3) $\{1, -1, -3\}$. **2.138.** (1) $\{-3, 2, -1\}$; (2) $\{2, 3, 1\}$; (4) $\{(-7c-20, 4c+10, c)/c \in \mathbf{R}\}$; (5) no solution; (6) no solution; (7) $(1, -1, 0, 2)$; (8) $\{1, 2, 3, 4\}$. **2.141.** (2) $a-5$; (7) $(m-n)$ for $n>m$; $(n-m)$ for $n<m$. **2.162.** (7) 1; (8) $9\sqrt{2x}$. **2.174.** (a) (2) 6; (3) $4-3\sqrt{7}+11$; (b) (3) $\sqrt{a(1-a)}$; (4) $2b(a-b)$; (c) (1) $2a(y\sqrt{2+a})$; (2) 3; (3) $2\sqrt[4]{ax}$; (4) $\sqrt{x}+\sqrt{y}$; (d) (2) 1; (3) $x+y$; (4) $0.5(\sqrt{2+a}+\sqrt{2-a})$; (5) 2. **2.175.** (a) (1) -6 for $x>0$; $(6-2x)$ for $0<x<6$; 6 for $x<0$; (2) 8 if $x>9$; $2x-10$ for $3<x<9$; -4 for $x<3$; (3) $\sqrt{1-m}-\sqrt{m}$ for $m \in [0; 1/2]$; $\sqrt{m}-\sqrt{1-m}$ for $m \in [1/2, 1]$; (5) $2\sqrt{x-1}$ for $1<x<2$; 2 for $x>2$; (6) $\sqrt{5-2}$; (8) $2\sqrt{2-\sqrt{3}}$; (b) (1) $1/(x-x^2)$ for $x \in (0, 1)$; $1/(x^2-x)$ for $x \in (1, +\infty)$; (2) -2 for $x \in (-\infty, 0)$; -2 for $x \in (0, +\infty)$; (3) $(x+1)/(1-x)$ for $x \in (-\infty, -1)$; $(x+1)/(x-1)$ for $x \in [-1, 0)$; $(x-1)/(x+1)$ for $x \in [0, +\infty)$. **2.182.** (1) $40az^2$. **2.183.** (a) (1) $\sqrt{3-\sqrt{3}}$; (2) $3\sqrt[3]{ab}$; (3) $\sqrt{a}-\sqrt{b}$; (4) $-2\sqrt[4]{a}$; (b) (2) 0; (3) $(1-\sqrt{x})/(1-x)$; (4) y^2; (c) (2) $\sqrt[3]{a^2}$; (4) 10. **2.191.** (c) (1) 0.2; 5; (2) 18/7, 6; (3) $-28/5$, 4; (4) 5.2; 10; (5) 8, 25, 12; (6) 9, 2, 14. **2.192.** (2) $-4/3$; (3) $-1/3$; (4) 2.2; (5) $-23/3$; (6) 1/6. **2.193.** (1) $3, 3 \pm 2\sqrt{5}$; (2) $-4, 2$; (4) 1, 3; (5) $-1, 1$; (6) 0; (7) -5; $-1-\sqrt{6}$, 1, $1+\sqrt{6}$; (8) -3, 1. **2.195.** (1) $a, 1/a$; (5) $-a, b+3$; (6) $a/3b, a/b$. **2.194.** (b) (1) $a+b, a-2b$; (2) $3a-2b, 2b-3a$; (4) 1. **2.196.** (a) (5) $12x^2-25x+12=0$; (7) $9x^2-32x+144=0$; (8) $x^2-20x+86=0$; (b) $9x^2-96x+130=0$. **2.197.** (a) (8) $(a^2-b^2)x^2-2(a^2+b^2)x+a^2-b^2=0$; (b) (5) $bx^2-2a\sqrt{ax}+a^2=0$. **2.210.** (a) (1) 81; (b) (1) 100/243. **2.204.** (1) -54;

(5) -3. **2.205.** (a) (3) -2; 18; (b) (3) ± 6. **2.206.** (a) (1) $\pm 35/8$; (b) (4) 24. **2.208.** (1) 3; (5) 1. **2.209.** (a) -16; (b) (1) -6.75; (c) (5, 7); $(-5, -7)$. **2.213.** (a) (1) No; (3) yes; (b) (2) yes. **2.217.** (4) 1; (7) 0.2; (11) ± 2. **2.219.** (a) (1) ± 1; ± 3; (3) ± 2; ± 5; (7) ± 0.5, ± 1; (10) $\pm\sqrt{2}$, $\pm\sqrt{3}/3$; (b) (1) $\pm a/b$, $\pm b/a$; (2) ± 5, $\pm m$. **2.221.** (1) $(x^2-1)/(x^2-4)$; (3) $(x^2-5)/(x^2-6)$; (7) $(x^2-a)/(x^2-1)$; (8) $(x^2-m^2)/(x^2-c^2)$; (9) $(9x^2-1)/(4x^2-1)$; (10) $(25x^2-a^2)/(4x^2-a^2)$. **2.222.** (3) $q = 9, \pm 1, \pm 3$. **2.223.** (a) (2) $x^4 - 37x^2 + 36 = 0$; (7) $x^4 - 2(a+b) \times x^2 + 4ab = 0$; (b) (5) $x^4 - 30x^2 + 216 = 0$; (9) $a^2b^2x^4 - (a^4+b^4) \times x^2 + a^2b^2 = 0$. **2.226.** (2) 20; (5) 4; (9) 10; (14) 2; (17) 4; (21) 64; (23) ± 1; (24) -1; (25) 2; (30) ± 1. **2.227.** (1) $2\sqrt{2}$; (2) 1024; (3) 4; (5) $\pm 2\sqrt{2}$; (6) 32; (7) 0.4; (8) 64; (9) 64; (13) $-3\sqrt[3]{9}$, 1, $2\sqrt[3]{4}$; (15) 2, $-1/511$; (18) 1; (19) $-(b+1)$; (22) 1, $-1/3$; (23) -5, 2; (28) 8; (29) -3.4. **2.228.** (1) 9; (3) 7.26; (10) -2. **2.231.** (a) (1) $(-22/3, -14/3)$; (6, 2); (2) $(-5, -4)$; (4, 5); (3) (3, 19); (3, 1); (5) $\{(192, -58); (6, 4)\}$; (b) (1) $\{(0, 1); (-11, -10)\}$; (c) (3) $\left\{\left(-\frac{a}{a+1}, -\frac{1}{a-1}\right); \left(\frac{a}{a-1}, \frac{1}{a+1}\right)\right\}$. **2.232.** (b) (1) $\{(4, 12); (12,4)\}$; (5) $\{(-a, 2a); (2a, -a)\}$. **2.233.** (a) (1) $\{(\pm 0.5, \pm 0.5); (\pm 2, \pm 1)\}$; (2) $\{(\pm 2, \pm 8); (\pm 8.5, \pm 5)\}$; (4) $\{(\pm 3, \pm 5); (\pm 1\frac{2}{3}, \pm 4\frac{1}{3})\}$; (b) (3) $\{(\pm 2, \pm 3); (\pm 3, \pm 2)\}$. **2.234.** (b) (1) (8, 2); (2, 8); (3) (3, 3/2); (6, 3); (c) (1) (5, 1); (3) (5, 4); $(-9, 25)$; (4) $(0, a)$; $(2-a, 2)$ for $a \neq 0$; (2, 2) for $a = 0$. **2.235.** (c) (3) $(0.2, -0.6)$; (1, 3); (4) $(\pm 1, \pm 2, \pm 3)$. **2.246.** (b) (1) (5, 0); (7) $\left(-2\frac{11}{40}, 2.05\right)$. **2.263.** (3) 4. **2.264.** (3) $-5/4$. **2.308.** 2, -16, 24. **2.309.** -4, 4, 24. **2.310.** 3, -36, 96. **2.311.** 2, 8, 15. **2.316.** (1) $(-2, 1)$, $(1, 2)$. **2.323.** (9) $(56, \infty)$; (11) $(-\infty, 6.5)$. **2.326.** (a) (1) (5, 27); (3) $(0, 16^2/_3)$; (5) (1, 4); (8) no solution; (10) no solution. **2.327.** (b) (1) $\{-2, -1, 0\}$; (4) $\{1, 2\}$; (6) 1; (7) $\{-24, -23, -22\}$; (10) 3. **2.328.** 6. **2.329.** 10. **2.331.** -9. **2.332.** (b) (7) $(-\infty, 4/3)$. **2.341.** (4) $k < 0$, $k > 1$; (5) $k < 3$, $k > 5$. **2.345.** (2) $(10^2/_3, 10)$; (3) (28, 3). **2.353.** (a) (1) $(-\infty, 5) \cup (0, \infty)$; (2) (0.4, 1); (6) R; (b) (1) $(1, 2) \cup (3, \infty)$; (3) $(-\infty, 1)$; (6) $(-4.5, -2) \cup (3, +\infty)$; (c) (2) $(-\infty, -3) \cup (1, \infty)$; (3) $(-4, 6)$; (4) $(-2, -1) \cup (3, 6)$; (5) $(-2, -1) \cup (-1, 2)$; (d) (1) $(-\infty, -1) \cup (0, 2)$; (4) $(-\infty, 1) \cup (4/3, 2)$; (7) $(-\infty, -5) \cup (-3, -1) \cup (1, 2)$; (e) (1) $(-\infty, 1) \cup (2, \infty)$; (2) $(-1, 1)$; (3) $(-1, 1)$; (4) $(-8, 1)$; (f) (1) $(-1, 2) \cup (3, 6)$; (2) (2, 4); (3) [0, 1.6]; (4) $(-\infty, -5/6) \cup (3, \infty)$. **2.354.** (1) $m < 1$; (3) $m < 2\frac{1}{24}$; (4) (1, 6); (7) (0, 28); (8) $(-\infty, -6.25)$. **2.355.** (a) (1) $(-\infty, 8/7)$; (2) $(-\infty, -0.25)$; (3) $(0.2, \infty)$; (b) (1) $(-0.4, 6)$; (3) $(-13/12, \infty)$; (c) (1) R; (3) $(-3/11, 3)$. **2.356.** (1) (1.75, 4); (2) $(-1, \infty)$; (4) $(-\infty, -5/6) \cup (3, \infty)$; (5) $(20/9, 4) \cup (5, \infty)$; (10) $(-3, 1)$; (11) $(-\infty, 2\sqrt{5} - 4)$; (12) $\left(\frac{\sqrt{34}-1}{2}, \infty\right)$; (13) $(1, 2\sqrt{3})$; (14) $(-\infty, 2) \cup \left(5, 5\frac{9}{13}\right)$; (18) [2, 8). **2.362.** (a) (1) [1.75; 4); (2) [5,

7] $\cup$ {4}; (3) $(-\infty, -1) \cup (2, 3]$; (b) (3) (3.75, 4); (4) 5. **2.407.** $\mathbf{Z} \cap \mathbf{N} = \mathbf{N}$. **2.410.** (1) [6, 12]. **2.412.** (b) (1) $(0, \infty)$. **2.443.** (1) $2^{300} > 3^{200}$. **2.445.** (1) $x > y$; (3) $x < y$; (6) $x < y$. **2.446.** (2) $x \geqslant 2$; (9) $0 < x < 1$. **2.449.** (2) $3\sqrt{3}$. **2.454.** (9) $(-\pi/2 + 2\pi n, \pi/2 + 2\pi n) \cup (3\pi/2 + 2k\pi, 2\pi(k+1))$. **2.455.** (8) $x > 1$; (11) [2, 3) $\cup$ (3, 4). **2.456.** (1) $x < y$; (5) $x > y$. **2.457.** $\log_8 9 > \log_9 8$. **2.458.** (6) 0.1; (12) 8. **2.459.** (1) 10; (2) 890. **2.464.** (2) $1/b$; (4) $3(1-a)(1+b)$; (5) $a(3+b)$. **2.465.** (1) 3.5; (2) 4.5; (3) 1; (4) 5; (5) 2.5; (6) 2.5; (7) 10; (8) 11; (9) 4; (10) $2\sqrt[4]{3}$. **2.475.** (a) (3) 180°; (b) (1) 0.4; (c) (3) 50. **2.477.** (c) (1) [−0.75, 0.25]; (3) [−0.21, 0.79). **2.482.** (1) 1; (7) 1. **2.484.** (1) 30.89; (2) 4.972; (3) 148900; (4) 87.7; (6) 4.821; (7) 2.242; (8) 186.5; (12) −62.34; (15) 0.6998. **2.485.** (5) −5; (6) −1.5; (8) −2/3; (9) 3; (12) 1.5; (13) 35; (15) 3; (16) 3; (17) 5/3; (18) 81; (19) −0.2; 3; (20) 25. **2.486.** (3) 2; 3; (9) 1. **2.488.** (1) 3; (7) 3; (10) 2; (11) $\log_{3/5}(31/13)$; (13) $\pm\sqrt{3}$; (15) −0.5. **2.489.** (1) 3; (4) 3; (5) 1; (7) 0.25; (8) 2; 10; (10) 3; (11) ± 1; (13) $\pm\sqrt{2}$; (14) 1; (16) 1; $\log_3(3 + \sqrt{29}) - 1$; (17) −1/2; 3/2; (18) −2; (20) 1; 3. **2.490.** (1) 0; (5) −2; (6) 0; 1/2. **2.491.** (1) 0; 1; 2; (4) 3; (5) 1/4; (8) ± 5; (9) ± 2; (10) 9; (11) 7. **2.493.** (1) 1, 2; (2) 2, 15; (3) (3, −2), $(\log_2(1/9), \log_2 8)$; (7) 3/14, 1/14; (8) (3, 3), (1, 5). **2.494.** (1) $\sqrt{3}$; (7) 3, 10; (9) 2; (11) $\pm\sqrt{3}$. **2.495.** (7) 100, 1000; (9) 3; (12) 2; (14) 0.99; (16) 1.1, 1.01; (17) ± 3; (21) 15; (22) 13; (23) 7, 15; (25) 3, 5. **2.496.** (a) (1) 10^{10}; (2) 64; (3) 169; (4) 81; (5) 49; (6) 31; (7) 1.5, 2.5; (8) 9; (b) (1) 0.1; 10; (3) 100; (4) 30, 100; (5) 0.0001, 10; (6) $10^{\pm\sqrt{7}}$; (7) 1/32, 2; (8) 0.01, 100; (9) −1; (10) 0.01, 0.1, 10, 100; (11) 1/9, 9; (12) $1/a$, a^2; (13) 2, 64; (14) 10, 15; (15) 0.1, 1000; (17) 0.1, $10^{\frac{1\pm\sqrt{3}}{2}}$; (c) (1) 1; (2) 1; (3) 0.5; (4) 9; (5) 36; (6) 2; (7) 0, 3; (8) 1; (9) 1; (10) 2. **2.497.** (a) (1) 16; (2) 64; (3) $\sqrt[10]{10}$; (4) $\sqrt{3}$; (5) 16; (6) 9; (b) (1) 1/3, 3; (2) 1/9, 9; (3) 0.1, 10; (4) 1/6, 6; (c) (1) −64; (2) −100; (3) −0.5; (4) 1000. **2.498.** (1) $\{0.04; 5\sqrt[3]{5}\}$; (2) {2, 16}; (3) {1/27, 9}; (4) {1/9, 9}; (5) $\{3, \sqrt{3}\}$; (6) {1/12}; (7) $\{0.5\sqrt[3]{4}, 4\}$; (8) {1/625, 5}; (9) $\sqrt[3]{2/2}$; (10) 2. **2.499.** (1) 100; (2) 5; (3) {−1.7}; (4) $\sqrt{a}$, a^9; (6) {2/3, 3}; (7) a^6; (8) {5, 15}; (9) $\{8, 1/\sqrt[3]{4}\}$; (10) 6; (11) 3; (12) {−5, 5}; (13) 17/12; (14) {0, 4}; (15) 3; (16) 100; (17) 18; (18) 32; (19) 9; (20) {2, 11}; (22) {4, 36}; (23) {1/9, 9}; (24) {2, 6}; (25) 12. **2.500.** (2) (10, 10 000); (3) {(10, 4), (4, 10)}; (4) {(10, 100), (100, 10)}; (5) (11, 9); (6) {(16, 25), (25, 16)}; (7) (5, 5); (8) {(1, 2), (16, −28)}; (9) {(9, 3), (3, 9)}; (10) (100, 10). **2.501.** (1) $(a^2/b, b^2/a)$; (2) (81, 4); (4) {(1/2, 8), (8, 1/2)}; (5) (5.0005, 4.995); (6) {(5, 1), (5, −1)}; (7) $(2^{\sqrt[3]{4}}, 4^{\sqrt[3]{2}})$; (9) {(27, 4), (1/81, −3)}; (10) {(2, 18), (18, 2)}. **2.502.** (1) (8, 6); (3) (8, 4); (4) (5, 2); (5) {(4, 2), (1, 1)}; (7) $(ab^2, a/b^2)$; (8) $((-1+\sqrt{5})/2, (3-\sqrt{5})/2)$; (9) {(16, 3), (1/64, −2)}; (10) {(6, 8), (8, 6)}. **2.503.** (a) (4) $(-\infty, 1) \cup (3, \infty)$; (5) $[(3-\sqrt{5})/2, (3+\sqrt{5})/2]$, (9) (1, 4); (13) (0, 1); (b) (3) (−1, 1); (5) $(-\infty, 0.5) \cup (1, \infty)$. **2.504.** (a) (2) $(-\infty, -2)$; (6) $(-\infty, 1)$; (7) $(-\infty, 0) \cup (0.5, \infty)$; (8) (0, 1); (9) $(0, \infty)$; (10) $(2, \infty)$; (16) $(\log_5 11, \infty)$. **2.505.** (1) $(-\infty, -1) \cup (0, 1) \cup (1, \infty)$; (4) $(-\infty, 3)$; (6) $(-\infty, -1) \cup (0, 2)$; (7) (3, 3.5) $\cup$ (4, ∞). **2.506.** (b) (10) (−1, 1) $\cup$

(3, 5). **2.507.** (a) (1) $(-\infty, -1) \cup (1, \infty)$; (2) $(-3, -1/3) \cup (1.5, \infty)$; (5) (1/3, 3/4); (7) (5, 10); (8) (1, 3); (b) (1) $(1, \infty)$; (7) $[-3, 1]$; (8) $(-\infty, -4)$; (9) $(2, \infty)$; (10) $(-\infty, -0.5)$; (12) $(0, 0.75) \cup (1.25, 2)$. **2.512.** (a) (1) Increasing. **2.513.** Yes. **2.516.** (a) $3n - 2$. **2.517.** (b) (3) Unbounded; (6) Bounded. **2.518.** (b) For $a = 2b$ the sequence is constant; for $a > 2b$ and $b \notin [-1, 0)$ the sequence is increasing; for $a < 2b$ and $b \notin [-1, 0)$ the sequence is decreasing. **2.527.** (1) 11, 15, 19, 23, 27, 31; (2) 5, 9, 13, 17; 21. **2.529.** (3) 1, 3; (4) 6, 4; (6) 1/3, 1/4; (8) $-4/3$, 1/3 or 4/3, $-1/3$. **2.530.** (1) (2, -3); $(-10, 3)$; (3) $(-17, 2)$; (3, 2); (4) (0, 3); $(-12, 4.2)$; (6) 8, -3; (8) -8, 3. **2.531.** (a) (1) 1275; (2) 5050; (3) 7260; (b) 2550; (c) 1715; (d) 70 336; (e) 82 350. **2.532.** (1) 4, 1430; (4) $a + b$; $45a + 36b$. **2.536.** (1) 5, 15, 25; (2) 1, 9, 17; (4) 8, 18, 28. **2.537.** (1) 25; (2) 55; (3) 1; (5) 6; (6) 7. **2.538.** 1, 2, 3, 4; . . . **2.539.** (3) 44. **2.540.** (1) 1/3, 2/3, 1 or 1, 2/3, 1/3; (2) $n_1 = 9$; $n_2 = 31$. **2.541.** (3) $\sqrt{3}$; (4) $\sqrt[10]{10}$; (5) $\sqrt{3}$; (7) $\sqrt[a]{a}$. **2.548.** (1) $\pm b_3/b_1$. **2.550.** (1) $\frac{1}{4}$; $1365\frac{1}{4}$; (2) 3584; 7161. **2.552.** (1) 27, 81; (2) 80, 40, 20, 10; (3) $\sqrt[7]{7}$, $\sqrt[7]{49}$, $\sqrt[7]{343}$, $\sqrt[7]{2401}$, $\sqrt[7]{16\,807}$, $\sqrt[7]{117\,649}$. **2.554.** (2) 1, 2; (3) 1, 2; (4) 1, 3. **2.556.** (1) 7; -28; 112; -448; (2) $-\frac{35}{3}$; $-\frac{140}{3}$; $-\frac{560}{3}$, $-\frac{2240}{3}$. **2.557.** (1) -56, 28, -14, 7, (2) 8, 12, 27; (3) 24, 12; 6, 3; (4) 3, 6, 12, 24. **2.558.** (1) 5, 405; (2) 2, -6, 18, -54. **2.559.** (1) 48, 0.25 or 3, 4; (2) 6, 24, 96. **2.561.** (5) $(5 + \sqrt{3})/2$; (7) 45/64; (8) $4 + 3\sqrt{2}$. **2.562.** 2, 1/2; . . . **2.563.** 6, -0.5. **2.564.** 0.2. **2.565.** 6, 3, 3/2. **2.566.** (1) 18; (2) 243 or 486; (3) 9; (4) 8.4. **2.567.** (a) 15; (b) (1) $\frac{a}{(a-1)^2}\ (na^{n+1} - a^n(n+1) + 1)$. **2.568.** (1) {1/3, 2/3}; (2) {3, 4} or {48, 1/4}; (3) $\{a^2, 1/a^2\}$. **2.570.** 5103 or 7/81. **2.571.** 4, 8, 16 or 4/25, $-16/25$, 64/25. **2.572.** 3, 6, 12, 18 or $18\frac{3}{4}$, $11\frac{1}{4}$, $6\frac{3}{4}$, $2\frac{1}{4}$. **2.575.** (1) $n + 1 - 1/2^{n-1}$; (3) $(7/81)(10^{n+1} - 10 - 9n)$; (6) 4. **2.579.** (a) (2) ∞; (4) 0; (c) (1) 1; (4) 0.6; (d) (3) a/c; (e) (2) 0; (3) -2.5; (f) (1) 0; (2) -0.5; (3) 1. **2.580.** (1) 3/4; (2) 1/2; (4) 1/2. **2.582.** (8) 0.25; (9) 0.25; (10) 2/3; (11) 0; (12) 0.25; (13) 2; (20) ∞. **2.583.** (8) 0.25; (12) 0.25; (14) $1/\sqrt{a}$; (15) 2/3; (17) $-145/42$; (18) -5; (19) 1; (20) 0.25; (21) 4; (22) -1; (23) 1; (24) -0.5. **2.584.** (1) 2; (3) 9; (4) 1; (5) 1; (7) -10; (9) -0.9; (10) 48; (11) $-1/(4\sqrt{2})$; (14) 1; (19) a/b. **2.586.** (3) e^4; (9) $e^{1/4}$. **2.590.** (a) (2) $-2/x^3$; (4) $3/2\sqrt{x}$; (5) $-4/x^2$; (b) (1) $1/3\,(\sqrt{x^2}) + 2/(3x\sqrt{x}) - 6/x^3 + 3/(125x^4)$; (4) $16x^4/3\sqrt[3]{x}$; (c) (5) $3/\sqrt[5]{x^2}$; (d) (4) $5 + 1/x^2$. **2.592.** (1) $-20x^3 - 4x$; (2) $5x^2(3 - 4x)$; (3) $4x^3 + 2x$; (4) $-140x^3 + 45x^2 - 24x + 3$; (5) $2x(1 - 2x^2)$. **2.593.** (a) (1) $-2/(1 + x)^2$; (2) $-(1 + x^2)/(x^2 - 1)^2$; (3) $-15/(2(3x - 2)^2)$; (4) $-2/(5 - 4x)^2$; (5) $(4x - 14x^2)/(6 - x)^2$; (7) $5/(6 - x)^2$; (8) $-70/(3 - 10x)^2$; (b) (1) $(ad - bc)/(cx + d)^2$. **2.594.** (a) (7) $\cos 2x$; (b) (2) $\cos^2 x \cos x - x^3 \sin x$. **2.596.** (a) (1) $x + \ln x$; (2) $(1 - \ln x)/x^2$; (4) $-1/x$; (5) $2a/(a^2 - x^2)$; (6) $2/(x(1 - x^2))$; (7) $\ln \sqrt{x}/x$; (b) (1) $\cot x - (1/2)\sin 2x$; (2) $1/(1 - x)^2$; (3) $\tan^3 x$; (4) $2/(1 - 4x^2)$; (c) (1) $a/x \ln x$; (2) $x^{m-1}(m \ln x +$

1); (e) (1) $(3\cos 3x + 2^x \ln 2)/((\sin 3x + 2^x)\ln 10)$; (2) $(2x\sqrt{x} + 2)/(\sqrt{x}(x^2 + 4\sqrt{x} + 5)\ln 11)$; (3) $3x^2 \ln x + x^2$; (4) $(x\cos x \cdot \ln(7x) - \sin x)/(x \ln^2(7x))$. **2.597.** (f) (2) $-4 \cdot 5^{-4x} \ln 5$; (3) $-6 \cdot 7^{-2x} \ln 7$; (4) $(2^x(\ln 2 \cdot \cos x + \sin x))/\cos^2 x$; (g) (3) $3^x(\ln 3 \cdot \ln(5x) + 1/x)$; (4) $-2 \ln 0.3 \cdot 0.3^{7-2x}$. **2.599.** (a) (1) Increases for $x \in (-\infty, 2)$; decreases for $x \in (2, \infty)$; (2) decreases for $|x| < 1$; increases for $|x| > 1$; (3) increases for $x \in (-1, 0)$; decreases for $x \in (-\infty, -1) \cup (0, 1)$; (4) decreases for $x \in \mathbf{R}$; (5) decreases for $x \in (-1, 0)$ and $(0, 1)$; increases for $|x| > 1$; (6) increases for $x \in (-\infty, 1/3)$ and $(1/3, \infty)$; (7) the function is constant (for $x \neq 1/2$); (b) (1) increases for $-\pi/3 + 2\pi n < x < \pi/3 + 2\pi n$; decreases for $\pi/3 + 2\pi n < x < 5\pi/3 + 2\pi n$; (2) decreases for $x \in \mathbf{R}$; (6) increases on the intervals $[x_0 - \pi + 2\pi k, x_0 + 2\pi k]$, $k \in \mathbf{Z}$; decreases on the intervals $[x_0 + 2\pi k, x_0 + \pi + 2\pi k]$, $k \in \mathbf{Z}$; maxima at points of the form $x_0 + 2\pi k$, $k \in \mathbf{Z}$; minima at points of the form $\pi + x_0 + 2\pi k$, $k \in \mathbf{Z}$; (c) (4) decreases for $x \in (-\infty, -1)$; increases for $x \in [-1, \infty)$, for $x = -1$ the function has a minimum; (5) increases on the half-interval $(0, \infty)$; decreases on the half-interval $[e, \infty)$. **2.600.** (a) (1) $-37/4$ (min); (3) $-a^2/8$ (min); (5) $y_{\min} = 110$, $y_{\max} = 174$; (6) $y_{\min} = -177$, $y_{\max} = 39$; (7) $y_{\min} = -1$; (d) (1) $y_{\min} = -\sqrt{2}$, $y_{\max} = \sqrt{2}$; (2) $y_{\min} = -2$, $y_{\max} = 2$. **2.602.** (1) (0, 5); (2) (0, 3). **2.603.** (1) (1, 4); (5) (0, -1). **2.604.** (a) $y_{\min} = -16$; $y_{\max} = -9$; (b) $y_{\min} = -25$; $y_{\max} = 0$; (4) $y_{\min} = 17$, $y_{\max} = 0$; (5) $y_{\min} = 1$, $y_{\max} = 3$; (6) $y_{\min} = -10/3$, $y_{\max} = -2$; (7) $y_{\min} = 1$, $y_{\max} = 2.125$. **2.606.** (3) $y_{\max} = 3\sqrt{3}/8$, $y_{\min} = 0$; (4) $y_{\min} = 1$, $y_{\max} = \pi/2$; (5) $y_{\max} = 2\sqrt{3}/3$, $y_{\min} = 1$; (6) $y_{\min} = 0.5$; $y_{\max} = 0.75$. **2.614.** $3\pi/4$. **2.615.** $\pi/4$. **2.616.** (1) {1, 4}; (2) 70 m/s; (3) 8 m/s, 10 m/s, 24 m/s, 22 m/s; (4) 14 m/s^2, 18 m/s^2. **2.617.** (1) $7x - x^2 + C$, (2) $3x + 5/6x^2 + C$; (3) $0.5kx^2 + bx + C$; (9) $5x + (1/7)\cos 7x + C$. **2.618.** (10) $e^{3x}/3 + C$. **2.619.** (1) $-(e^{-3x}/3 + 5/3)$; (3) $2e^{x/2} + 1$; (5) $(4 \cdot 7^{x/4} - 195)/\ln 7$; (7) $(3 - \cos 2x)/2$; (8) $-(1/3)(\cot 3x + 2)$. **2.621.** (3) $x^4 + C$; (11) $(1/3)\sin 3x + C$; (14) $x + (1/2)x^2 + (2/3)x^3 + (3/4)x^4 + C$. **2.622.** (3) $(2/3)(5\sqrt{3} - 8)$; (4) 42; (5) 3.75; (6) 2.25; (9) π. **2.627.** (2) 1/3. **2.628.** (1) 9; (4) $5\frac{1}{3}$; (5) $20\frac{5}{6}$; (6) 18; (7) $1\frac{1}{3}$. **2.629.** (4) $2/(2n + 1)$; (5) 18; (6) 2; (8) 4.5. **2.630.** (8) $12 - 5 \ln 5$. **2.631.** (1) $e^3 - 1$; (4) $4 - 3/(2 \ln 2)$; (5) $e^x - 5$; (6) $2/\ln 3 + 0.3/\ln 0.7$; (7) $e - 1 - 2/\pi$. **2.654.** (1) $x^4 + 10x^3 + 35x^2 + 50x + 24$; (5) $y^4 - 2y^3 - 13y^2 + 14y + 24$. **2.656.** (1) $101\,376x^7$; (2) $168, 2x^6$; (3) $C_{16}^8 x^{-4}$. **2.659.** (a) (1) C_{17}^8; (2) C_{15}^6; (3) C_7^4; (b) (1) 120; (2) $C_{12}^6 b^{-3}$; (3) No. **2.661.** $\{10, 1/\sqrt{100\,000}\}$.

Chapter 3.
Trigonometry

3.3. 45°; 60°; 75°. **3.18.** (1) $(0, \pi) \cup (\pi, 3\pi/2)$. **3.32.** (1) $\frac{p^2 - 1}{2}$; (3) $\frac{p(3 - p^2)}{2}$. **3.40.** (1) $y_{\max} = 3$; $y_{\min} = 2$. **3.46.** (b) (1) 0.5 sin 41°;

(c) (5) $17\sqrt{3}/3$. 3.62. (c) (1) 0. 3.83. (a) 43°. 3.96. $0.25\sqrt{2}(\sqrt{3}-1)$. 3.97. (a) 0.5; (f) −0.8. 3.98. (a) −56/65; (b) −33/65. 3.110. 24/25; 7/25; $3\frac{3}{7}$. 3.112. (1) 0.25. 3.116. (a) (1) $y_{max}=2$; $y_{min}=-1$. 3.117. (1) 0.8; −0.6; −4/3. 3.123. (4) 0. 3.125. (a) (1) $2\sin 16°30'; \cos 1°30'$. 3.126. (a) $4\sin(30°+\alpha)-\sin(30°-\alpha)$; (e) (3) $4\cos\alpha\cos(30°+\alpha/2)\cos(30°-\alpha/2)$. 3.127. (a) $\alpha-\beta=2\pi k$, $\alpha+\beta=\pi(2k+1)$, $k\in Z$; (b) $\beta\pm\alpha=2\pi k$, $k\in Z$; (c) $\alpha-\beta=\pi k$, $\alpha\in Z$. 3.128. (a) (1) $\sqrt{3}/2$; (2) $\sin 3\alpha$. 3.133. (d) (1) $\operatorname{arccot}(-4)+k\pi$; (2) $\pm\pi/3+\pi k/3$; $\pm\arctan\left(\frac{\sqrt{3}}{2}/3\right)+\pi k/3$. 3.134. (b) (3) $0.25\pi+\pi k$; $\arctan\frac{3}{4}+\pi k$; (c) (1) $-\pi/4+\pi k$; $\arctan 3+\pi k$; (2) $\pi/9\,(6k\pm 1)$; (d) (1) $0.25\pi\,(4k+1)$ $\arctan 0.5+k\pi$; (2) $\pi/4+k\pi$, $(k\in Z)$. 3.139. (a) (1) $\pm\pi/3+k\pi$, $k\in Z$, (3) $(-1)^{k+1}(\pi/16+\pi k/4)$. 3.142. (a) (1) $\pi n/4$; (8) $2k\pi$, $(\pi+4k\pi)/11$; (b) (3) $\pi(2k+1)/4$; (6) πn, $\pi(6k+1)/48$, $(n,\ k\in Z)$; (8) $\pi(2n+1)/8$, $\pi(3k+1)/3$, $(n,\ k\in Z)$; (9) $-20°+60°\,kn$. 3.143. (c) (1) $\pi/10+k\pi/5$, $(-1)^n(\pi/12+\pi n/3)$; (d) (1) $\pi\,(k+1)-\operatorname{arccot}4$, $\pi/4+\pi n$. 3.144. (d) $\{(\pi/6, 2\pi/3), (5\pi/6, 2\pi/3)\}$; (j) $\pi(6k+1)/6$, $\pi(1-6k)/6$. 3.145. (b) (2) $\pi k+\arctan(0.25+k)$. 3.147. (1) $a=8.49$, $b=3.93$; $\hat{B}=24°\,50'$, $S=16.7$; (3) $b=2.334$, $\hat{A}=50°\,10'$, $\hat{B}=39°\,50'$, $S=3.265$. 3.148. (b) $\hat{A}=\hat{C}=37°\,10'$, $\hat{B}=105°\,40'$, $S=36.9$. 3.154. (1) 765; (4) 17 801. 3.155. (1) 15. 3.156. $a\sin(\beta/2)\tan(45°-\beta/2)\frac{a}{2\cos(\beta/2)}$. 3.157. $b\tan\alpha\cot(\alpha/2)$.

Chapter 4.
Geometry

34.2. 70°. 4.3. 80°, 100°. 4.6. 35°45′, 146°45′. 4.7 (a) $5d/8$, $11d/8$; (b) $8d/15$, $22d/15$. 4.11. $1\frac{5}{8}d$. 4.12. $1\frac{5}{7}d$. 4.18. 36°, 144°. 4.19. 50° or 130°. 4.22. 10 m. 4.25. 10 cm, 10 cm, 1 cm. 4.26. 10 cm. 4.27. 37°16′30″. 4.32. $7d/15$, $16d/15$. 4.36. 30°, 60°, 90°. 4.37. $14d/19$. 4.56. $1\frac{4}{7}d$, $\frac{3}{7}d$, $1\frac{4}{7}d$. 4.57. $19d/22$, $25d/22$. 4.59. 0.6 m, 0.8 m. 4.60. 9 cm, 6 cm. 4.61. 3 cm, 2 cm, 3 cm. 4.62. Yes. 4.65. 25 cm and 10 cm or 18.75 cm and 7.5 cm. 4.68. 5.3 cm and 6.7 cm. 4.69. $2d/3$, $4d/3$. 4.70. 60° and 120°. 4.74. 6 cm, 10 cm. 4.76. $2d/3$, $4d/3$. 4.79. a. 4.82. 60° and 120°. 4.83. 30° and 150°. 4.84. (a) 60°, 120°; (b) $11d/27$, $16d/27$. 4.85. (b) 30° and 60°. 4.86. (a) 36° and 54°; (b) 150° and 30°; (c) 60° and 120°. 4.90. 2 m. 4.92. 4 m. 4.93. 2 cm. 4.95. 8 m. 4.97. 1 cm, 10 cm and $10\sqrt{2}$ cm. 4.107. 13. 4.108. Will increase by $10d$. 4.110. $1\frac{4}{11}d$, $1\frac{10}{11}d$, $\frac{6}{11}d$, $\frac{2}{11}d$. 4.112. (1) 5 cm and 25 cm; (2) 7 cm and 13 cm. 4.116. 120°. 4.117. 2 cm. 4.119. 1 cm. 4.120. 2.2 m. 4.121. 60°. 4.122. 2 cm. 4.124. 16 cm. 4.125. $15d/16$. 4.126. R, 60°. 4.128. 37°30′. 4.132. 144°. 4.135. 60 cm. 4.136. 1 m. 4.137. 2 cm. 4.138. 143°, 37°. 4.139. 3 cm. 4.141. 3/2 cm, 8/3 cm, 25/6 cm. 4.142. $6+4\sqrt{3}$ cm; $12+6\sqrt{3}$ cm. 4.143. 5 cm, 10 cm, 25 cm, 20 cm. 4.144. {60°, 120°},

{90°, 90°}, {108°, 72°}, {120°, 60°}, {135°, 45°}, {144°, 36°}, {150°, 30°}. 4.147. (a) n; (b) a regular polygon with an even number of sides. 4.154. $\frac{a}{\cos(180°/n)}$. 4.156. $48\sqrt{3}$ cm², 128 cm², $96\sqrt{3}$ cm², $128\sqrt{2}$ cm², $96\sqrt{2}$ cm². 4.165. 3 m and 2.4 m. 4.166. 1 m, 1.2 m. 4.168. $AM : AB = 1 : 3$, $MB : AB = 2 : 3$. 4.176. 20 cm. 4.177. 12 cm. 4.178. 13.5 cm, 17 cm, 27 cm. 4.180. $m(p+q)/q$; $m(p+q)/p$; $(p+q)$. 4.181. 15 cm, 15 cm, 10 cm. 4.182. (a) 8 m, 12 m; (b) 10 m. 4.183. 39 cm, 65 cm. 4.184. 8 cm. 4.193. 29 cm, 21 cm. 4.195. 13 cm. 4.196. 9 cm and 6 cm or 12.5 cm and 2.5 cm. 4.197. 12 cm and 18 cm. 4.206. 5. 4.213. Yes. 4.227. 5.2 m. 4.229. $4\sqrt{41}$ cm. 4.230. $a\sqrt{3}/(2+\sqrt{3})$. 4.231. m, $m\sqrt{3}$, $2m$. 4.233. 15 cm. 4.243. 30 cm. 4.244. 12 cm or 16 cm. 4.247. 540 m². 4.249. 288 cm². 4.250. 480 cm². 4.252. 2 dm. 4.253. 112 cm. 4.261. 68 cm. 4.262. 10.2 cm. 2.263. $\pi\sqrt{3}$ cm, $2\pi\sqrt{3}$ cm. 4.264. 4π cm, $4\pi\sqrt{2}$ cm. 4.266. 0.3π cm. 4.267. ≈ 8.0 cm. 4.268. ≈ 157 cm. 4.270. $3l/4$. 4.275. $\pi a^2/4$. 4.278. (1) $0.25R^2(\pi - 2)$; (2) $R^2(2\pi - 3\sqrt{3})/12$. 4.280. $\pi R^2/6$. 4.285. $\approx 0.18a^2$. 4.286. $1.9a^2$. 4.287. 6 cm. 4.288. 3 cm. 4.289. 36 cm or 44 cm. 4.300. (1) (1, 2, −3); (2) (8, 0, 1); (3) (0, 0.5, $-\sqrt{3}$). 4.301. (1) (−6, −2, 4); (2) (2/3, 0, 1); (3) (12, 7, 23). 4.302. (−2, 4, 0). 4.303. $\sqrt{3}$. 4.304. $\sqrt{3}$. 4.309. $63/\sqrt{6441}$. 4.310. (3) 45° or 135°. 4.313. (4) 0; (5) 0.5; (6) $\arccos(1/n)$; (7) $\arccos(4/9)$. 4.314. 45°, 90°, 45°. 4.319. −3. 4.321. 5, 52 m. 4.334. $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$. 4.344. $3x + 5y + 2z - 32 = 0$. 4.345. (a) $(x-2)^2 + (y+3)^2 + (z-3)^2 = 36$; (b) $x^2 + y^2 + z^2 = 49$; (c) $(x-1)^2 + (y-4)^2 + (z+7)^2 = 121$; (d) $(x-6)^2 + (y+8)^2 + (z-3)^2 = 100$. 4.347. 8 cm. 4.348. 3.5 cm. 4.349. 2 cm. 4.353. $5\sqrt{2}$ cm, 45°. 4.354. 211 cm. 4.355. (a) $a\sqrt{2}/2$; (b) $a/2$. 4.356. 45°. 4.357. $a\sqrt{2}$. 4.360. $a\sqrt{6}$. 4.361. $a\sqrt{2}$. 4.362. 13 cm. 4.363. 3.36 cm. 4.367. $\sqrt{6}$ cm. 4.369. 60°. 4.375. (a) 9 cm, 13 cm; (b) 15 cm, $\sqrt{277}$ cm. 4.378. $a\sqrt{6/3}$. 4.380. 288 cm². 4.381. 6 cm and 3 cm or 4 cm and 7 cm. 4.382. 24 cm. 4.383. 5 cm. 4.384. 5 cm, 6 cm. 4.385. 3 cm. 4.386. 12 cm. 4.387. $a/\cos\alpha$. 4.389. (1) $a\sqrt{3}(a + \sqrt{a^2 + 12n^2})$; (2) $a^2 + a\sqrt{a^2 + 4n^2}$. 4.390. $18a^2$. 4.393. $a^2(1 + \sqrt{3})$. 4.395. $3m^2\sqrt{3/2}$. 4.398. $3a^2\sqrt{3}\cos\alpha(\cos\alpha + \sqrt{1 + 3\sin^2\alpha})$. 4.399. $Q\left(1 + \cot\frac{\alpha}{2}\sqrt{3}\right)$. 4.404. 9 cm. 4.407. 576 cm², 3136 cm². 4.408. 6 cm. 4.412. 168 cm². 4.415. 90°. 4.424. R^2. 4.425. $0.5R^2\sqrt{3}$. 4.427. $\arcsin\frac{P-S}{S}$. 4.429. ≈ 1.2. 4.430. $m/\sqrt{H^2 + 2m}$. 4.431. 30 dm². 4.432. 20 cm. 4.433. a, $2a$. 4.434. $\pi(R^2 - r^2)/\cos\alpha$. 4.446. $abS/4(a+b)$. 4.448. $\sqrt{MNQ/2}$. 4.453. $nl^3 \sin\frac{360°}{n}\sin 2\alpha\cos\alpha/16$. 4.454. $0.5Q\sqrt{S\sin 2\alpha}$. 4.455. $9d^2/64$. 4.456. 4. 4.461. $3\pi a^3/4$. 4.462. $\pi a^2 H/4\sin^2\frac{\alpha}{2}$. 4.465. (a) $a^2\sqrt{3b^2 - a^2}/12$;

(b) $a^2\sqrt{4b^2 - a^2}/6$; (c) $a^2\sqrt{3\,(b^2 - a^2)}/2$. 4.466. $a^3/6$. 4.468. $b^3\sqrt{3}/24$. 4.471. $4l^3 \sin^2 \frac{\alpha}{2} \sqrt{\cos \alpha}$. 4.472. $3a^2$, $a^3\sqrt{3/6}$. 4.447. 60.375 cm^3. 4.478. $b^3 \sin \frac{\alpha}{2} \sqrt{\cos \alpha}/6$. 4.479. $\frac{360^\circ}{n}$ m^3. 4.480. 229 m^3. 4.483. $(a^3 - b^3) \tan \alpha/24$. 4.484. $(a^3 - b^3)\sqrt{\tan^2 \alpha - 1}/6$. 4.486. 872 cm^3. 4.488. 12π cm^2. 4.489. 96π cm^2. 4.490. $c^2\sqrt{4\pi^2 l^2 - c^2}/(24\pi)^2$. 4.491. $\sqrt{Q\,(S^2 - Q^2/\pi/3)}$. 4.494. 9π m^3. 4.495. 18. 4.497. 56π. 4.498. 63π cm^3. 4.499. $\pi^2\,(R^3 - r^3)/3$. 4.500. 2 m, 5.5 m, 12.5 m. 4.502. 8 cm. 4.504. 1600π cm^2. 4.505. 3 : 4. 4.511. $3\pi R^2$. 4.512. 100π cm^2. 4.513. $\sqrt{36\pi V^2}$. 4.514. The area of the greater surface is equivalent to the sum of areas of the two others. 4.517. $4\pi Q$. 4.522. $4\pi a^2\sqrt{2}$.

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